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Passivity-based Proportional Integral Tuning Method for a Simple Heat Exchanger Network

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ABSTRACT

A more realistic heat exchanger model was developed by implementing a bypass control loop. Under the decentralized unconditional stability (DUS) condition, proportional integral (PI) tuning parameters were designed based on the passivity concept to maintain closed-loop stability. The controller's performance was tested with a simple heat exchanger network (HEN) under various inlet flow rates of hot stream using an Aspen Dynamics simulator.

The results indicated that the proposed controller can reject the disturbance and give faster and better setpoint tracking than conventional PI controllers based on the Ziegler-Nichols method. In addition, this proposed controller could handle the entire process at the target setpoint even with sluggishness in one control loop whereas the conventional controller could not operate properly and all responses were too slow.

Keywords: heat exchanger networks, passivity theorem, closed-loop stability, fault tolerance

INTRODUCTION

A heat exchanger network (HEN) plays an important role in dealing with energy recovery in many chemical process industries (Jäschke and Skogestad, 2012). Generally, a HEN consists of a number of heat exchangers where the hot process streams are integrated with the cold process streams in order to achieve the highest energy recovery. The energy integration introduces interactions, and may make the process more difficult to control and operate (Mathisen, 1994). Interactions and other effects such as disturbance or setpoint variations, which are normally encountered in the HEN operations, may deteriorate the performance and also may lead to the instability of the HEN which, thus, results in undesirable process temperatures. Consequently, the control of a HEN is of interest. Although work on HENs have been published over the last two decades (Furman and Sahinidis, 2002), it has not focused on control but has concentrated mostly on steady state optimal design and the operation of the HEN.

State space has been commonly used to describe the dynamic behavior of the system (Ogata, 1967) and was adopted throughout this work. A state space equation is a set of equations that describes the unique relations between the input, output and state (Chen, 1984). In practice, models of HEN are formidable. Many works

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(Mathisen, 1994; Varga *et al*., 1995; Hangos and Cameron, 2001) were concerned only with a single heat exchanger which is not realistic in industrial processes that regularly combine a bypass loop with the heat exchanger. Glemmestad *et al*. (1996) also mentioned that the bypasses should be manipulated in order to maintain the target temperatures and obtain the optimal operation conditions of a HEN.

As mentioned before, the stability of the HEN is indispensable in an energy integration process. All of the HEN may fail if one heat exchanger fails, Therefore, the stability analysis of these systems is important. One of the most useful techniques to analyze the stability of a process is the passivity theorem (Van Der Schaft, 2000) which states that the closed-loop system of a strictly passive system combined with the feedback of another passive system is stable without necessarily satisfying the small gain condition. Therefore, the passivity theorem provides another approach to robust control which may be less conservative than the small-gain based approaches (Bao, 1998). Moreover, the stability analysis of a HEN using the passivity theorem has not been researched yet. Bao and Lee (2007) found that a single heat exchanger is inherently passive because the passivity condition is valid for all design parameters, types of fluid and operating conditions. However, when a heat exchanger is accompanied with a bypass which is commonly found in many chemical process industries, for example, Glemmestad *et al*. (1996), Westhalen *et al.* (2003) and Escobar and Trierweiler (2009), it may produce some different results from Bao and Lee (2007).

The objective of this paper was to develop a single heat exchanger model by incorporating the bypass term. In addition, a design proportional integral (PI) controller based on the passivity theorem was proposed and tested with a simple HEN example.

MATERIALS AND METHODS

Heat exchanger model

The basic dynamic equations of a countercurrent configuration heat exchanger based on an ideal mixing tank are presented in Equations 1 and 2:

$$\rho_H V_H C_{pH} \frac{dT_{1H}}{dt} = \rho_H (1 - f_H) F_H C_{pH} (T_{Hin} - T_{1H}) + UA(T_{1C} - T_{1H}) \quad (1)$$

$$\rho_C V_C C_{pC} \frac{dT_{1C}}{dt} = \rho_H F_C C_{pC} (T_{Cin} - T_{1C}) + UA(T_{1H} - T_{1C}) \quad (2)$$

where this system has two state variables; outlet cold and hot temperature (T_{1H}, T_{1C}), and two manipulated variables; cold flowrates F_C and the bypass fraction on the hot side f_H by assuming a constant hot flowrate F_H.

Consider the output hot temperature after the split stream from the inlet and the exchanged stream from the exchanger are mixed. The mixed temperature equation on the hot side is presented in Equation 3:

$$T_H = (1 - f_H) T_{1H} + f_H T_{Hin} \quad (3)$$

In this work, the lower and upper bounds of heat exchanged are shown in Equations 4 and 5 respectively. At the upper bound, thermal efficiency (P_H) and a heat capacity flow rate are assumed at constant value.

$$-Q \leq 0 \quad (4)$$

$$Q \leq P_H(\rho_H V_H C_{pH})(T_{Hin} - T_{Cin}) \quad (5)$$

Passivity concept

Passivity is a property of the system which is used in a variety of engineering disciplines and it is frequently used to design stable control systems or to show stability in control systems. It is especially important in the design of large, complex control systems.

The implementation of the passivity concept can be summarized by Figure 1. First, the transfer function for a heat exchanger with a bypass is found by applying process modeling techniques. The combination of control loops around the heat exchanger is an arbitrary pairing designed using the transfer functions. Therefore, the application of the passivity theorem is introduced as a means of finding the best controller loop pairings. Non-passivity systems can be driven into the passive region by introducing a weighting function. After the system is passive, the decentralized unconditional stability (DUS) system for PI controller tuning is implemented and all of the parameters are calculated.

