

การออกแบบจำนวนเทอร์โมอิเล็กทริกและขนาดของแหล่งระบายความร้อนเพื่อให้
ทำงานร่วมกันอย่างเกิดประโยชน์สูงสุด

TO INVESTIGATES THERMOELECTRIC MODULES WITH FINNED HEAT SINK
PERFORMANCE ON OPTIMUM SELECTION DESIGN

ดร. จิรประภา กิมสุนทร
Dr. Jiraprabha Kimsunthorn

คณะวิศวกรรมศาสตร์ มหาวิทยาลัยธรรมศาสตร์
kjiapra@engr.tu.ac.th และ jiraprabha_4@hotmail.com

บทคัดย่อ

งานวิจัยนี้จัดทำขึ้นเพื่อแนะนำการออกแบบจำนวนเทอร์โมอิเล็กทริกที่นำมาใช้งานเพื่อให้ได้ประโยชน์สูงสุด ซึ่งจะกล่าวถึงประสิทธิภาพการทำงานของเทอร์โมอิเล็กทริกและการออกแบบ การออกแบบเพื่อเลือกเทอร์โมอิเล็กทริก ประสิทธิภาพการทำงานของแหล่งระบายความร้อนและการออกแบบเพื่อเลือกแหล่งระบายความร้อนที่เหมาะสม จากผลงานวิจัยจะพบว่าการเพิ่มแรงดันไฟฟ้าและจำนวนเทอร์โมอิเล็กทริกนั้นสามารถที่จะเพิ่มความสามารถในการทำความเย็นได้ แต่จะมีผลต่อค่าประสิทธิภาพไม่มากนัก โดยทั่วไปแล้วเทอร์โมอิเล็กทริกจะทำงานร่วมกับแหล่งระบายความร้อน ดังนั้นขนาดของแหล่งระบายความร้อนจึงมีผลต่อค่าประสิทธิภาพของระบบ ผลจากงานวิจัยพบว่าเมื่อความยาวของครีระบายความร้อนมากกว่า 0.08-0.09 เมตร จะทำให้ความสามารถในการทำความเย็นลดลง อีกทั้งความสามารถในการทำความเย็นจะลดลงเมื่อจำนวนของครีระบายความร้อนเพิ่มขึ้น ดังนั้นจะเห็นได้ว่าการเลือกจำนวนเทอร์โมอิเล็กทริกและขนาดของแหล่งระบายความร้อนให้เหมาะสมนั้นเป็นเรื่องที่สำคัญเพราะจะมีผลต่อค่าประสิทธิภาพ

คำสำคัญ: เทอร์โมอิเล็กทริก ความสามารถในการทำความเย็น

ABSTRACT

This paper theoretically investigates thermoelectric module (TECs) performance on optimum selection design. It will cover the thermoelectric module performance and design model, validation of the optimum

selection model, finned heat sink performance and modelling of a finned heat sink. The results from the design model reveal that an increase in voltage and in the number of TECs can increase the cooling capacity. However, the Coefficient Of Performance (COP) seems stable. TECs always work with finned heat sinks. Therefore, the size of finned heat sink will effect to the COP of the system. After modelling a finned heat sink, it was found that if the fin lengths were above 0.08-0.09 meters, the cooling capacity would decrease. Moreover, cooling capacity would decrease when the number of fins was increased. Therefore, the right number of TECs and right size of finned heat sink are quite important for COP of the system.

Keywords

Thermoelectric module, finned heat sink

INTRODUCTION

Thermoelectric coolers (TECs) are solid devices. They can convert electric energy into a temperature gradient D.M. Rowe (2006) After applying power to a TEC, one side of the TEC will decrease in temperature while the other will increase. TECs have been widely used in many types of applications. For example, for cooling systems, some applications use TECs to produce cooling system. It makes the system environmental friendly. However, TECs still have a high cost even though the COP is not so high. There are some investigations which have showed that the operating COP of TECs was around 0.4. Also, TECs can deliver a cooling capacity of 25 W Goldsmid H. J. (1995) However; the price trend of TECs is expected to be lower [3]. The design model should be able to select the right number of TECs and right size of finned heat sink to reach the optimum COP. Also, the design model should reduce cost.

So before applying TECs to any application, it would be a good idea to do a design model to assess how many pieces of TECs should be applied to each application, and to ascertain what voltage should be applied on

application of TECs in order to get optimum results. The design model can be done by using mathematic equations. Creating a design model helps to reduce cost by avoiding using too many TECs on application. Having a higher number of TECs does not mean that the TECs will give a better COP. Normally, TECs do not work alone. TECs always work with a finned heat sink and a fan. Finned heat sinks and fans transfer temperature from TECs both to the hot side and cold side, and to the surroundings. The size of the finned heat sink is also important. The size of the finned heat sink should be related to the number of TECs. Fins which have a long size do not provide a good heat transfer and COP. Also, thermal grease improves the heat sink's performance by filling the air gap between the finned heat sink and the application.

Performance of thermoelectric module and optimum selection model (design)

Cold side heat transfer: The combination of the Peltier effect, the Joule effect and heat conduction give a cooling effect generated from a thermoelectric device. Its thermal performance can be addressed as follows Webb-R L, et al (1998) and Riffat S.B. and Xiaoli Ma (2003)

The thermoelectric heat pump on the cold side (Peltier effect):

$$Q_{sb} = 2N\alpha IT_c \quad (1)$$

There is Joule heat (Q_J) due to the current passing through the thermoelectric material, 50% of the Joule heat on the hot side and 50% on the cold side:

$$Q_J = 2NI^2\rho/G \quad (2)$$

The geometric factors will affect the amount of Joule Heat as the thicker the object, the lesser the amount of Joule heat there is. Heat is conducted from the hot side to the cold side of the module during operation in the thermoelectric material:

$$Q_{cd} = 2Nk(T_h - T_c)G = 2Nk\Delta T G \quad (3)$$

This equation (4) is a combination of equations (1), (2) and (3) which shows the cold side's energy balance:

$$Q_c = 2N(Q_{sb} - 0.5Q_J - Q_{cd}) = 2N[\alpha I T_c - I^2 p / (2G) - k \Delta T G] \quad (4)$$

