

# Decision-Making Model for Public Facility Project Development Under Uncertainty

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## ABSTRACT

A water resource development project is studied with consideration to chronological structural change of water demand and supply, project cost, loss and penalty cost, time of implementation, time of service commencement, benefit, decision-making process as well as probability of relevant events. The Bayesian decision theory is adopted against uncertainty in planning data and it is revealed that both the single decision model and the continuous decision model give identical solution but the latter still indicates that project deferment as the optimum solution as being affected by supplementary information available during the decision-making processes.

**Key words:** decision-making process, Bayesian decision theory, demand and supply change, continuous decision model, supplementary information

## INTRODUCTION

Public construction projects enter an epoch of hardship nowadays. It is naturally learned that such projects cannot avoid but affect a large number of people and project managers, both directly and indirectly. Therefore, it is usually said that planning procedures of the project seem to consume longer time than its detailed design, construction or monitoring.

Fundamental problems of public facility project plannings having been studied by the author so far, e.g. water resource development projects, are (1) evaluation of functions and constraints to the long lead time, (2) possibility of occurrence of uncertainty and prediction of its effect on the project as well as its countermeasures. The mathematical model of decision-making related to the project adoption or postponement is, therefore, established under these conditions.

By the way, since various public facility projects are observed to be implemented after a series of repetitive and reluctant evaluation of overall benefit and loss among parties concerned, a mathematical model in this stage is needed to be developed. Consequently, the purpose in this paper is that, we

intend to expand the simple mathematical model, in other words, the decision-making problem on project postponement or implementation. If a water resource development project is selected as a case study, the following features and definitions have been naturally observed and suggested. (1) "Water demand that increases with time can immediately paused and water demand in the future becomes constant". This situation is defined as "the structural change". (2) The project can function in reality if it has been decided to implement and sufficient lead time is placed prior to actual water use. Again, this situation shall be reflected within the model. (3) The project implementation criterion shall be based on the expected (or average) benefit.

Such the prior model has already been proposed by Erlerkotter et al (Erlerkotter and Okada 1981) on the  $\theta$ -P curve (Figure 1) with use of only the structural change probability and the penalty cost at the time of decision-making (time 0). The model will be extended in this paper. That is, since public projects must be reevaluated and criticized among a wide range of beneficiaries, decision-making on the project shall not be limited only to time 0 as conventionally adopted but we need to introduce the Bayesian decision theory to formulate steps of information availability as well

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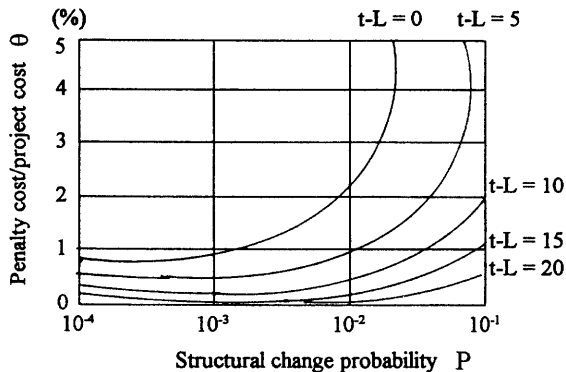


Figure 1 q-P curve.

as utilization until the final decision while updating data on judgment criteria with added-up information during any development interval. Again, in order to clarify our standpoint of analysis, this model is to be particularized by defining it as “the continuous decision model” which obeys the previous work of Shimizu (Shimizu, 1985).

## MATERIALS AND METHODS

Since this paper is proposed to offer methodology on decision-making related to the public facility project with data of water resource development project, the data have long been collected from a number of projects in the Royal Irrigation Department, Ministry of Agriculture and Cooperatives. That is, data on events and probability in the following tables and figures has already been adjusted for sake of model simplicity and nature.

We will establish necessary terminologies, symbols and scenario before discussing on the Bayesian decision model as follow.

(i) Time point. *Time zero* (0) is set as initial time to decide on project implementation or postponement and *Present time* ( $v_0$ ) as time to decide on project implementation or postponement.