1. Passivity index

The passivity index measures how far a system of interest is from being passive and is defined as a frequency-dependent function by Equation 6 (Wen, 1988):

$$v\left(\mathbf{G}(s),\omega\right) = -\lambda_{\min}\left(\frac{1}{2}\left[\mathbf{G}(j\omega)+\mathbf{G}^{*}(j\omega)\right]\right) \quad (6)$$

where v is passivity index, $\mathbf{G}(s)$ is the transfer function of the process, $\mathbf{G}^{*}(s)$ is the conjugate transpose transfer function of the process, λ_{min} is

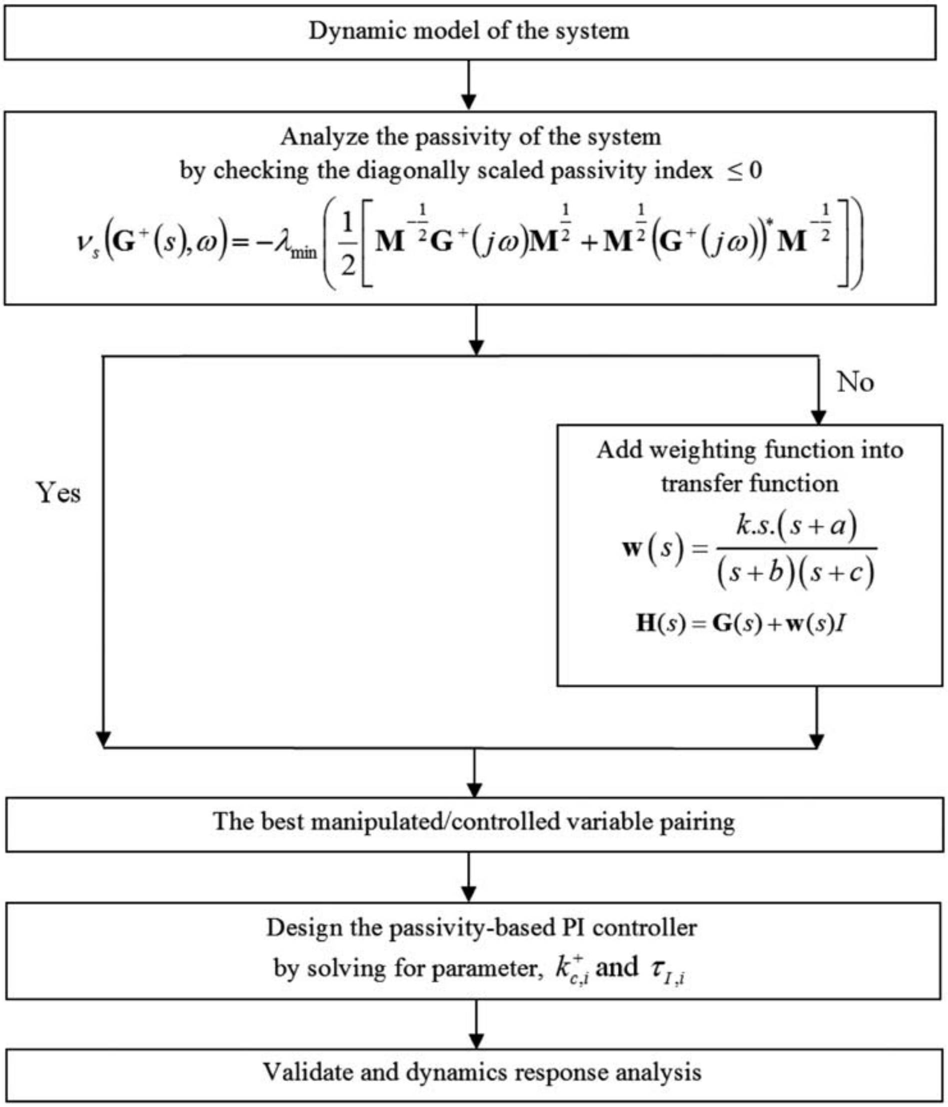


Figure 1 Methodology of process control design based on passivity technique.

the minimum eigenvalue and ω is the frequency. The passivity index can be made less conservative (called the diagonally scaled passivity index) as shown by Equation 7 (Bao *et al*., 2000);

$$v_s\left(\mathbf{G}^+(s),\omega\right) = -\lambda_{\min}\left(\frac{1}{2}\left[\mathbf{M}^{-\frac{1}{2}}\mathbf{G}^+(j\omega)\mathbf{M}^{\frac{1}{2}} + \mathbf{M}^{\frac{1}{2}}\left(\mathbf{G}^+(j\omega)\right)^*\mathbf{M}^{-\frac{1}{2}}\right]\right) \quad (7)$$

where v_s is the diagonal scaling passivity index, $\mathbf{G}^+(s)$ is equal to $\mathbf{G}(s)\mathbf{U}$, $\mathbf{U}$ is a diagonal matrix with either 1 or -1 and the signs of each element are determined such that the diagonal of $\mathbf{G}^+(s)$ is positive. For each frequency (ω), a diagonal and real matrix of $\mathbf{M}(\omega)$ can be obtained by solving Equations 8 to 10:

$$\min_M \{t\} \quad (8)$$

subject to:

$$\begin{bmatrix} -\mathbf{X}(\omega)\mathbf{M} - \mathbf{M}\mathbf{X}^{\mathbf{T}}(\omega) & \mathbf{Y}(\omega)\mathbf{M} - \mathbf{M}\mathbf{X}^{\mathbf{T}}(\omega) \\ -\mathbf{Y}(\omega)\mathbf{M} + \mathbf{M}\mathbf{Y}^{\mathbf{T}}(\omega) & -\mathbf{X}(\omega)\mathbf{M} - \mathbf{M}\mathbf{X}^{\mathbf{T}}(\omega) \end{bmatrix} < t\begin{bmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{M} \end{bmatrix} \quad (9)$$

$\mathbf{M}$ is diagonal and $\mathbf{M} > 0$ (10)

where t is time, $\mathbf{X}$ is the space of the state variable and $\mathbf{Y}$ is the space of the output variable.