Equation (4) shows the thermal module performance standard equation. From this equation, we can conclude that a thermoelectric module is not functional ($Q_c=0$) when the total of half the Joule heat and the conducted heat equals the Peltier heat. Q_c is zero when $(0.5Q_J + Q_{cd} = Q_{sb})$. The module's power consumption (electrical energy) Q_E is:

$$Q_E = IV = 2N(I^2 p / G + \alpha I \Delta T) \quad (5)$$

Thermoelectric module cooling COP:

$$\begin{aligned} \epsilon &= Q_c / Q_E \\ &= [\alpha I T_c - k \Delta T G - I^2 p / (2G)] / [I^2 p / G + \alpha I \Delta T] \end{aligned} \quad (6)$$

There is an optimum current for maximum COP for specific modules and working temperatures.

The equation by $\delta \epsilon / \delta I = 0$:

$$I_{opt} = [k \Delta T G (1 + (1 + Z T_m)^{1/2})] / (\alpha T_m) \quad (7)$$

Replacing I in Equation (6) by I_{opt} :

$$\epsilon_{opt} = (T_m / \Delta T) [((1 + Z T_m)^{1/2} - 1) / ((1 + Z T_m)^{1/2} + 1)] - 1/2 \quad (8)$$

$$T_m = \frac{1}{2}(T_h + T_c) \quad (9)$$

The dimensionless figure of merit, ZT . The value of Z is given by

$$Z = \alpha^2 / (p k) \quad (10)$$

where a , p and k represent the Seebeck coefficient (Volts/K), electrical resistivity (ohm-cm) and thermal conductivity (W/cm-K) respectively.

The material property coefficient of thermoelectric are from Electronics cooling December 2014 electronics-cooling.com and Laird Smart Technology Delivery company.

Material Property Coefficients

$$a = (a_0 + a_1 T_m + a_2 T_m^2) \times 10^{-9} \quad (\text{volts/Kelvin})$$

$$a_0 = 22224.0$$

$$a_1 = 930.6$$

$$a_2 = -0.9905$$

$$\rho = (\rho_0 + \rho_1 T_m + \rho_2 T_m^2) \times 10^{-8} \quad (\text{ohm cm})$$

$$\rho_0 = 5112.0$$

$$\rho_1 = 163.4$$

$$\rho_2 = 0.6279$$

$$k = (k_0 + k_1 T_m + k_2 T_m^2) \times 10^{-6} \quad (\text{watt/(cm Kelvin)})$$

$$k_0 = 62605.0$$

$$k_1 = -277.7$$

$$k_2 = 0.4131$$

Hot side heat transfer: The energy balance on the thermoelectric hot side

$$Q_h = 2N(\alpha I T_h + (I^2 \rho / (2G)) - k \Delta T G) \quad (11)$$

From equations (4) and (5) we get:

$$Q_h = 2N[\alpha I T_c - (I^2 \rho / (2G)) - k \Delta T G] + 2N(I^2 \rho / G + \alpha I \Delta T) \quad (12)$$

$$= 2N[\alpha I T_h + (I^2 \rho / (2G)) - k \Delta T G] \quad (13)$$

This shows that equation (11) is the same as equation (13)

Thermoelectric module for heating COP:

$$\begin{aligned} \eta &= Q_h / Q_E \\ &= (Q_c + Q_E) / Q_E \\ &= 1 + Q_c / Q_E \end{aligned}$$

i.e.

$$\eta = 1 + [(\alpha I T_c - k \Delta T G - I^2 \rho / (2G))] / [I^2 \rho / G + \alpha I \Delta T] \quad (14)$$

Given $\partial \eta / \partial I = 0$, the optimum current for the maximum COP for heating equals that for cooling (see equation (7)).

Put I in equation (14) by I_{opt} , to get maximum COP:

$$\eta_{max} = 1 + T_m / \Delta T [((1 + ZT_m)^{1/2} - 1) / ((1 + ZT_m)^{1/2} + 1)] - 1/2 \quad (15)$$

As shown in equations (4) and (11), to obtain the cooling/heating capacity Q_c / Q_h required for a room/building, a specific operating current is required. It depends on the temperatures of T_h and T_c . By using a suitable numbers of thermocouples or suitable geometry factor, the needed operating current can be the optimum current. In the case studied, it is assumed that all the thermocouples (thermoelectric modules) are

connected in a series. All the details of the material property and geometry factor can be obtained from manufactory.

Optimal section model

Figures 1-5 show the comparison of the best thermoelectric parameter by varying the hot side temperature under the conditions of fixed cold side temperature and cooling capacity.

For thermoelectric model DT 12-6-01L, geometry factor (G) equals 0.061 and it contains 127 thermocouples.

Results of modelling

Every case should be based on acceptable values (cooling capacity and thermoelectric cold side and hot side temperatures), such as the lowest temperature at cold side, highest temperature on the hot side and acceptable cooling capacity.

Cooling system

Figures 1 to 3 show the relation of the optimum thermoelectric parameters, cooling capacity and geometry factor. Figure 1 and 2 show the components to needed gain the optimum of COP for cooling systems. The thermoelectric device should be designed to work under some conditions such as optimum current. The results from the figures show that the optimum current and optimum COP are unchanged when varying cooling capacity when working temperatures (T_c and T_h) are fixed.

This then entails that an increased cooling capacity requires an increased number of thermocouples (thermoelectric numbers are increased). An increased number of thermocouples require a higher voltage (Figure 3). The optimum current and COP can be obtained by using thermoelectric material property coefficients, T_h , T_c , temperature and geometry factor. This means the specific working temperatures; the optimum current and optimum COP of modules have definite values.

