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Autocorrelation, Power Spectrum Analysis and Eddy Viscosity Coefficient of Tidal Current

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ABSTRACT

Twenty-records of current velocity were obtained at two coastal stations in the Seto Inland sea. It was observed that water movement was characterized by east-west flow and north-south flow. Autocorrelation analysis showed that the characteristics of water motion were periodic in semidiurnal period. At frequencies about 2 cycles per day where power spectrum of current velocity had slopes of -5/3, the horizontal eddy viscosity coefficient were about 1.8×10^4 cm^2/sec in east-west component and about 1.2×10^4 cm^2/sec in north-south component.

Key words : viscosity, autocorrelation, eddy

INTRODUCTION

The purpose of the present paper was to make clear hydraulic characteristics on the basis of the observation data obtained in and out of the bay. The autocorrelation, power spectrum and eddy viscosity of flow would be considered. On the basis of the Taylor (1921) hypothesis, the horizontal eddy viscosity coefficient of this area could be calculated in east-west component and in north-south component.

The observed area was located at the eastern part of the Seto Inland sea Takamatsu Prefecture, in Shikoku Island, Japan. The average depth at the station was about 20 m and the maximum depth was about 22 m.

MATERIALS AND METHODS

Field Experiment

The observation had been carried out by using two Onoshiki current meters set at Kamano station 1 and 2 for 20 days as shown in Figure 1. The mean sea level at each stations was about 20 m. As for annually tide of this area, there are 2 tide cycles per day, the period for one tide cycle is about 12 hours 50 minutes. Sea water was westward flow during flood tide and became eastward flow during ebb tide.

The water speed and water direction at Kamano station 1 (34°23.37'N, 134°09.49'E) and Kamano station 2 (34°23.36'N, 134°09.45'E) during May 16, 1994 to June 6, 1995 were measured by two Onoshiki current meters. These meters were fixed at 2 m below the sea surface by floats and anchors as shown in Figure 2.

Time of operation was about 20 days for every 10 minutes sampling. The variations in tidal current at Kamano station 1 and 2 were shown in Figure 3. The variations of fluctuation velocity were shown in Figure 4.

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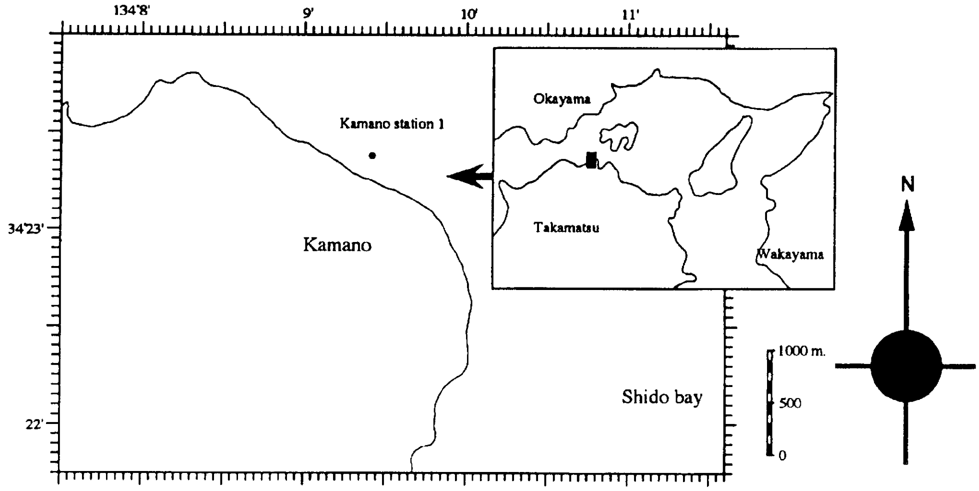


Figure 1 Map of drogue tracking at Kamano station 1 on July 7, 1994.