Decentralized integral controllability (DIC) is another property of a plant. DIC analysis determines whether a multivariable process can be stabilized by multi-loop controllers, that is, whether the controller can have integral action to ensure a zero steady-state error. If a system is subject to DIC, then it is possible to achieve stability and to offset-free control by tuning every loop separately. For this work, DIC was used to screen out unworkable manipulated/controlled variable pairings based on the conditions developed by Morari and Zafiriou (1989) who state that an $m \times m$ linear time invariant stable process $\mathbf{G}(s)$ is DIC if and only if $\lambda_{ii}(\mathbf{G}(0)) \geq 0, \forall i = 1,...,m$ where $\lambda_{ii}(\mathbf{G}(0))$ is the i^{th} diagonal element of the relative gain array (**RGA**) matrix. **RGA** is an analytical tool used to determine the optimal variable pairings for a multi-input-multi-output system. Equations 11 and 12 show the formulation of the **RGA** using the steady-state gain matrix.

$$\mathbf{y} = \mathbf{G}\mathbf{u} \quad (11)$$

$$\mathbf{RGA} = \mathbf{G}(0)\left(\mathbf{G}(0)^{-1}\right)^{\mathrm{T}} = \begin{bmatrix} \lambda_{11} & \lambda_{11} & \cdots & \lambda_{1m} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2m} \\ \vdots & & & \\ \lambda_{m1} & \lambda_{m2} & \cdots & \lambda_{mm} \end{bmatrix} \quad (12)$$

where $\mathbf{y}$ is a vector of the controlled variables, $\mathbf{u}$ is a vector of the manipulated variables, $\mathbf{G}$ is the gain matrix and $\mathbf{G}(0)$ is the steady-state gain matrix.

Therefore, the passivity index profile can be obtained using the following steps for each pairing scheme (Bao and Lee, 2007):

1) Determine the transfer function $\mathbf{G}(s)$ for each possible pairing scheme.

2) Screen out the non-DIC pairing schemes by using the necessary DIC condition.

3) Find the sign matrix $\mathbf{U}$ and obtain $\mathbf{G}^+(s)$ such that $\mathbf{G}^+_{\mathbf{ii}}(0) > 0$ $(i = 1,...,m)$.

4) Calculate the diagonally scaled passivity index $v_s\left(\mathrm{G}^+(s),\omega\right)$ at a number of frequency points and compare the passivity index profiles of different pairings. The best pairing should correspond to the one with the largest frequency bandwidth ω_b such that $v_s\left(\mathbf{G}^+(s),\omega\right) \leq 0$ for any $\omega \in [0,\omega_b]$. This pairing scheme would present integral action and a fast dynamic response.

2. Weighting function

According to the passivity concept, this non-passive process can be shifted to the passive regions by adding a weighting function $\mathbf{w}(s)$. The process transfer function after absorbing the weighting function is called $\mathbf{H}$(s) as in Equation 13:

$$\mathbf{H}(s) = \mathbf{G}(s) + \mathbf{w}(s)I \quad (13)$$

The weighting function has the form shown in Equation 14:

$$\mathbf{w}(s)=\frac{k.s.(s+a)}{(s+b)(s+c)} \qquad (14)$$

in which the parameters *a, b, c* and *k* as decision variables can be obtained from solving Equations 15 and 16:

$$\min_{a,b,c,k} \sum_{i=1}^{m}\left(\mathrm{Re}\left(\mathbf{w}(j\omega_i)\right)\upsilon_s\left(\mathbf{G}^+(s),\omega_i\right)\right)^2 \qquad (15)$$

subject to:

$$\mathrm{Re}\left(\mathbf{w}(j\omega_i)\right)>\upsilon_s\left(\mathbf{G}^+(s),\omega_i\right),\quad \forall i=1,\ldots,m \qquad (16)$$

where Re(**w**) is the real part of the weighting function.

Passivity-based proportional integral controller design

A multi-loop PI controller can be tuned based on the proposed stability conditions. To achieve DUS of the closed-loop system as well as providing good control performance, a controller tuning method was introduced to minimize the sensitivity function of each loop. For decentralized PI controller synthesis, this tuning problem was converted into Equations 17 to 19 (Bao *et al*., 2002):

$$\min_{k_{c,i}^+,\tau_{I,i}} \left(-\gamma_i\right) \qquad (17)$$

subject to:

$$\left|\frac{\mathbf{w_i}(j\omega)\gamma_i}{1+\mathbf{G_{ii}^+}(j\omega)k_{c,i}^+\left[1+\dfrac{1}{\tau_{I,i}\times j\omega}\right]}\right|<1 \qquad (18)$$

and:

$$\tau_{I,i}^2\geq\frac{k_{c,i}^+v_s(\omega)}{\left[1-k_{c,i}^+v_s(\omega)\right]\omega^2}\qquad \forall\omega\in\mathrm{R},\ i=1,\ldots,n \qquad (19)$$

where γ is the sensitivity function, k_c^+ is the proportional gain of the PI controller and τ_I is the time integral of the PI controller.

For a given stable process **G**(s), a multi-loop PI controller can be obtained by the design procedure as follows.

1) Determine the pairing scheme for controlled and manipulated variables according to the procedure to find the best pairing in the passivity-based pairing section.

2) For each subsystem $\mathbf{G}_{ii}^+(s)$ (i = 1,…,m), solve for $k_{c,i}^+$ and $\tau_{I,i}$.