(ii) Time interval. Here, *Total observation duration* ( $0 < v < v_0$ ) is duration from time zero to present time where data are available and *Unit observation interval* ( $v$ ) is duration from present time to the next decision-making time where the nature is unknown. Again, water demand trend on the unit observation duration at the present time (5 years in this situation) is observed to have the following two possibilities:

(iii) Natural state of event to occur which is

free from control ( $\theta$ ) shall be indirectly induced from the observation results to conclude on occurrence and the two situations can be considered as follow.  $\theta_1$ : Water demand continues to increase until the next observation time and  $\theta_2$ : Water demand pauses from the present time. In this situation, actions to be selected at present time shall be the following two scenario.

(iv) Action to be taken by the project developers ( $\alpha$ ) also include  $\alpha_1$ : The project is implemented from the present time, and  $\alpha_2$ : The project is postponed at the present time

The above natural state  $\theta$  governs the action  $\alpha$ , being selected to results in loss evaluation data as in Table -1. These data are illustrated with use of the loss functions as follow (Table 2).

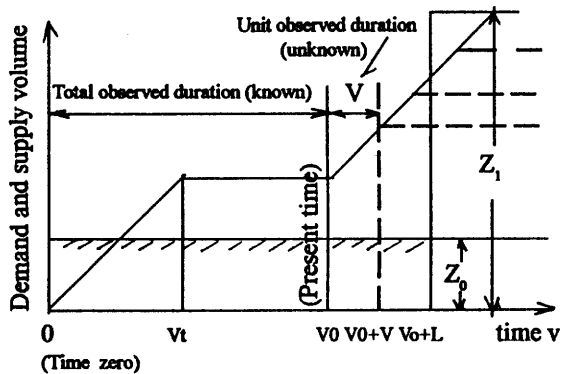
Table 1 Loss table.

State	Action	
	$\alpha_1$	$\alpha_2$
	$L(\theta_1, \alpha_1) = 616$	$L(\theta_1, \alpha_2) = 634$
	$L(\theta_2, \alpha_1) = 551$	$L(\theta_2, \alpha_2) = 58$

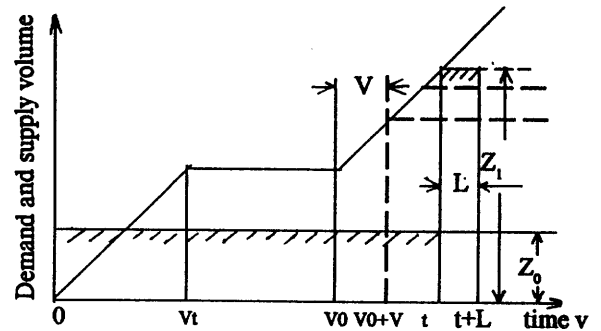
(v) Penalty related loss function with regard to action upon decision-making being selected for the project  $L(\theta, \alpha_i)$  is classified into four categories.  $L(\theta_1, \alpha_1)$ : Loss at state  $\theta_1$  if  $\alpha_1$  is selected (Figure 2-1).  $L(\theta_1, \alpha_2)$ : Loss at state  $\theta_2$  if  $\alpha_1$  is selected (Figure 2-2).  $L(\theta_2, \alpha_1)$ : Loss state  $\theta_2$  if  $\alpha_1$  is selected (Figure 2-3) and  $L(\theta_2, \alpha_2)$ : Loss at state  $\theta_2$  if  $\alpha_2$  is selected (Figure 2-4).

The decision-maker shall predict for the natural state  $\theta$  to some extents since it is unobvious. In this instance, the known and observed data measured at the entire interval until the present time shall be effective. Therefore, the following water demand patterns has been selected as the data on the natural state  $\theta$

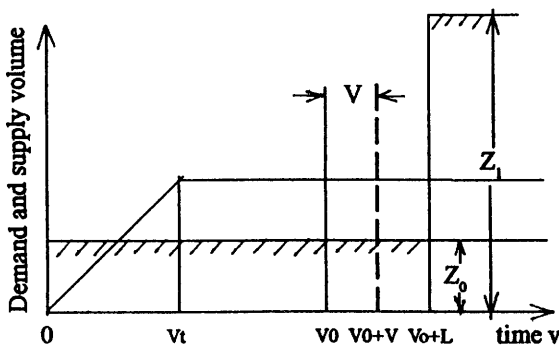
(vi) Data from actual observations and measurements ( $\xi$ ) are classified to four states, that is,  $\xi_1$ : Water demand pattern becomes constant immediately after time point 0,  $\xi_2$ : Water demand pattern becomes constant at the first half of the total observation duration until the present time,  $\xi_3$ : Water demand pattern becomes constant at the second half of the total observation duration until the present time, and,  $\xi_4$ : Water demand pattern increases continuously all over the total observation duration until the present time.



