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Mathematical Modelling Development of Packed Bed Bioreactors in Solid-state Fermentation

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ABSTRACT

A two-dimensional heat transfer model was developed on the microbial growth on starchy materials in packed bed bioreactors. The model comprised of two sets of equations: (i) the growth and death kinetics and (ii) the energy balance. The system is assumed to be pseudo-homogeneous and microbial growth and death are influenced by the system temperature. The model predicts the temporal temperature and biomass concentration profiles at any location of the bed in two dimensions.

The model was verified by comparing with experimental data and also with simulating results of a one-dimensional model. The model fitted reasonably well with the experimental data and the comparison of these two models were discussed. From the basis of heat transfer, the model was flexible to various sizes of bioreactors and other microorganisms of packed bed solid-state fermentation. The model was proven to be useful and could be used as an investigated tool on either the bioreactor performance and in control, optimization and scale-up.

Key words: bioreactor design, heat transfer, packed bed bioreactor

INTRODUCTION

Solid-state fermentation (SSF) has been widely used in Asia and Africa (Hesseltine, 1983) in the production of fermented foods for many years. SSF defines as the growth of microorganisms on water insoluble solid substrate in the absence of free water (Mitchell and Lonsane, 1992). From engineering point of view SSF may be described as exothermic heterogeneous catalysis as a large amount of heat (typically around 2500 kJ/kg-substrate) being generated during the fermentation (Saucedo-Castaneda *et al.*, 1990). Overheating is a crucial problem in SSF which affects the microbial growth or even kill microorganisms.

SSF is in general carried out in bioreactor of simple construction and operation. Basically, there are five bioreactors used in SSF: tray, packed bed, rotating drum, stirred and fluidized bed bioreactors. Packed bed is relatively simple to design and operate but has the potential to allow some control over fermentation parameters. Some research has been done experimentally into how aeration rates affect performance but otherwise theoretical investigation of the effects of design and operational parameters in packed bed SSF were found to be relatively poor (Ghildyal *et al.*, 1994; Yang *et al.*, 1994). Saucedo-Castaneda et al (1990) proposed a one-dimensional model on conductive heat removal which was adequate to debolic

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scribe their system because the high geometric ratio (H/D) of 5/1 for their bioreactor reduced the importance of axial heat transfer. The two-dimensional model proposed in this paper should have the flexibility to describe heat transfer in systems whose geometry favor either axial or radial heat transfer.

MATERIALS AND METHODS

System

The transient two-dimensional of packed bed bioreactor has been proposed to describe the microbial growth and death kinetics and heat transfer phenomena in the system of solid-state fermentation. Figure 1 shows the prototype of the system. These following assumptions were made for this study.

1 The system is pseudo-homogeneous.

2 The thermal and physical properties of the bed are independent of temperature.

3 There is no radial or angular air velocity, only air movement in the axial direction is considered.

4 The axial air velocity is constant across the cross section of the bioreactor.

5 Porosity is uniform and constant throughout the fermentation.

6 Growth is not limited by substrate availability.

7 The change in weight of dry matter in the bed is ignored.

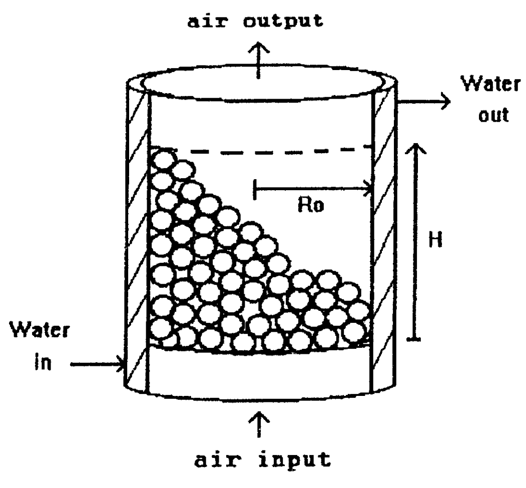

Figure 1 Packed bed bioreactor in solid-state fermentation system.

MATHEMATICAL MODEL

The macroscopic balance over the cylindrical geometry was performed. The model comprises of essentially two sets of equations (Sangsurasak, 1996). The first set of equations described growth and death kinetics, while the second set of equations described the heat transfer phenomena. These two sets of equations would be described separately.