RESULTS AND DISCUSSION

Considering the extension of the heat exchanger models to a simple HEN, the passivity theorem can be implemented to propose the robust control configuration following the procedure in Figure 1. The HEN example used in this work is from Glemmestad *et al*. (1996) as shown in Figure 2.

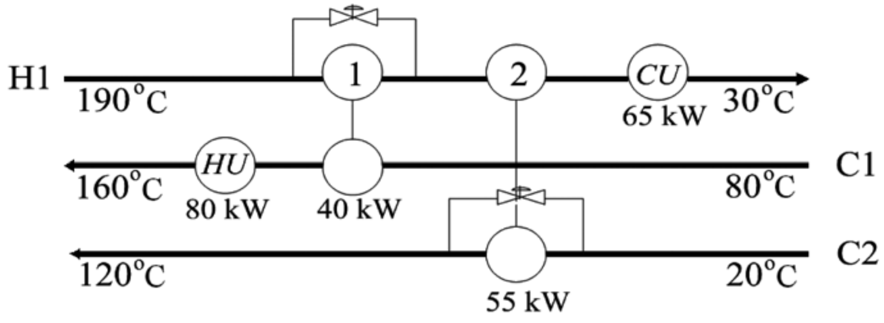


Figure 2 Grid diagram of a heat exchanger network from Glemmestad *et al*. (1996). (HU = Heat utility; CU = Cold utility; H1 = Hot stream 1; C1, C2 = Cold stream 1 and 2, respectively).

The HEN example contains two process exchangers and two utilities. There are three target temperatures which are the outlet temperatures of streams H1, C1 and C2. The manipulations are bypasses of exchangers 1 and 2 and the flow rate of the cooler and heater. It was designed by the pinch method (Glemmestad *et al*., 1996) and the minimum temperature approach was 20 ºC.

This network was considered in two groups, being firstly the two process exchangers and secondly the two utilities. Then, the following passivity-based decentralized controller synthesis procedure was started.

The state space for the two process exchangers is shown in Equations 20 and 21. Exchanger number 1 has a single bypass on the hot side and the other has a single bypass on the cold side.

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{22} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{B}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{22} \end{bmatrix} \mathbf{u} \tag{20}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{22} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{D}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{22} \end{bmatrix} \mathbf{u} \tag{21}$$

where $\mathbf{x} = \left[T_{1C}\ T_{1H}\ T_{2C}\ T_{2H} \right]^T$,

$\mathbf{u} = \left[F_{C1}\ f_{H1}\ f_{C2}\ F_{H2} \right]^T$

$\mathbf{y} = \left[T_{1C1}\ T_{1H1}\ T_{2C2}\ T_{2H2} \right]^T$ and the partitioned matrices are shown in Equation 22:

$$\mathbf{A}_{11} \equiv \begin{bmatrix} \dfrac{-U_1A_1 - \tau_{C1}\bar{F}_{C1}}{\xi_{C1}} & \dfrac{U_1A_1}{\xi_{C1}} \\ \dfrac{U_1A_1}{\xi_{H1}} & \dfrac{-U_1A_1 + \tau_{H1}F_{H1}\bar{f}_{H1} - \tau_{H1}F_{H1}}{\xi_{H1}} \end{bmatrix},$$

$$\mathbf{A}_{22} \equiv \begin{bmatrix} \dfrac{-U_2A_2 + \tau_{C2}F_{C2}\bar{f}_{C2} - \tau_{C2}F_{C2}}{\xi_{C2}} & \dfrac{U_2A_2}{\xi_{C2}} \\ \dfrac{U_2A_2}{\xi_{H2}} & \dfrac{-U_2A_2 - \tau_{H2}\bar{F}_{H2}}{\xi_{H2}} \end{bmatrix}$$

$$\mathbf{B}_{11} \equiv \begin{bmatrix} \dfrac{\tau_{C1}T_{Cin1} - \tau_{C1}\bar{T}_{1C1}}{\xi_{C1}} & 0 \\ 0 & \dfrac{\tau_{H1}F_{H1}\bar{T}_{1H1} - \tau_{H1}F_{H1}T_{Hin1}}{\xi_{H1}} \end{bmatrix},$$

$$\mathbf{B}_{22} \equiv \begin{bmatrix} \dfrac{\tau_{C2}F_{C2}\bar{T}_{1C2} - \tau_{C2}F_{C2}T_{Cin2}}{\xi_{C2}} & 0 \\ 0 & \dfrac{\tau_{H2}T_{Hin2} - \tau_{H2}\bar{T}_{1H2}}{\xi_{H2}} \end{bmatrix}$$

$$\mathbf{C}_{11} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1-\bar{f}_H \end{bmatrix} \mathbf{C}_{22} \equiv \begin{bmatrix} 1-\bar{f}_C & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{D}_{11} \equiv \begin{bmatrix} 0 & 0 \\ 0 & T_{Hin} - \bar{T}_{1H} \end{bmatrix}, \mathbf{D}_{22} \equiv \begin{bmatrix} T_{Cin} - \bar{T}_{1C} & 0 \\ 0 & 0 \end{bmatrix} \tag{22}$$

where $\tau_i = \rho_i C_{Pi}$ and $\xi_i = \rho_i V_i C_{Pi}$.

The steady-state results after simulation in the Aspen Plus software (Version 7; Aspen Technology Inc.; Burlington, MA, USA) are shown in Table 1.

Four possible pairing schemes of two heat exchangers in the networks are classified in Table 2.

Table 1 Steady-state results of heat exchanger network using the Aspen Plus simulator.