Figures 4 to 6 show the relation of temperatures on the hot side, thermoelectric parameters for cooling capacity, temperatures on the cold side and geometry factor. The modelling results are:

The optimum current is increased and COP is decreased when hot side temperature is increased. Lower hot side temperature gave a higher COP for the system. Increasing the number of thermoelectric modules number results in a lower temperature on the hot side and achieves a higher optimum COP. A number of thermocouples are required for the optimum current. A higher number of thermocouples then require a higher voltage.

An increase in operating current for thermoelectric modules decreased the optimum COP. The optimum current was increased, which resulted in more Joule heat and thermal conductive energy being transferred back to the cold side. Therefore, it decreases the total net energy absorbed at the cold side. These phenomena allow for a minimum number of thermoelectric couples to satisfy the requirement of the cooling capacity.

Figure 7 shows the relation of the geometry factor and thermoelectric parameters for cooling capacity and T_h , T_c . The modelling results are: different thermal modules have different geometry factors. The total area and dimension of the heat sink has to be considered when selecting a module. The required number of thermocouples varies with the geometry factor to achieve the optimum COP. Fewer thermocouples are required with a bigger geometry factor. The optimum current increases and voltage decreases with the increase of the geometry factor, but the optimum COP remains the same under various geometry factors because the optimum COP is determined by T_h , T_c and the figure of merit of the material. There is no relationship with the geometry factor.

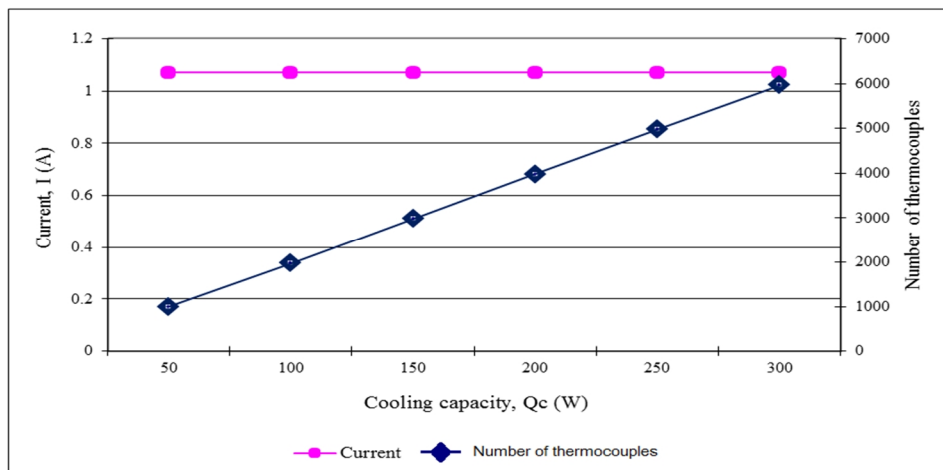


Figure 1 Relation Of Optimum Current And Number Of Thermocouples Against Cooling Capacity ($T_c=16^\circ\text{C}$ and $T_h=45^\circ\text{C}$)

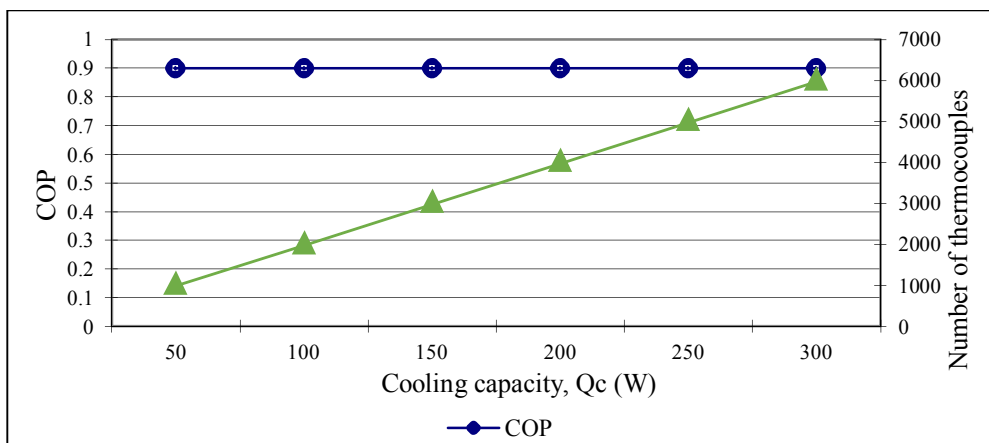


Figure 2 Relation of Optimum COP and Number of Thermocouples against Cooling Capacity ($T_c=16^\circ\text{C}$ and $T_h=45^\circ\text{C}$)

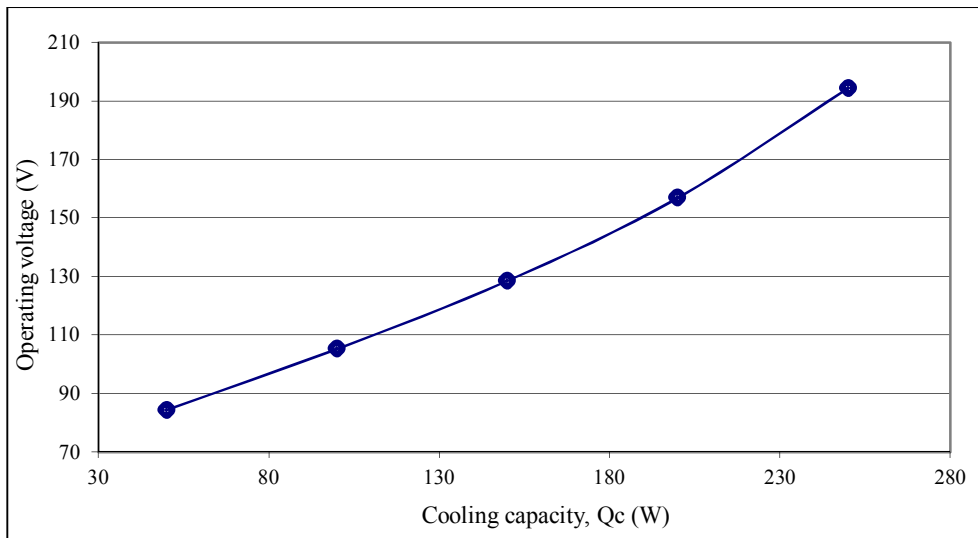


Figure 3 Cooling Capacity against Operating Voltage ($T_c=16^\circ\text{C}$ and $T_h=45^\circ\text{C}$)