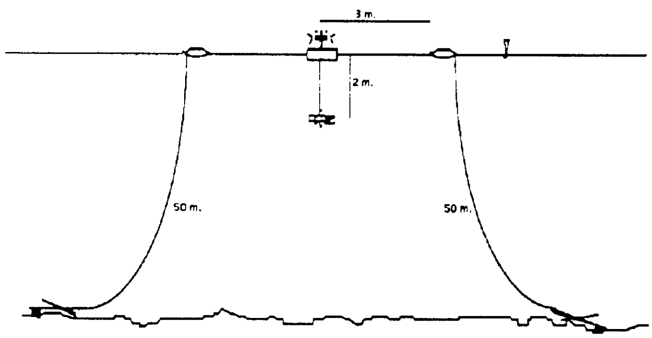


Figure 2 An experiment design used for observing tidal current at kamano station 1 and 2.

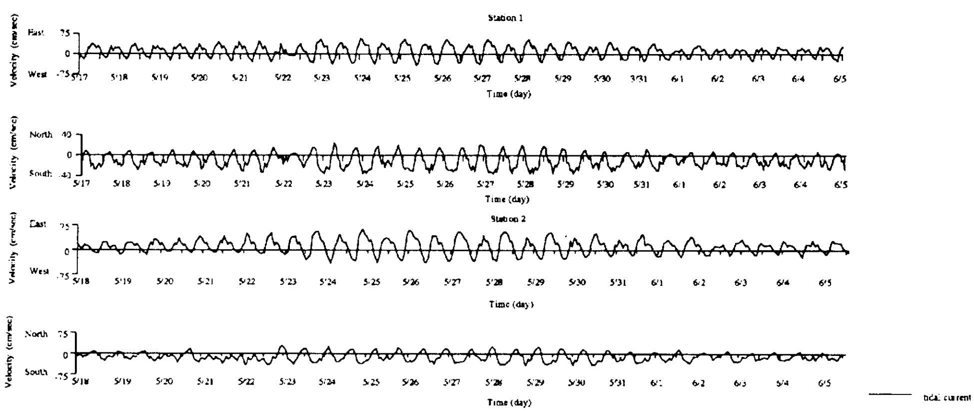
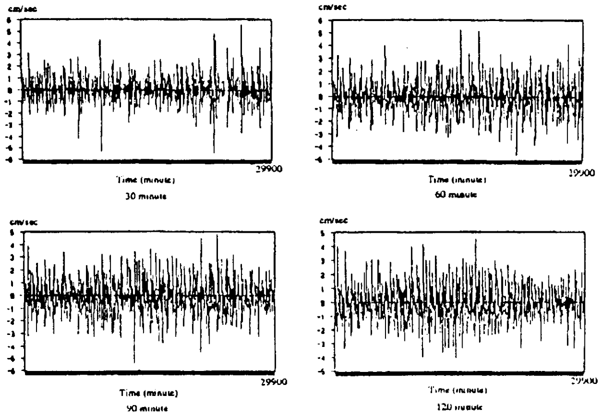
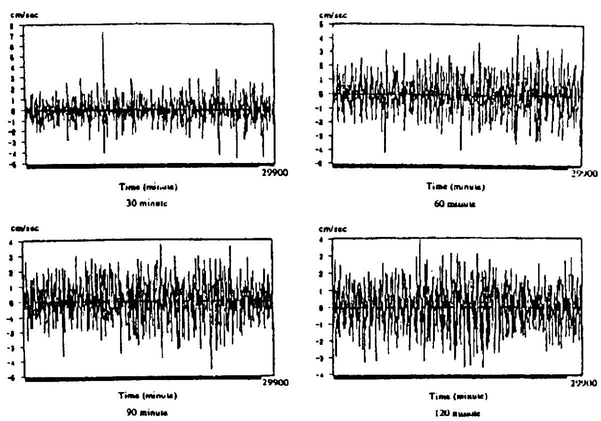


Figure 3 Variations of tidal current in north-south component and east-west component at kamano station 1 and 2 in May 1994.



Variation of fluctuation velocity in east-west component at Kamano station 1 during May 1994 when changed moving average at 30, 60, 90 and 120 minutes



Variation of fluctuation velocity in north-south component at Kamano station 1 during May 1994 when changed moving average at 30, 60, 90 and 120 minutes

Figure 4 Variation of fluctuation velocity.