Parameter	Symbol	Value
Output hot temperature of HE 1 (ºC)	T_{1H1}	155.9
Output hot temperature of HE 2 (ºC)	T_{2H1}	106.9
Output cold temperature of HE 1 (ºC)	T_{1C1}	104.4
Heat exchanger area of HE 1 (m^2)	A_1	0.617
Heat exchanger area of HE 2 (m^2)	A_2	1.14
Overall heat transfer coefficient of HE 1 ($kW.m^{-2}$ ºC)	U_1	0.85
Overall heat transfer coefficient of HE 2 ($kW.m^{-2}$ ºC)	U_2	0.85

HE = Heat exchange.

For definition of the terms, see Equations 21 and 22 in the text.

Table 2 Possible pairing schemes of two heat exchangers in the heat exchanger network example.

Pairing scheme	Pairing	Manipulated variable (MV)	Controlled variable (CV)
(1) 1-1/2-2/3-3/4-4	1-1	F_{C1}	T_{1C1}
	2-2	f_{H1}	T_{1H1}
	3-3	f_{C2}	T_{2C2}
	4-4	F_{H2}	T_{2H2}
(2) 1-2/2-1/3-3/4-4	1-2	F_{C1}	T_{1H1}
	2-1	f_{H1}	T_{1C1}
	3-3	f_{C2}	T_{2C2}
	4-4	F_{H2}	T_{2H2}
(3) 1-1/2-2/3-4/4-3	1-1	F_{C1}	T_{1C1}
	2-2	f_{H1}	T_{1H1}
	3-4	f_{C2}	T_{2H2}
	4-3	F_{H2}	T_{2C2}
(4) 1-2/2-1/3-4/4-3	1-2	F_{C1}	T_{1H1}
	2-1	f_{H1}	T_{1C1}
	3-4	f_{C2}	T_{2H2}
	4-3	F_{H2}	T_{2C2}

For definition of the terms, see Equations 20 and 21in the text.

For each pairing scheme, the transfer functions $\mathbf{G}(s)$ were analyzed to determined the non-DIC pairing schemes by using the necessary DIC condition. After applying the DIC conditions, it was shown that all pairing schemes satisfied this condition. Then, the sign matrix **U** was determined. The diagonally scaled passivity index of each pairing scheme was calculated and plotted as shown in Figure 3.

Comparing the diagonally scaled passivity index for all options, pairing scheme 1 (1-1/2-2/3-3/4-4) was the most passive. Hence, pairing scheme 1 is the best for these two heat exchangers in the HEN. Equations 23 and 24 show $\mathbf{G}(0)$ and the sign matrix **U** of pairing scheme 1.

$$\mathbf{G}(0) = \begin{bmatrix} -35.75 & -14.84 & 0 & 0 \\ -12.29 & 6.66 & 0 & 0 \\ 0 & 0 & -50.17 & 28.26 \\ 0 & 0 & 25.15 & 42.81 \end{bmatrix} \quad (23)$$

$$\mathbf{U} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (24)$$

After the best pairing scheme had been chosen, the passivity based PI tuning parameters for each loop of the two heat exchangers in a HEN were found through Equations 17 to 19. The performance of the passivity based PI controller was considered by comparison with a conventional controller calculated using the Ziegler-Nichols method—one of the most common classical controller tuning methods. Table 3 shows their tuning parameters.

The state spaces for the two utilities are shown in Equations 25 and 26:

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{A_{33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{A_{44}} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{B_{33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{B_{44}} \end{bmatrix} \mathbf{u} \quad (25)$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{C_{33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{C_{44}} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{D_{33}} & \mathbf{0} \\ \mathbf{0} & \mathbf{D_{44}} \end{bmatrix} \mathbf{u} \quad (26)$$

where $\mathbf{x} = \left[T_{3C}\ T_{3H}\ T_{4C}\ T_{4H} \right]^T$,

$\mathbf{u} = \left[F_{3C}\ F_{3H}\ F_{4C}\ F_{4H} \right]^T$

$\mathbf{y} = \left[T_{3C3}\ T_{3H3}\ T_{4C4}\ T_{4H4} \right]^T$ and the partitioned matrices are shown in Equation 27:

$$\mathbf{A_{33}} \equiv \begin{bmatrix} \dfrac{-U_3A_3 - \tau_{C3}\bar{F}_{C3}}{\xi_{C3}} & \dfrac{U_3A_3}{\xi_{C3}} \\ \dfrac{U_3A_3}{\xi_{H3}} & \dfrac{-U_3A_3 - \tau_{H3}\bar{F}_{H3}}{\xi_{H3}} \end{bmatrix},$$

$$\mathbf{A_{44}} \equiv \begin{bmatrix} \dfrac{-U_4A_4 - \tau_{C4}\bar{F}_{C4}}{\xi_{C4}} & \dfrac{U_4A_4}{\xi_{C4}} \\ \dfrac{U_4A_4}{\xi_{H4}} & \dfrac{-U_4A_4 - \tau_{H4}\bar{F}_{H4}}{\xi_{H4}} \end{bmatrix}$$

$$\mathbf{B_{33}} \equiv \begin{bmatrix} \dfrac{\tau_{C3}T_{Cin} - \tau_{C3}\bar{T}_{1C3}}{\xi_{C3}} & 0 \\ 0 & \dfrac{\tau_{H3}T_{Hin3} - \tau_{H3}\bar{T}_{1H3}}{\xi_{H3}} \end{bmatrix},$$

$$\mathbf{B_{44}} \equiv \begin{bmatrix} \dfrac{\tau_{C4}T_{Cin} - \tau_{C4}\bar{T}_{1C4}}{\xi_{C4}} & 0 \\ 0 & \dfrac{\tau_{H4}T_{Hin4} - \tau_{H4}\bar{T}_{1H4}}{\xi_{H4}} \end{bmatrix}$$