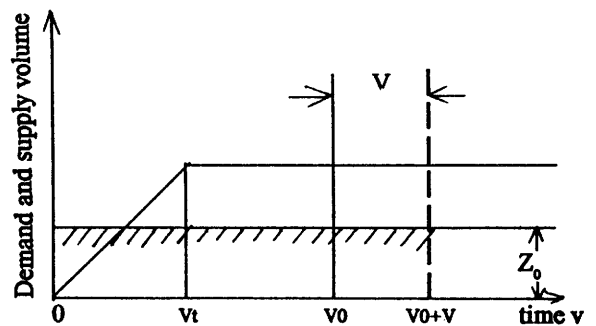
$$L(\theta_2, \alpha_1) = \int_0^\infty B_0(v) e^{-\alpha_1 v} dv + B_0(v_0) \int_{v_0}^\infty e^{-\alpha_1 v} dv + \int_{v_0}^{v_0+V} B_0(v - v_0 + v_0) e^{-\alpha_1 v} dv + \int_{v_0}^{v_0+V} B_0(v - v_0 + v_0) e^{-\alpha_1 v} dv + B_0(t - v_0 + v_0) \int_t^{t+L} e^{-\alpha_1 v} dv + B_1(t - v_0 + v_0) \int_t^{t+L} e^{-\alpha_1 v} dv + \int_t^{t+L} B_0(v - v_0 + v_0) e^{-\alpha_1 v} dv + \int_t^{t+L} B_1(v - v_0 + v_0) e^{-\alpha_1 v} dv + B_1(t - v_0 + v_0) \int_t^{t+L} e^{-\alpha_1 v} dv + C e^{-\alpha_1(t+L)} \quad (1)$$

Figure 2-1 Loss function  $L(\theta_2, \alpha_1)$ .

$$L(\theta_2, \alpha_2) = \int_0^\infty B_0(v) e^{-\alpha_2 v} dv + B_0(v_0) \int_{v_0}^\infty e^{-\alpha_2 v} dv + \int_{v_0}^{v_0+V} B_0(v - v_0 + v_0) e^{-\alpha_2 v} dv + \int_{v_0}^{v_0+V} B_0(v - v_0 + v_0) e^{-\alpha_2 v} dv + B_0(t - v_0 + v_0) \int_t^{t+L} e^{-\alpha_2 v} dv + B_1(t - v_0 + v_0) \int_t^{t+L} e^{-\alpha_2 v} dv + \int_t^{t+L} B_0(v - v_0 + v_0) e^{-\alpha_2 v} dv + \int_t^{t+L} B_1(v - v_0 + v_0) e^{-\alpha_2 v} dv + B_1(t - v_0 + v_0) \int_t^{t+L} e^{-\alpha_2 v} dv + C e^{-\alpha_2(t+L)} \quad (2)$$

Figure 2-2 Loss function  $L(\theta_2, \alpha_2)$ .

$$L(\theta_2, \alpha_1) = \int_0^\infty B_0(v) e^{-\alpha_1 v} dv + B_0(v_0) \int_{v_0}^\infty e^{-\alpha_1 v} dv + B_1(v_0) \int_{v_0}^\infty e^{-\alpha_1 v} dv + C e^{-\alpha_1(t+L)} \quad (3)$$

Figure 2-3 Loss function  $L(\theta_2, \alpha_1)$ .

$$L(\theta_2, \alpha_2) = \int_0^\infty B_0(v) e^{-\alpha_2 v} dv + B_0(v_0) \int_{v_0}^\infty e^{-\alpha_2 v} dv \quad (4)$$

Figure 2-4 Loss function  $L(\theta_2, \alpha_2)$ .

**Table 2** Symbol for loss function.

$v$	: Arbitrary time ( $v \geq 0$ )
$t$	: Structural change timing ( $t \geq 0$ )
$B_0(v)$	: Benefit at time $v$ if the project is not implemented.
$B_1(v)$	: Benefit at time $v$ if the project is implemented.
$r$	: Time discount rate
$re^{-rt}$	: Probability density function related to structural change
$L$	: Lead time
$C$	: Project cost
$t+L$	: Water demand constraint duration in total observation time until the present time
$v_t$	: Time that water demand becomes constant until the present observation time

(Vii) Probability of response  $P\theta(\delta)$  is used to explain that the above data  $\zeta$  are not certain to occur to any decision-makers. Then, although various factors can explain their behavior, this situation is considered as a series of probability. That is, for simplicity, these data are concluded to be observed with experience to appear as the natural state  $\zeta$  at certain probability (Table 3).