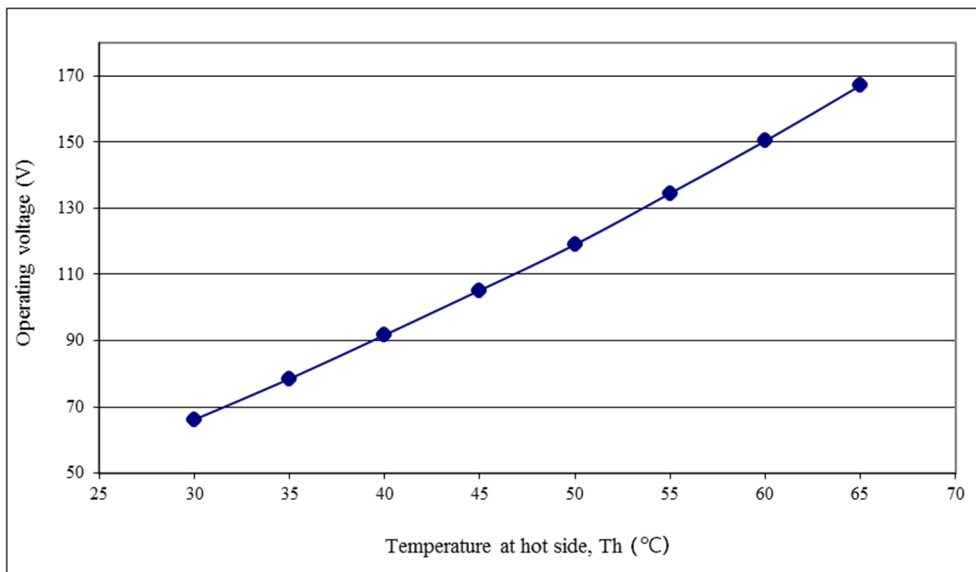


Figure 4 Temperature on the Hot Side Against Voltage ($Q_c = 100\text{W}$, $T_c = 16^\circ\text{C}$)

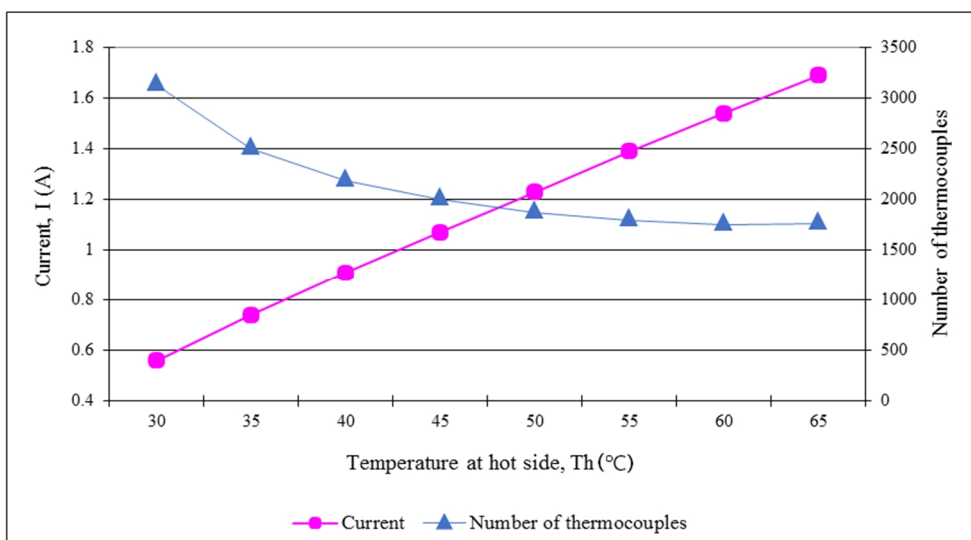


Figure 5 Relation of Optimum Current and Number Of Thermocouples Against Temperature Outlet On The Hot Side ($Q_c = 100\text{W}$, $T_c = 16^\circ\text{C}$)

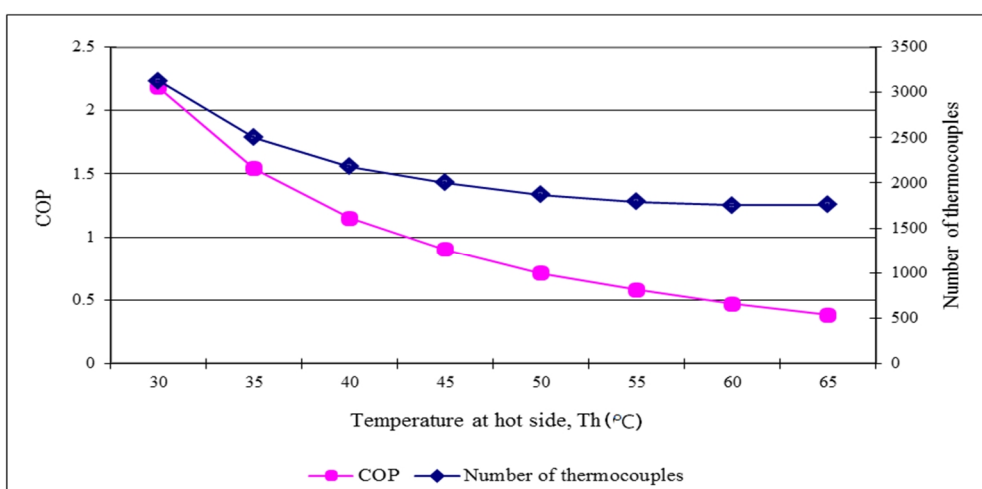


Figure 6 Relation of Optimum COP And Number Of Thermocouples against Temperature Outlet On The Hot Side ($Q_c = 100\text{W}$, $T_c = 16^\circ\text{C}$)