Statistical analysis

Autocorrelation function

The autocorrelation function $R_{xx}(\tau)$ of a quantity x(t) was the average of the product of the quantity at time t with the quantity at time t+τ for an appropriate averaging time T:

$$R_{xx}(\tau) = \frac{1}{T}\int_0^T x(t)x(t+\tau)dt \quad (1)$$

For the finite function, the autocorrelation function could be written as:

$$R_{xx}(\tau) = \frac{\Sigma(x_t - \bar{x})(x_{t+\tau} - \bar{x})}{\Sigma(x_t - \bar{x})^2} \quad (2)$$

For all τ, the quantity $\rho_{xy}(\tau)$ satisfied $-1 \leq \rho_{xy}(\tau) \leq 1$.

Power spectrum function

The power spectrum function X(f) for a stationary record represented the rate of change of mean square value with frequency. It was estimated by computing the mean square value in a narrow frequency band at various center frequencies, and then dividing by the frequency band. The total area under the power spectrum function over all frequencies, therefore the total mean square value of the record. The partial area under the power spectral function from f_1 to f_2 represented the mean square value of the record associated with that frequency range. By Blackman and Tukey (1958), the computation of power spectrum may be written as:

$$X(f) = x_0 + 2\sum_{q=1}^{M-1} x_q \cos\left(\frac{qp\pi}{M}\right) + x_M \cos p\pi \quad (3)$$

where $x_0, x_1, ..., x_M$ = correlation values at lag p
M = maximum value of the lag p
$X(f)$ = raw power spectrum
f = frequency
p = 0,1,2,3,...,M

It may be smoothed value by using the formulation as:

$$G(f) = 0.23 \cdot X_{i-1}(f) + 0.54 \cdot X_i(f) + 0.23 \cdot X_{i+1}(f) \quad (4)$$

where $X_{-1}(f) = X_1(f)$ and $X_{M+1}(f) = X_{M-1}(f)$
$G(f)$ = smoothed power spectrum.

Eddy viscosity coefficient or Eddy diffusion coefficient (ε)

The Taylor (1921) hypothesis for expressing diffusivity in terms of Lagrangian statistics was used as the basis for this study. Taylor showed that the eddy diffusion coefficient, ε (cm_2/sec.), of substances could be expressed as the product of the mean square of the fluctuation velocity, v', and the Lagrangian integral time scale of large diffusion times:

$$\varepsilon = \overline{v'^2}\int_0^\infty R_L(\tau)d\tau \quad (5)$$

where τ = dummy lag time variable
$R_L(\tau)$ = Lagrangian correlation coefficient
$\int_0^\infty R_L(\tau)d\tau$ = Lagrangian integral time scale.

Since the Lagrangian integral time scale in case of fluid was very complicated to determine. In this study, we used the Eulerian integral time scale to estimate the eddy diffusion coefficients which the relation were investigated by Hay and Pasquill (1959). This relation may be expressed as:

$$\varepsilon = \overline{u'^2}\beta\int_0^\infty R_E(\tau)d\tau \quad (6)$$

where $\sqrt{\overline{u'^2}}$ = intensity of the turbulence in x component
$R_E(\tau)$ = Eulerian correlation coefficient
$\int_0^\infty R_E(\tau)d\tau$ = Eulerian integral time scale
β = ratio of the Lagrangian to Eulerian integral time scales.

In this study, β from the results of Sasaki and Inoue (1984)'s observations were used and may be expressed as:

$$\beta \cong 0.8\left(\frac{U_E}{\sqrt{\overline{u_E'^2}}}\right). \quad (7)$$

RESULTS AND DISCUSSION

Autocorrelation and power spectrum analysis

The autocorrelation coefficients and power spectrum of velocity had been determined. The autocorrelation coefficient curves and power spectrum curves of velocity at station 1 and 2 in east-west component and north-south component were shown in Figure 5 and Figure 6. Figure 7 and Figure 8 showed the curves of autocorrelation coefficient and power spectrum of fluctuation velocities.

The autocorrelation coefficient curves of velocity seemed to be cosine curve which had period about 12 hours 50 minutes. The autocorrelation coefficient curves of fluctuation velocity were decreased rapidly to zero. Power spectrum curves of velocity and fluctuation velocity had slope about $-\frac{5}{3}$ which satisfied the Kolmogoroff hypothesis (1941).