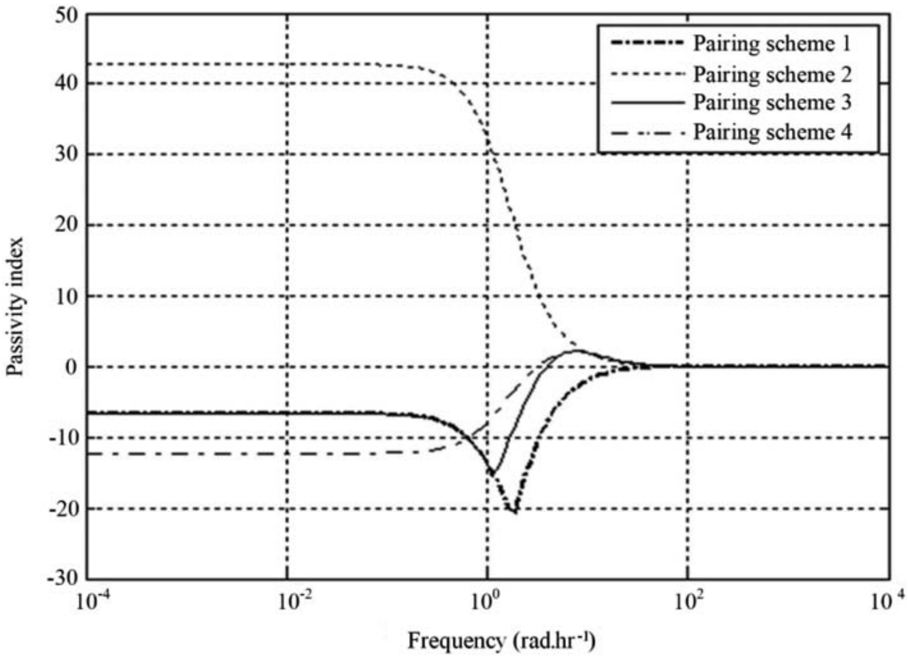


Figure 3 Passivity indices of four pairing schemes of two heat exchangers.

Table 3 Proportional integral (PI) tuning parameters for two heat exchangers in the heat exchanger network example.

Control type	Tuning Parameter	1-1 pairing	2-2 pairing	3-3 pairing	4-4 pairing
Passivity-based PI	k_c^+	606.38	38.45	35.43	270.62
	τ_I	10.83	15.00	15.00	14.30
Conventional PI	k_c^+	454.50	30.45	33.18	245.16
	τ_I	6.82	9.60	13.49	12.04

k_c^+ = Proportional term.
τ_I = Integral term.

$$\mathbf{C_{33}} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \mathbf{C_{44}} \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ \mathbf{D_{33}} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\mathbf{D_{44}} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (27)$$

The procedure of passivity based pairing was followed to find the best controlled-manipulated variable pairing of utility units.

Although there are four pairing schemes for these two utility models as shown in Table 4, pairing scheme 4 of 1-2/2-1/3-4/4-3 was the only possible pairing scheme in this system since it satisfies the main goal of using utility in a heat exchanger system to take the stream which does not reach the target temperature for that target.

After screening out for the non-DIC, this pairing scheme 4 also satisfies the necessary DIC condition. The steady state $\mathbf{G}(0)$ and sign matrix $\mathbf{U}$ of pairing scheme 4 are calculated as in Equations 28 and 29:

$$\mathbf{G}(0) = \begin{bmatrix} -1.45 & 6.78 & 0 & 0 \\ -0.33 & 38.08 & 0 & 0 \\ 0 & 0 & -31.495 & 0.88 \\ 0 & 0 & -2.51 & 3.08 \end{bmatrix} \quad (28)$$

$$\mathbf{U} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (29)$$

After the best pairing scheme had been chosen, the passivity based PI tuning parameters for each loop of two utilities in a HEN were found through Equations 17 to 19. The performance of the passivity based PI controller was considered by comparison with a conventional controller calculated using the Ziegler-Nichols method. Table 5 shows their tuning parameters.

As shown in Figure 4, a HEN example accompanied with controllers was tested to study the disturbance rejection response. The controllers were named TCC1, TCH1, TCC2, TCH3 and TCC4.

Table 4 Possible pairing schemes of two utility units in the heat exchanger network example.

Pairing scheme	Pairing	Manipulated Variable (MV)	Controlled Variable (CV)
(1) 1-1/2-2/3-3/4-4	1-1	F_{C3}	T_{3C3}
	2-2	F_{H3}	T_{3H3}
	3-3	F_{C4}	T_{4C4}
	4-4	F_{H4}	T_{4H4}
(2) 1-2/2-1/3-3/4-4	1-2	F_{C3}	T_{3H3}
	2-1	F_{H3}	T_{3C3}
	3-3	F_{C4}	T_{4C4}
	4-4	F_{H4}	T_{4H4}
(3) 1-1/2-2/3-4/4-3	1-1	F_{C3}	T_{3C3}
	2-2	F_{H3}	T_{3H3}
	3-4	F_{C4}	T_{4H4}
	4-3	F_{H4}	T_{4C4}
(4) 1-2/2-1/3-4/4-3	1-2	F_{C3}	T_{3H3}
	2-1	F_{H3}	T_{3C3}
	3-4	F_{C4}	T_{4H4}
	4-3	F_{H4}	T_{4C4}

For definition of the terms, see Equations 25 and 26 in the text.

Table 5 Proportional integral (PI) tuning parameters for two utility units in the heat exchanger network example.