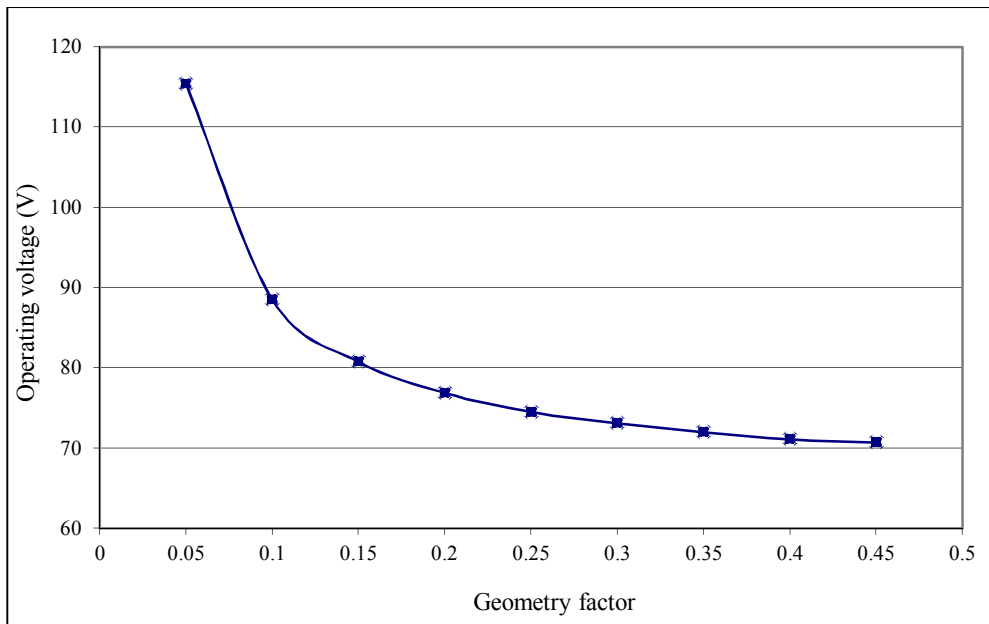


Figure 7 Geometry Factor against Operating Voltage ($Q_c = 100\text{W}$, $T_c = 16^\circ\text{C}$ and $T_h = 45^\circ\text{C}$)

The performance of finned heat sink

The thermoelectric modules do not work effectively without a finned heat sink due to high heat flux. The finned heat sink helps by improving heat transfer efficiency and is widely used in thermoelectric systems. Modelling a finned heat sink provides a basic design which shows the relationship between finned heat sinks, geometry parameters and convective coefficients.

Finned heat sink performance analysis mode

The analytical model was set up based on the following assumption: heat conducts across the fins in one dimension. In practice, an error of less than 1% exists. The accuracy increases with the thinning of the fin. Lienhard John H (2001) There are conditions for the modelling:

- The thermal conductivity of a fin has a constant value
- Fin has a constant cross-sectional area
- Constant convective heat transfer coefficient on the surface of fins

Heat transfer, efficiency and thermal resistance of a finned heat sink

To analyze finned heat sink's performance, one monitored a single finned heat sink performance.

A finned heat sink has a fixed length and can conduct heat through convection from its surface area. The heat transfer from the fin is expressed by the following equation Lienhard John H. (2001)

$$Q_f = [(hPk_f A_c)^{1/2} (T_r - T_s) (\sinh(mL) + (h/mk_f) \cosh(mL))] / [\cosh mL + (h/mk_f) \sinh(mL)] \quad (16)$$

$$\text{where} \quad m = (hP/k_f A_c)^{1/2} \quad (17)$$

However, equation (16) is inconvenient to use. Below is the corrected fin length equation which can be substituted into equation (16):

$$L_c = L + (A_c / P) \quad (18)$$

To calculate the heat transfer from the finned heat sink with convective tip by: $Q_f = (hPk_f A_c)^{1/2} (T_r - T_s) \tanh(mL_c)$ (19)

Finned heat sink efficiency can be defined as the actual heat transferred ratio from the finned heat sink to heat that would be transferred if the entire fin were $T = T_r$. Finned heat sink efficiency can show performance of a finned heat sink. The efficiency of finned heat sink is calculated below Lienhard John H (2001):

$$\eta = Q_f / hPL_c(T_r - T_s) = [(hPk_f A_c)^{1/2} (T_r - T_s) \tanh(mL_c)] / [hPL_c(T_r - T_s)] \\ = \tanh(mL_c) / mL_c \quad (20)$$

The equations mentioned above give an error of less than 8% if:

$$(h\delta / 2k_f)^{1/2} \leq 1/2 \quad (21)$$

The following equation (22) is a different form of equation (20) (rearranged): $Q_f = \eta_f hPL_c(T_r - T_s)$

$$= \eta_f hA_f(T_r - T_s) \quad (22)$$

where $A_f = PL_c$

The thermal resistance of a fin (R_f) is as therefore,

$$R_f = (T_r - T_s) / Q_f = 1/(\eta_f A_f h) \quad (23)$$

Heat transfer, efficiency and thermal resistance of finned heat sinks

Figure 3.8 shows the practical finned heat sink number extruded from the base plate. The total area exposed to coolant can be calculated from:

$$A_t = A_o + NA_f \quad (24)$$

The overall heat transfer from a finned heat sink can be calculated by Lienhard John H (2001):

$$Q_{fs} = hA_o(T_r - T_s) + N\eta_f hA_f(T_r - T_s) \quad (25)$$

The first term in equation (25), $[(hA_o)(T_r - T_s)]$, shows the heat transfer from the base plate exposed regions whereas the second term, $[N\eta_f hA_f(T_r - T_s)]$, shows the heat transfer from the fins (Refer to equation 22).