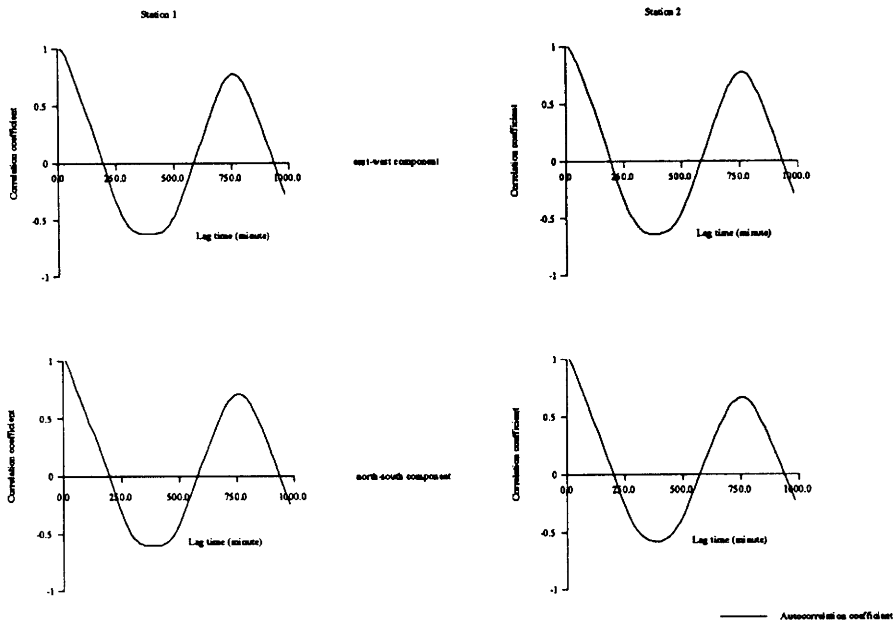


Figure 5 Autocorrelation coefficient curves of velocity in east-west component and north-south component at Kamano station 1 and 2 during May 1994.

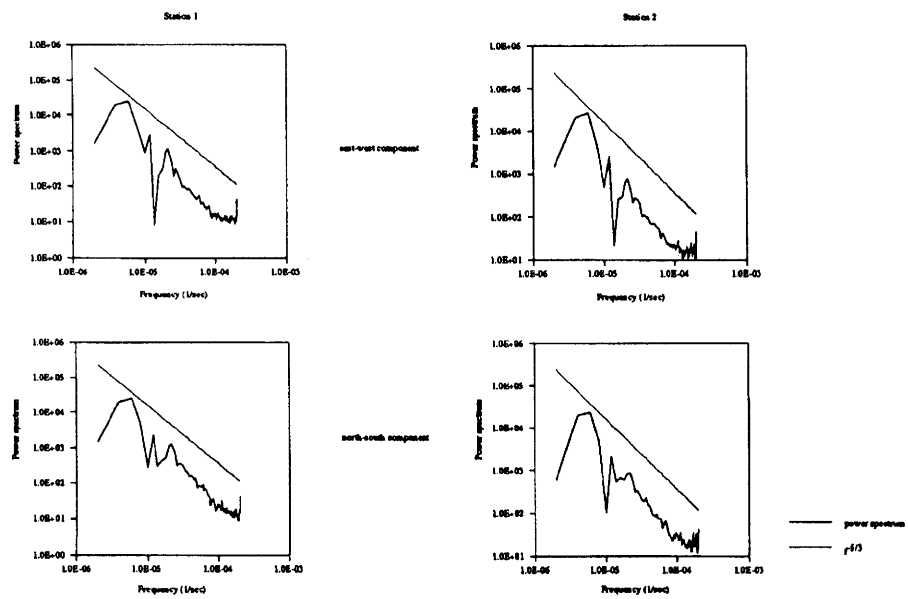


Figure 6 Power spectrum curves of velocity in east-west component and north-south component at Kamano station 1 and 2 during May 1994.