Microbial growth and death kinetics

Biomass is segregated into two types, viable biomass and dead biomass. The total biomass is then the sum of viable and dead biomass. Growth is described by the logistic equation, and empirical kinetic model that has commonly been used to model growth in SSF (Mitchell, 1992; Sargantanis *et al.*, 1993). The rate of total biomass production is:

$$\frac{dXt}{dt} = \mu g Xv(1 - \frac{Xt}{Xm})$$

The specific growth rate of *Rhizopus oligosporus* was fitted to the data of Mitchell (1989) and described as the function of system temperature by the quadratic equation.

$$\mu_g = -39.599 + 0.2566T - 0.0004T^2$$

Non-viable biomass has no metabolic activity at all and therefore does not contribute to metabolic heat generation. Non-viable biomass is generated through the death of viable biomass according to the following equation.

$$\frac{dX_d}{dt} = \mu_d X_v$$

The specific death rate depends on temperature according to the Arrhenius equation (Muck *et al*., 1991).

$$\mu_d = \mu_{d,m} \exp\left[-\frac{E_a}{R_g}\left(\frac{1}{T} - \frac{1}{T_m}\right)\right]$$

Combining the equations for the increase in total biomass and the increase in dead biomass, the expression for the increase in viable biomass with time is:

$$\frac{dX_v}{dt} = [\mu_g(1 - \frac{X_t}{X_m}) - \mu_d]X_v$$

The maintenance heat was assumed to be comparatively small and negligible in this study. The metabolic heat generated during growth was assumed to be directly proportional to the rate of production of total biomass.

$$q = \frac{dX_t}{dt}\rho_b g$$

Heat transfer phenomena

With the assumptions about the system geometry and operation outlined above, the transient energy balance of heat transfer with the introduction of the convection term can be written in two dimensions as:

$$\rho_b C_{p,b}\left(\frac{\partial T}{\partial t}\right) + (\rho_{ma} C_{p,ma}) v_z\left(\frac{\partial T}{\partial z}\right) = \frac{k_b}{r}\left(\frac{\partial T}{\partial r}\right) + k_b\left(\frac{\partial^2 T}{\partial r^2}\right) + k_b\left(\frac{\partial^2 T}{\partial z^2}\right) + q$$

The initial temperature of the bed is set at the optimum growth for *R. oligosporus*.

$t = 0, \quad T = T_i$

The boundary conditions are as follow:

$$z = 0,\ T = T_a$$

$$z = H,\ \frac{\partial T}{\partial z} = 0$$

$$r = 0,\ \frac{\partial T}{\partial r} = 0$$

$$r = R_0,\ \frac{\partial T}{\partial r} = \frac{h}{k_b}(T_{surr} - T)$$

Where the thermal properties of the substrate bed were calculated as weighted average of the moist air and starch values as shown in the following equations.

$$\rho_b = (e r_{ma}) + (1-\varepsilon)\rho_s$$

$$C_{p,b} = (\varepsilon C_{p,ma}) + (1-\varepsilon)C_{p,s}$$

$$k_b = (\varepsilon k_{ma}) + (1-\varepsilon)k_s$$

MODEL VERIFICATION

In this study the orthogonal collocation method was applied as a two-dimensional problem with cylindrical geometry (Villadsen and Michelsen, 1978; Finlayson, 1980). The collocation points were chosen. By this method, the set of partial differential equations were discretized into ordinary differential equations. The equations were then solved simultaneously by the Backward Differential Formula (BDF) using the GEAR package (Hindmash, 1974).

The predictions of the two-dimensional model with evaporative heat removal were compared against the experimental results and the predictions of the one-dimensional model of Saucedo-Castaneda *et al.* (1990) (Figure 2). This was done for the dimensionless height of 0.5 and at the dimensionless radii of 0.8 and 1.0 which corresponded to points at which thermocouples were located in the experiment of Saucedo-Castaneda *et al.* (1990).

CONCLUSION

The model is useful for a study of bioreactor performance and design for any scale packed bed bioreactors. Superficial air velocity, inlet air temperature and geometric ratio could be varied to see the effects on the temperature and growth in the packed bed. The model could be simulated and aids in process design, optimization and control of packed bed solid-state fermentation.

The two-dimension model gave a better fit to the experimental data during the early temperature rise compared to Saucedo-Castaneda *et al.*' s model (Figure 2). However, overall predictions from those two models gave similar results. The difference of the predictions would be more prominent when the geometric ratio (H/D) is lower. In most cases the geometric ratio (H/D) of packed bed bioreactor was about 2 (Ghildyal *et al.*, 1994; Yang *et al.*, 1994; Sargantanis *et al.*, 1993).