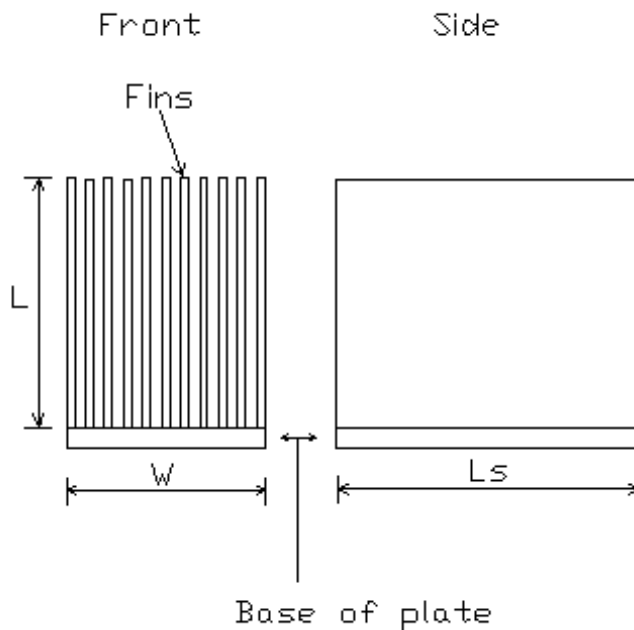


Figure 8 Schematic Diagram of Finned Heat Sink

Combining equation (24) and (25), one gets

$$Q_{fs} = hA_t [1 - (NA_f / A_t)(1 - \eta_f)] (T_r - T_s) \quad (26)$$

The finned heat sink efficiency can be obtained by comparing equation (22) with equation (26):

$$\eta_{fs} = 1 - (NA_f / A_t)(1 - \eta_f) \quad (27)$$

Combining equation (26) with equation (27) gives

$$Q_{fs} = hA_t\eta_{fs}(T_r - T_s) \quad (28)$$

Equation (3-28) gives the next equation (29):

$$R_{fa} = (T_r - T_s) / Q_{fs} = 1 / (\eta_{fs}hA_t) \quad (29)$$

The combination of finned heat sink thermal resistance with base plate conductive thermal resistance is as follows:

$$R_{fs} = (\delta_b/k_bA_b) + (1/\eta_{fs}hA_t) \quad (30)$$

Results of Modelling and Discussion

The relationship between finned heat sink performance; fin length, convective coefficient and fin number are analysed by modelling. In the modelling, certain parameters are set up as:

- Fins thickness is 3mm,
- Heat sink width (W) is 0.25m,
- Heat sink length (L_s) is 0.25m,
- Base plate thickness is 0.01m and
- Finned heat sink thermal conductivity (made of aluminium) is 236 W/(m°C) Lienhard John H (2001)
- The convective coefficient can be found from [7].
- The temperature difference between surrounding fluid over the finned heat sink and heat source is 15°C.
- Number of fins is 22

Heat transfer rate increased quite quickly when the fin length was 0.07m (Figure 9), but there is not much difference after 0.14 m. This means that the increased length of fins does not mean better heat transfer. The increased length of fins will decrease the fin efficiency.

Thermal resistance is dramatically decreased when the fin's length is increased to 0.12m (Figure 10), but not obviously when the length of fins is

greater than 0.12m it can be seen that the an increased length of fins does not mean decreased thermal resistance.

Assuming the length and performance of fins to be: length of fins – 0.07m, heat transfer from heat sink increased directly with increasing number of fins (Figure 11). The increased number of fins decreased the efficiency. As we can see from Figure 11, the efficiency decreased until it reached a point at which number of fins was 16, whereby after this point, there was no longer much difference in the efficiency.

The increased number of fins decreased thermal resistance, but thermal resistance does not differ much with an increased number of fins (Figure 12).

The thermal resistance is dramatically decreased when the length of fins is about 0.08-0.09m (Figure 12). It does not make much difference when the length is more than 0.08-0.09m. An increase in the fin's length decreases the fin's efficiency.

Increased in convective coefficient increased the heat transfer from fins, but decreased the fin's efficiency (Figure 13) and thermal resistance (Figure 14).