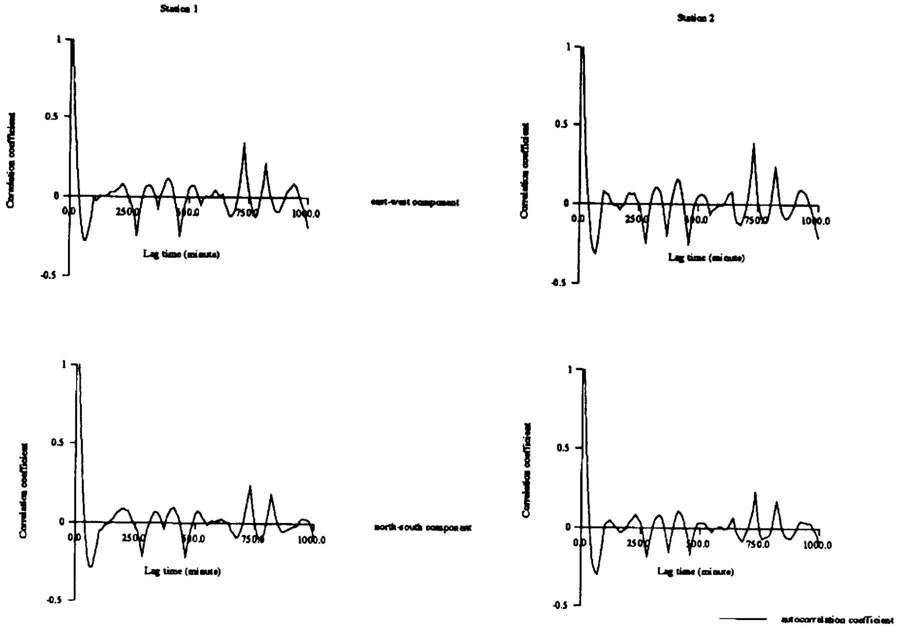


Figure 7 Autocorrelation coefficient curves of fluctuation velocity in east-west component and north-south component at Kamano station 1 and 2 during May 1994.

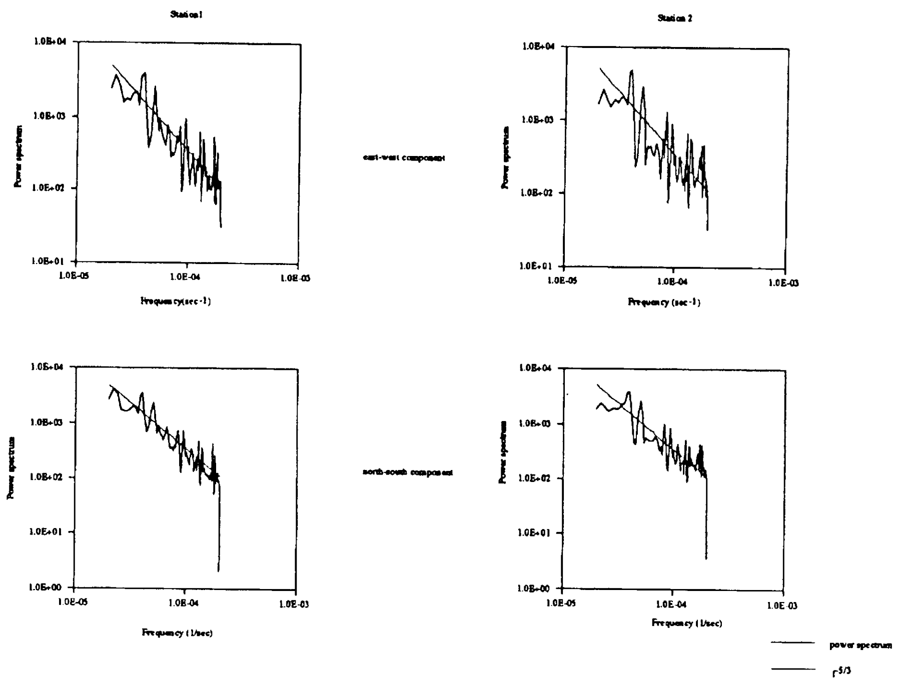


Figure 8 Power spectrum curves of fluctuation velocity in east-west component and north-south component at Kamano station 1 and 2 during May 1994.