(viii) Determination equation (d) is used in the final stage of planning because these are obligated to decide on the behavior of project development, e.g. to implement immediately or to defer or to quit. Yet, it takes account of information on benefit, loss as well as results from structural change of water demand and supply pattern.

By all means, action  $\alpha$  under available data is determined. Hence, the action patterns to be undertaken by the decision-makers against data  $\zeta$  can be listed as equations (Table 4).

## METHOD OF ANALYSIS

## 1 No Data Problem in Bayesian Decision Theory

In this section, it is observed that “no data problem (unavailable information)” is a nature which the decision-maker must experience. Some events for prior decision-makings must be assumed as follow:

**Table 3** Response probability.

		Data			
		$\xi_1$	$\xi_2$	$\xi_3$	$\xi_4$
Natural	$\theta_1$	$P\theta_1(\xi_1)=0$	$P\theta_1(\xi_2)=1/100$	$P\theta_1(\xi_3)=9/100$	$P\theta_1(\xi_4)=9/10$
State	$\theta_2$	$P\theta_2(\xi_1)=6/10$	$P\theta_2(\xi_2)=3/10$	$P\theta_2(\xi_3)=9/100$	$P\theta_2(\xi_4)=1/100$

**Table 4 Decision method.**[illegible]

(1) It is assumed that the prior probability against natural state  $\theta$  are  $1-\omega_2$  and  $\omega_2$

( $\omega_2$  = probability of structural change), respectively.

(2) Loos at state  $\theta$  when action  $\alpha$  is selected will be as shown in table -1.

(3) It is unobvious about occurrence of natural state  $\theta_1$  or  $\theta_2$ . Then, each prior probability are considered and calculated for the expected prior loss against action  $\alpha_i$  as follows:

$$EL_i = (1-\omega_2)L(\theta_1, \alpha_i) + \omega_2 L(\theta_2, \alpha_i) \quad (i=1,2)$$

Here, if action  $\alpha_i^*$  is adopted at  $EL_i^* = \text{Min} \{EL_i\}$ , it shall be selected as optimum.

(4) Figure 3 is illustrated from equations in (3).  $T(\alpha_1)$  and  $T(\alpha_2)$  are the prior probability in selecting of action  $\alpha_1$  and  $\alpha_2$  acceptably.

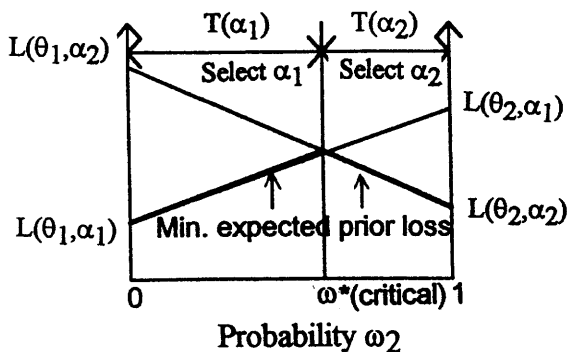


Figure 3 Expected prior loss against action  $\alpha_1, \alpha_2$ .

(5) Prior probability at the no data problem is equivalent to probability of structural change at the single decision model. If this probability is within  $T(\alpha_1)$  and  $T(\alpha_2)$ , the action to minimize loss will be selected, respectively.

## 2 Bayesian Decision Equation

(1) We use probability distribution ( $P\theta_i(\xi)$ ) of data  $\xi$  at the natural state  $\theta_i (i=1,2)$  and prior probability  $\omega$  to find the posterior probability  $\omega_i(\xi)$  from Bayesian theory as follow:

$$\omega_i'(\xi) = \frac{\omega_i P\theta_i(\xi)}{\omega_1 P\theta_1(\xi) + \omega_2 P\theta_2(\xi)} \quad \dots\dots(i=1,2)$$

(2) The posterior probability  $\omega_i'(\xi)$  against data  $\xi$  observed all over the duration is substituted in the loss function  $L(\theta_i, \alpha_i)$  to formulate the new loss table.