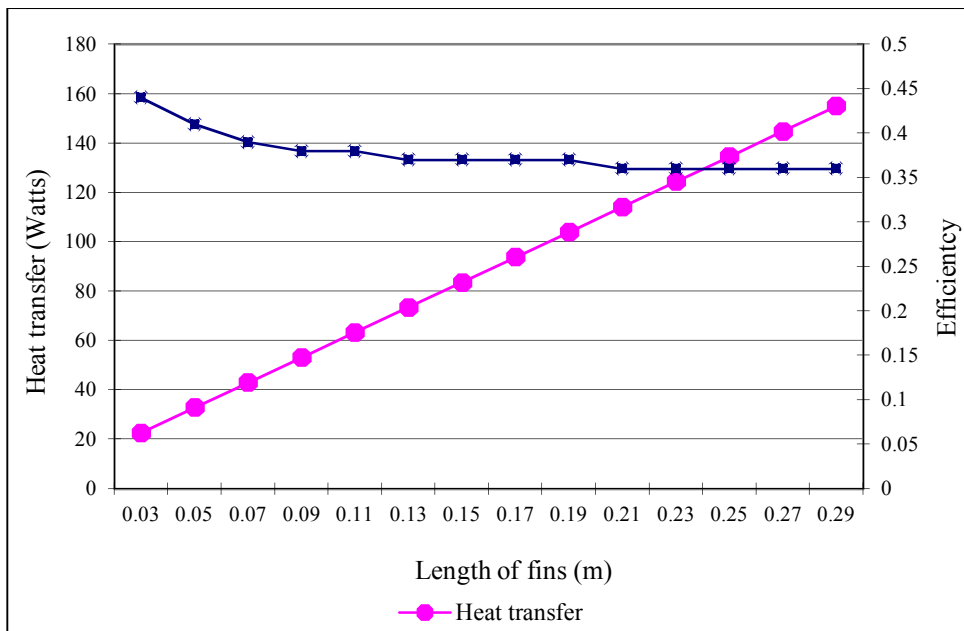


Figure 9 Heat Transfer And Efficiency against Length of Fins

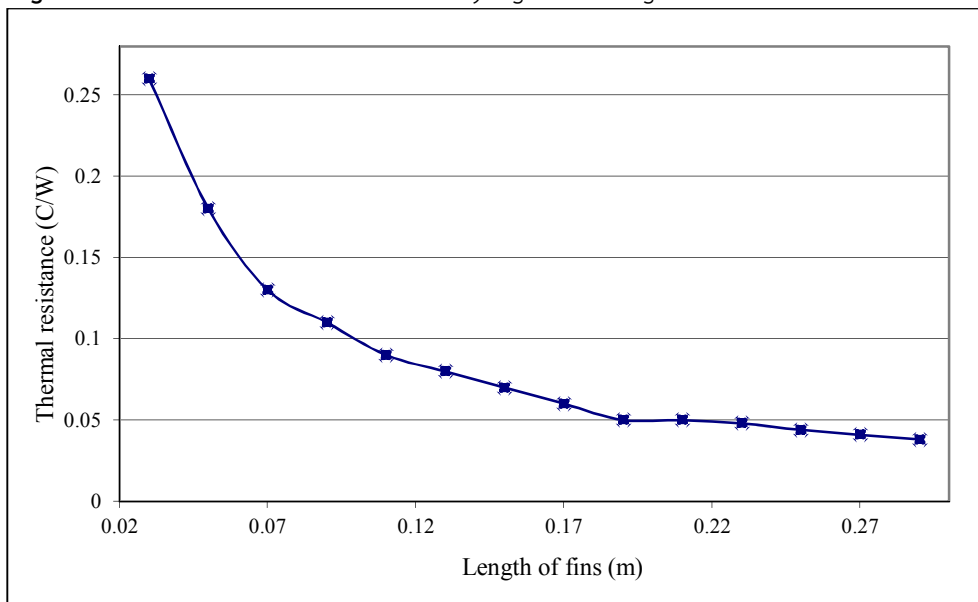


Figure 10 Thermal Resistances against Length of Fins

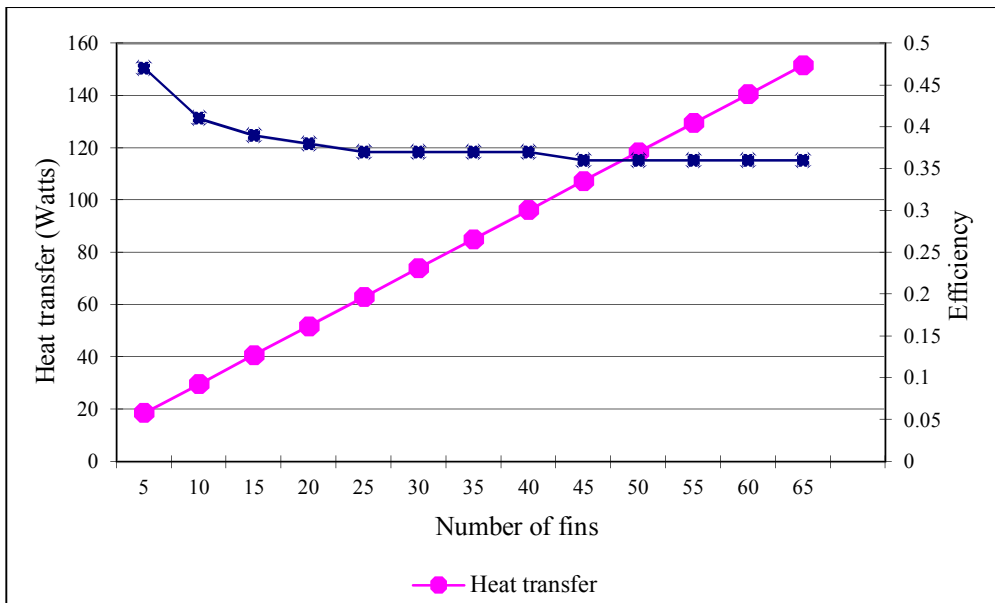


Figure 11 Heat Transfer And Efficiency against Number of Fins

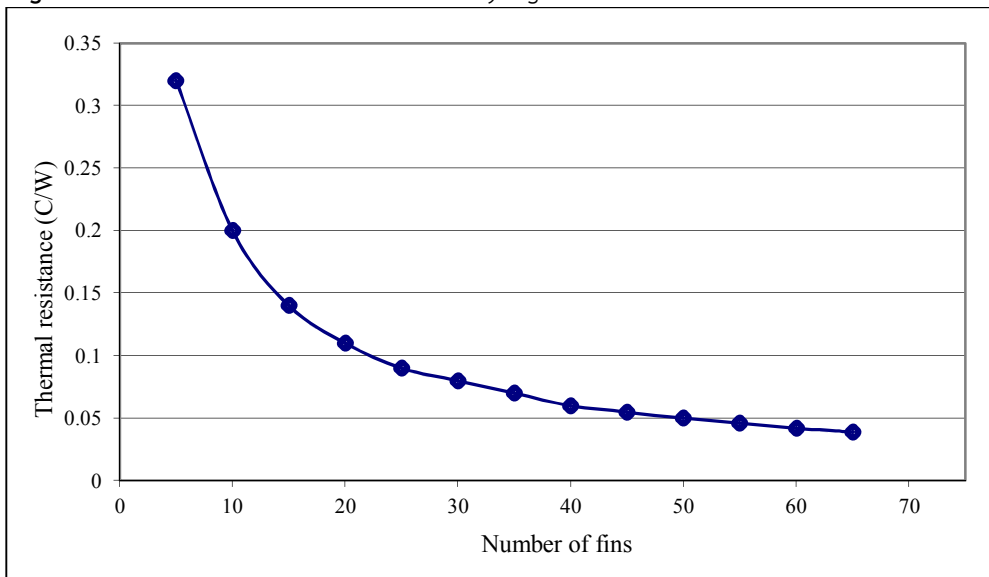


Figure 12 Thermal Resistances against Number of Fins

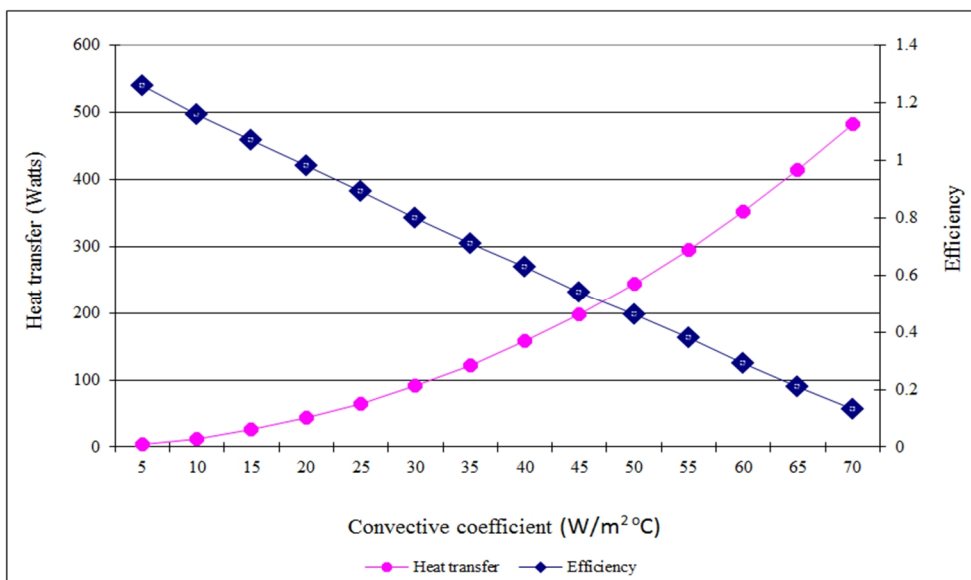


Figure 13 Heat Transfer And Efficiency against Convective Coefficient

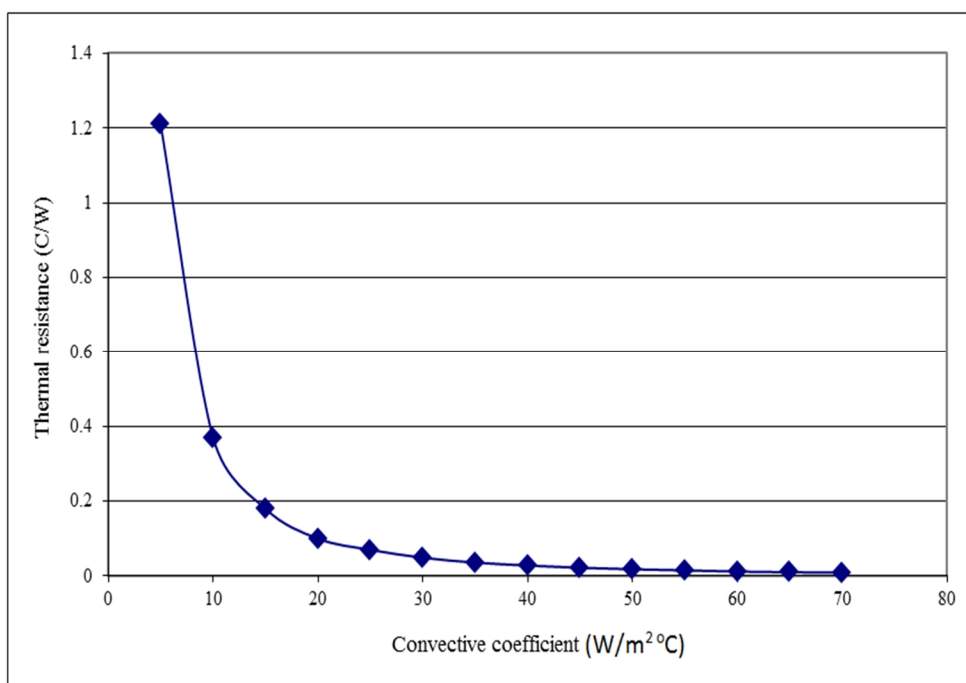


Figure 14 Thermal Resistances against Convective Coefficient

Conclusion

Results from modelling have shown that the cooling capacity is dependent on the number of thermocouples (or thermoelectric modules) with fixed temperatures on cold and hot sides. Also, there exist a minimum number of thermocouples needed to meet certain cooling capacity requirements when there is a fixed cold side temperature. To increase operating current is to increase hot side temperature. A lower operating current gives a higher optimum COP but lower cooling capacity. Theoretically, the system can be operated under the maximum cooling capacity output with a reduction of optimum COP to maintain a cooling capacity at certain level. The specific cooling requirement, the optimum number of thermoelectric modules for the optimum current and optimum COP vary with the geometry factor. The optimum COP value can be determined by the characteristics of the material and working temperatures. There is no relationship with the geometry factor and cooling capacity.

The relationship of finned heat sink performance, geometry parameter and convective coefficient can be developed by modelling. Modelling results showed that the thermal resistance and fin efficiency from finned heat sink decreases when there is an increase in the finned heat sink length, convective coefficient and finned heat sink number. On the other hand, an increase of these factors gives a lower COP. It has been shown that modelling can provide a good design for finned heat sinks. When integrating TEC modules with finned heat sinks, a higher number of thermocouples will result in a higher cooling capacity. However, an increase in thermocouples also requires a higher operating voltage. To design a heat sink system with good heat transfer efficiency, one should keep in mind that heat transfer is increased with increasing length, number and convective coefficient of fins. Yet, COP and thermal resistance seem to be decreased when those three components are increased.

References

- Lienhard John H. (2001). **A Heat Transfer Textbook**, Phlogiston Press.
Cambridge, MA : Phlogiston Press.
- Riffat S.B. & Xiaoli Ma. (2003). Review-thermoelectrics: a review of present and potential applications. **Applied Thermal Engineering**. 23(8), 913-935.
- Webb-R L, et al. (1998). Advanced heat exchange technology for thermoelectric cooling. **Journal of Electric Packing**. 120(1), 98-105.