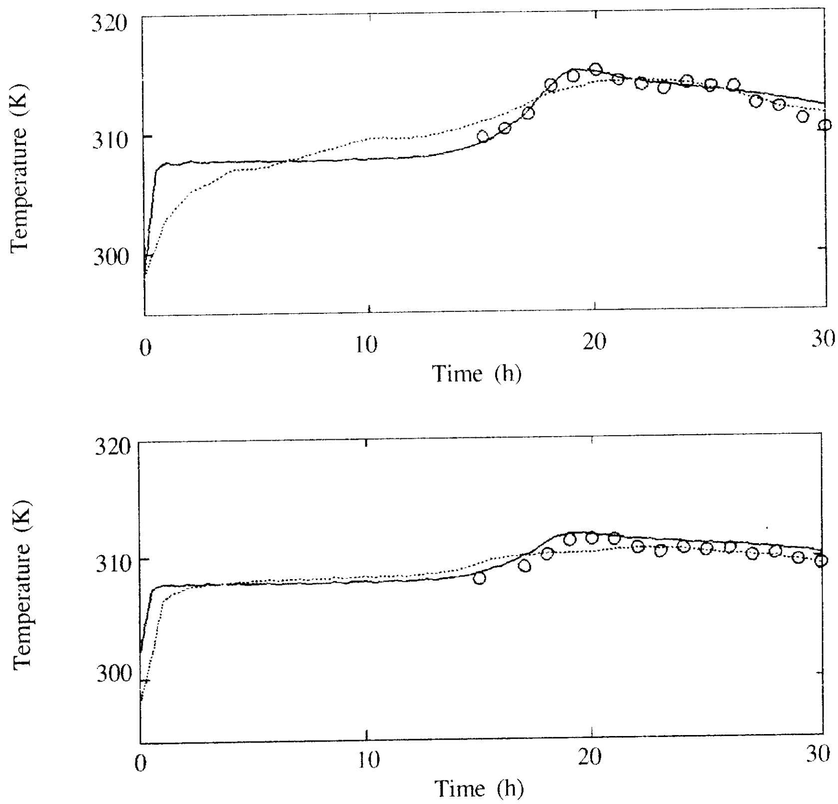

Figure 2 Comparison of experimental data of Saucedo-Castaneda *et al.* (1990) (o), with the predictions of two-dimensional heat transfer model (---) and the one-dimensional model of Saucedo Castane da *et al.* (1990) (---) at two different locations (a) at the central and (b) at the bioreac-tor wall.

Table 1 Parameter value used to validate data of Saucedo-Castaneda *et al.* (1990)

Constant	Description	Value		Source
$C_{p,ma}$	heat capacity of moist air	1180	J/kg.K	Sangsurasak (1996)
$C_{p,s}$	heat capacity of solid	2500	J/kg.K	Sweat (1986)
D	bed diameter	0.06	m	Saucedo-Castaneda *et al.* (1990)
d_p	particle diameter	0.0045	m	Saucedo-Castaneda *et al.* (1990)
E_a	activation energy	578.6	kJ/mol	Muck *et al.* (1991)
g	metabolic heat yield coefficient	8366	J/g-biomass	Sangsurasak (1996)
H	bed height	0.3	m	Saucedo-Castaneda *et al.* (1990)
k_{ma}	thermal conductivity of moist air	0.0206	W/m.K	Perry *et al.* (1984)
k_s	thermal conductivity of solid	0.3	W/m.K	Sweat (1986)
R_g	gas constant	8.314	J/mol.K	Muck *et al.* (1991)
T_a	air temperature	309	K	estimated
T_b	bed temperature	308	K	Saucedo-Castaneda *et al.* (1990)
T_i	initial temperature	298	K	Saucedo-Castaneda *et al.* (1990)
T_m	maximum temperature	318	K	Muck *et al.* (1991)
V_z	superficial velocity	0.01	m/s	Saucedo-Castaneda *et al.* (1990)
X_0	initial biomass concentration	0.874	g-biomass /kg-substrate	Saucedo-Castaneda *et al.* (1990)
X_m	maximum biomass concentration	125	g-biomass/kg-substrate	Saucedo-Castaneda *et al.* (1990)
e	bed void fraction	0.35		Terzic & Todorovic (1992)
ρ_{ma}	density of moist air	1.14	kg/m^3	Weast (1974)
ρ_s	density of solid	700	kg/m^3	Saucedo-Castaneda *et al.* (1990)
μ	specific rate	0.3	1/h	Saucedo-Castaneda *et al.* (1990)

Notations

C_p heat capacity, J/kg.K

d_p particle diameter, m

D bed diameter, m

E_a activation energy, kJ/mol

g metabolic heat yield coefficient, J/g-biomass

h heat transfer coefficient, W/m^2.K

H bed height, m

k thermal conductivity, W/m.K

q metabolic heat generation, W/m^3

r length in radial direction, m

R_g gas constant, J/mol.K

R_0 bed radius, m

t fermentation time, h

T system temperature, K

Tm temperature at which the maximum specific growth rate and specific death rate are equal, K

V superficial velocity, m/s

X biomass concentration, g-biomass/kg-substrate

X_0 initial biomass, g-biomass/kg-substrate

z length in axial direction, m

ε void fraction

ρ density, kg/m^3

μ specific rate l/h

Subscripts

a moist air

b bed

d dead

i initial

g growth

m maximum

r radius

s substrate

surr surroundings

t total

v viable

w wall

z axial direction

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