Control type	PI Tuning Parameter	1-3 pairing	2-1 pairing	3-4 pairing	4-3 pairing
Passivity-based PI	k_c^+	172.53	348.87	54.48	330.19
	τ_I	20.95	12.47	13.13	17.36
Conventional PI	k_c^+	156.41	295.29	43.57	247.52
	τ_I	17.74	9.45	9.11	10.94

k_c^+ = Proportional term.
τ_I = Integral term.

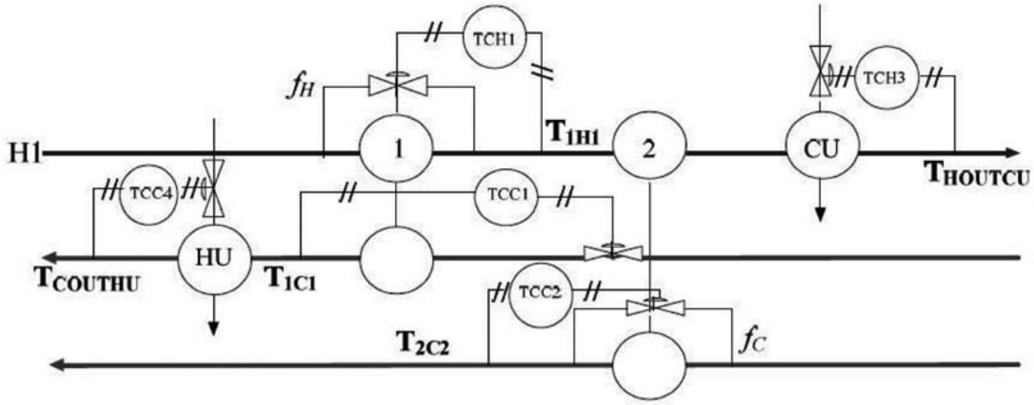
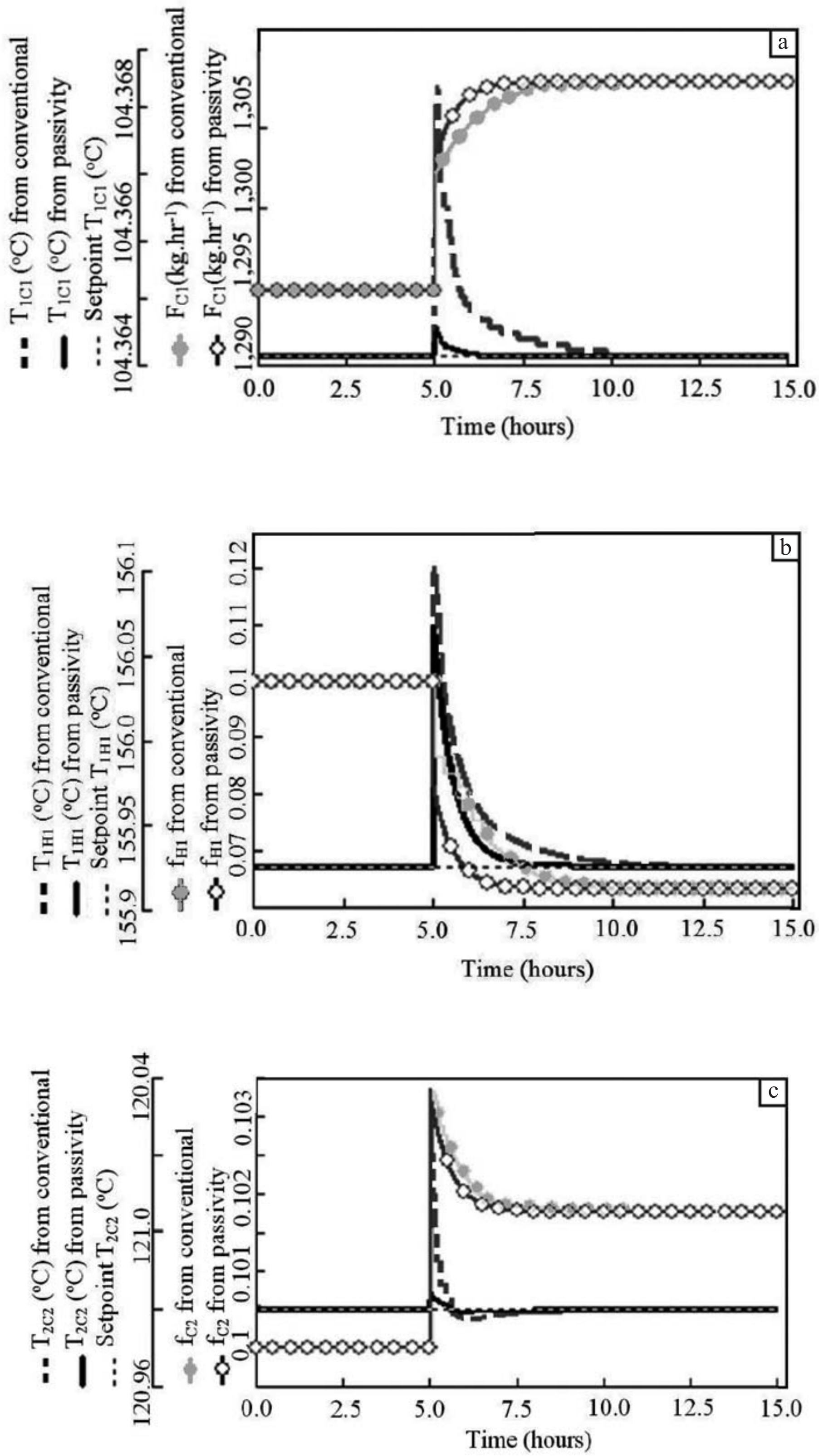