Eddy viscosity coefficients

Since the eddy viscosity coefficients varied when M and the moving averages were changed. In each case, the moving averages were changed at 30 minutes, 60 minutes, 90 minutes and 120 minutes, respectively. Similarly M was changed from 12 to 20, 50, 100, 180, 300, and 600, respectively. The comparison of eddy viscosity coefficients with changing M and moving averages were shown in Figure 9 and Figure 10. The ranges of eddy diffusion coefficient were about 1.1×10^4 to 2.4×10^4 cm^2/sec in east-west component and about 7.6×10^3 to 1.6×10^4 cm^2/sec in north-south component. Suitable values of horizontal eddy viscosity coefficient in Kamano were about 1.4×10^4 to 2.0×10^4 cm^2/sec.

CONCLUSION

The maximum velocity during ebb tide was about 72 cm/sec and during flood tide was about 43 cm/sec. The residue flow in Kamano was east-westward flow. The north-southward flow was extremely small. One cycle of tide was about 12 hours 50 minutes.

The spectrum had peak near the frequency, $f = 6.7 \times 10^{-5}$Hz. Each spectrum had a slope of -5/

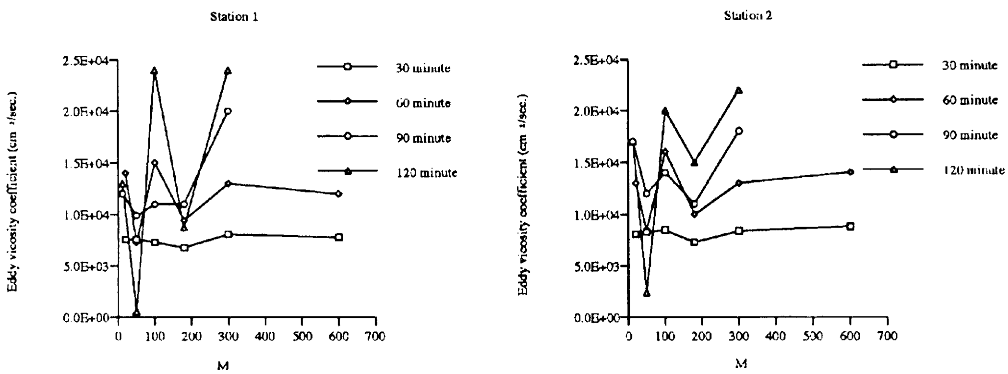


Figure 9 Compared values of Eddy vicosity coefficient in north-south component during May 1994 with changing M and moving average as 30, 60, 90 and 120 minutes at Kamano station 1 and 2.

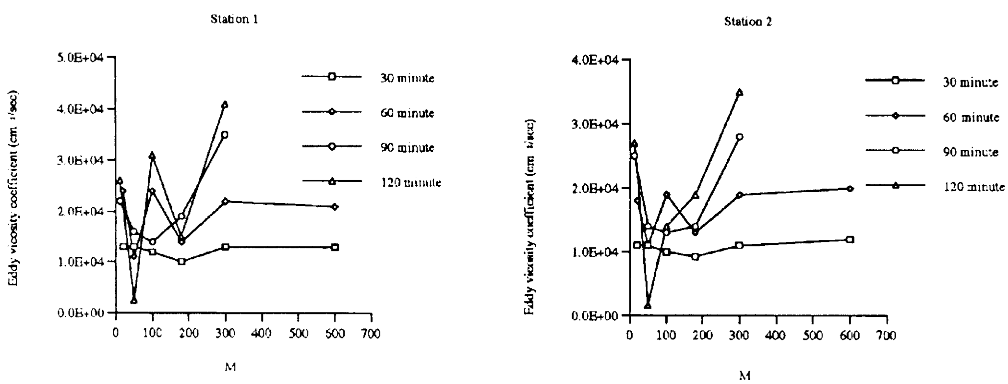


Figure 10 Compared values of Eddy vicosity coefficient in east-west component during May 1994 with changing M and moving average as 30, 60, 90 and 120 minutes at Kamano station 1 and 2.

3 for periods shorter than about 12 hours. The suitable value of eddy viscosity coefficient in Kamano was about 1.8×10^4 cm^2/sec (eastward flow) and about 1.2×10^4 cm^2/sec (northward flow).

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