(3) Prior probability  $\omega$  against lack of data will be replaced with  $\omega'$  and repeated as in 3.1 (3), 3.1(4) and 3.1 (5).

## RESULTS

The input data in this situation are adopted with standard data as in table 5. The single decision model and the continuous decision model are compared by using (1) probability of structural change, and, (2) different patterns of penalty cost. Table 6 and table 7 show results in these situations, that is, both the single decision model and the continuous (posterior) decision model indicate optimum solutions to defer or to start the project.

## DISCUSSIONS

In case the single decision model shows that starting the water resource development project at the present time is plausible, the continuous model still points out that the project is better to postpone until sometime in the future. The solution is worked out because data of  $P, \xi, \delta, \alpha$  and  $B_0, B_1$  have been taken into account continuously in a series of decision-makings which is, of course, different from those use of one set of data in the single model even under the similar basis of project cost and benefit.

It is hereby verified that a number of available information during the project planning procedures should still determine degree of project feasibility differently if Bayesian decision theory is applied which, in fact, is the matter being naturally experienced by decision-makers in many ranks of the government or the project entities. For instance, at penalty=3.4 million baht (0.5%) in table 7, the project is better to defer while the single decision model indicates to postpone until 7-8 years later.

**Table 5 Standard data.**

a	: Ratio of idle penalty to deficitly penalty	0.1
r	: Time discount rate	0.06
L	: Lead time	15 (years)
$\phi_0$	: Fundamental effective capacity	5 (non-dimension)
$\phi_1$	: Extended effective capacity	48 (non-dimension)
$\Delta$	: Annual demand increase	1.0 (1/year)
C	: Project cost	680 ( $10^6$ baht)
$\lambda$	: Average occurrence frequency of structural change per year	$\lambda = \ln 1/(1-\eta)$
$\eta$	: Probability that structural change occurs even in the successive year	$\theta = 0.01$
(Benefit function)		
$B_0(v) = P_{\max} \{0, \Delta v - \phi_1\} - aP_{\min} \{0, \Delta v - \phi_0\}$		
$B_1(v) = P_{\max} \{0, \Delta v - \phi_1\} - aP_{\min} \{0, \Delta v - \phi_1\}$		

**CONCLUSION**

Above is the example of analytic results based on a series of data with regard to water resource development project in this country. Using the continuous decision model, the followings will be summarized.

1 Project cost, change in demand structure as well as other penalty are reflected in the continuous model instead of the single model.

2 Bayesian decision theory is applied with the model to exploit available project information.

3 It is discovered that when supplementary information on the project is adopted, decision to implement will give various solutions that offer more rational decision on the public oriented project.

By all means, it is sufficient to say that this model is sufficiently effective to determine on commencement timing of large scale projects under uncertain environment based on physical, economical or social reasons.

**Table 6 Results of occurrence probability on structural change.**

Occurrence probability of structural change $\theta$		0.0001	0.01	0.1
Single decision model		Start at $t = 0$	2-3 year delay	10 year delay
No data problem		$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_2$ (defer)
Posterior decision		$\delta_{13}$	$\delta_{13}$	$\delta_{15}$
Observation data	$\xi_1$ (pending)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)
	$\xi_2$ (first half)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)
	$\xi_3$ (second half)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_2$ (start)
	$\xi_4$ (increase)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)

**Table 7 Results of penalty cost.**

Penalty P		3.4(0.5%)	6.8(1%)	13.6(2%)	27.2(4%)	34.0(5%)
Single decision model		7-8 year defer	2-3 year defer	Start at $t = 0$	Start at $t = 0$	Start at $t = 0$
No data problem		$\alpha_2$ (defer)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)
posterior decision		$\delta_{16}$	$\delta_{13}$	$\delta_{13}$	$\delta_{13}$	$\delta_{13}$
Observation data	$\xi_1$ (pending)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)
	$\xi_2$ (first half)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)	$\alpha_2$ (defer)
	$\xi_3$ (second half)	$\alpha_2$ (defer)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)
	$\xi_4$ (increase)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)	$\alpha_1$ (start)

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## ใบรับรอง

ข้าพเจ้า นาย/นาง/นางสาว.....  
 ตำแหน่ง.....สถานที่ทำงาน.....  
 จังหวัด.....รหัสไปรษณีย์.....ขอรับรองว่า นาย/นาง/นางสาว  
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(.....)

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