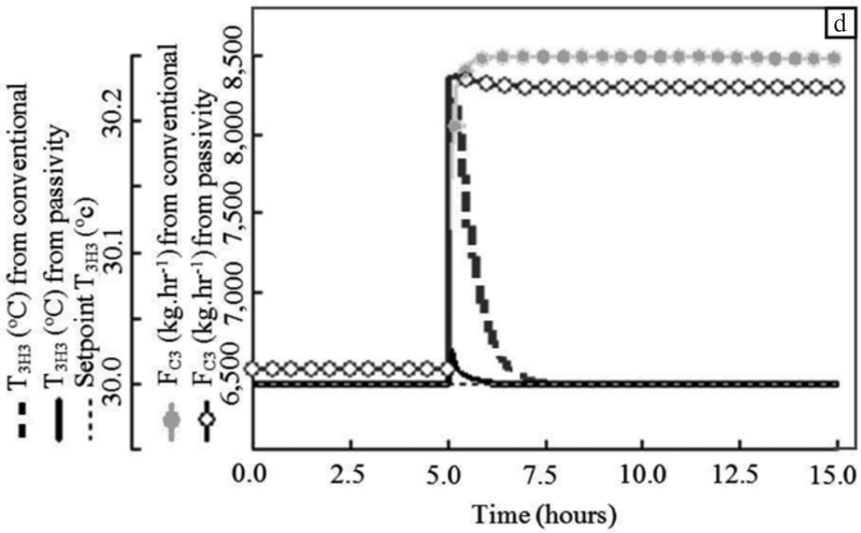
Figure 4 Heat exchanger network with controlled variables accompanied with controllers. (HU =Heat utility; CU = Cold utility; H1 = Hot stream 1; C1, C2 = Cold stream 1 and 2; TCH1, TCH3= Temperature controller of hot stream; TCC1, TCC2, TCC4=Temperature controller of cold stream.)

Figure 5 shows the dynamic responses of the system to a change in the inlet flow rate of the hot stream at the fifth hour. From this figure, the passivity based PI controllers and conventional controllers can adjust their manipulated variables to maintain their target temperatures. In conventional controllers the settling times were 5, 6.5, 3.75, 2.5 and 27.5 hr, whereas in the passivity based PI controllers the settling times decreased to 1.25, 3.75, 3, 1.1 and 9 hr respectively. It was clear that the passivity based PI controllers were much quicker than the conventional controllers.

After verifying the results, the HEN was also tested for fault-tolerance control to ensure that the system was able to continue operating properly in the event of the failure of some of its components. In this paper, controller TCC1 was assigned to fail and the system was disturbed by changing an inlet flow rate of the hot stream. The controller's performance was compared with the conventional controller.

Figure 6a reports that the failure of controller TCC1 and disturbance was applied directly affected the temperature at T1C1. However, the remaining passivity based PI controllers (TCH1, TCC2, TCH3 and TCC4) could control the outlet temperatures to their set points with settling times of 3, 3, 3 and 1 hr, respectively. Figure 6b shows that when controller TCC1 failed and disturbance was applied, controller TCC4 worked the overload and the temperature at TCOUTHU could not meet its setpoint within 50 hr. This implies that the remaining conventional controllers (TCH1, TCC2, TCH3 and TCC4) cannot operate and reject disturbance properly without controller TCC1.


a
b
c
Time (hours)
T1C1 (ºC) from conventional
T1C1 (ºC) from passivity
Setpoint T1C1 (ºC)
FC1(kg.hr-1) from conventional
FC1(kg.hr-1) from passivity
T1H1 (ºC) from conventional
T1H1 (ºC) from passivity
Setpoint T1H1 (ºC)
fH1 from conventional
fH1 from passivity
T2C2 (ºC) from conventional
T2C2 (ºC) from passivity
Setpoint T2C2 (ºC)
fC2 from conventional
fC2 from passivity



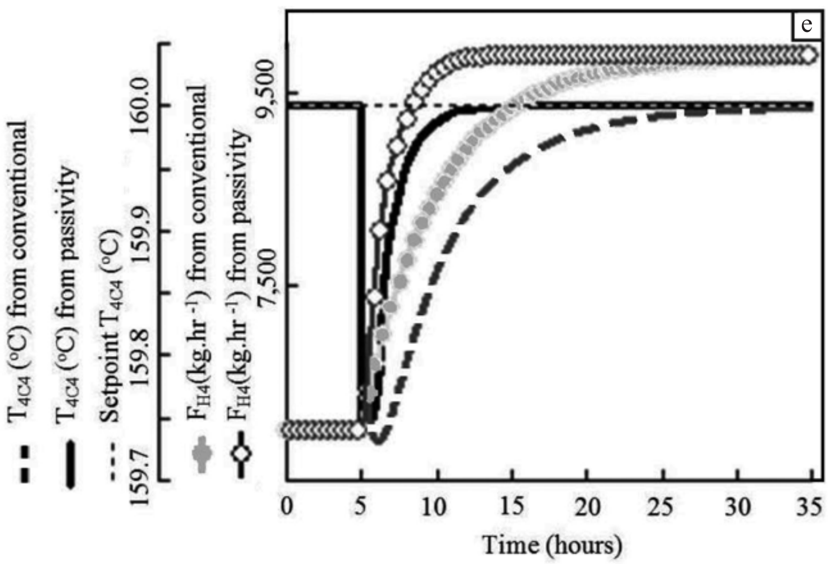


Figure 5 Dynamic responses of controllers: (a) TCC1; (b) TCH1; (c) TCC2; (d) TCH3; and (e) TCC4 when increased inlet flow rate of hot stream in the fifth hour. (See equations in the text for definition of the terms).

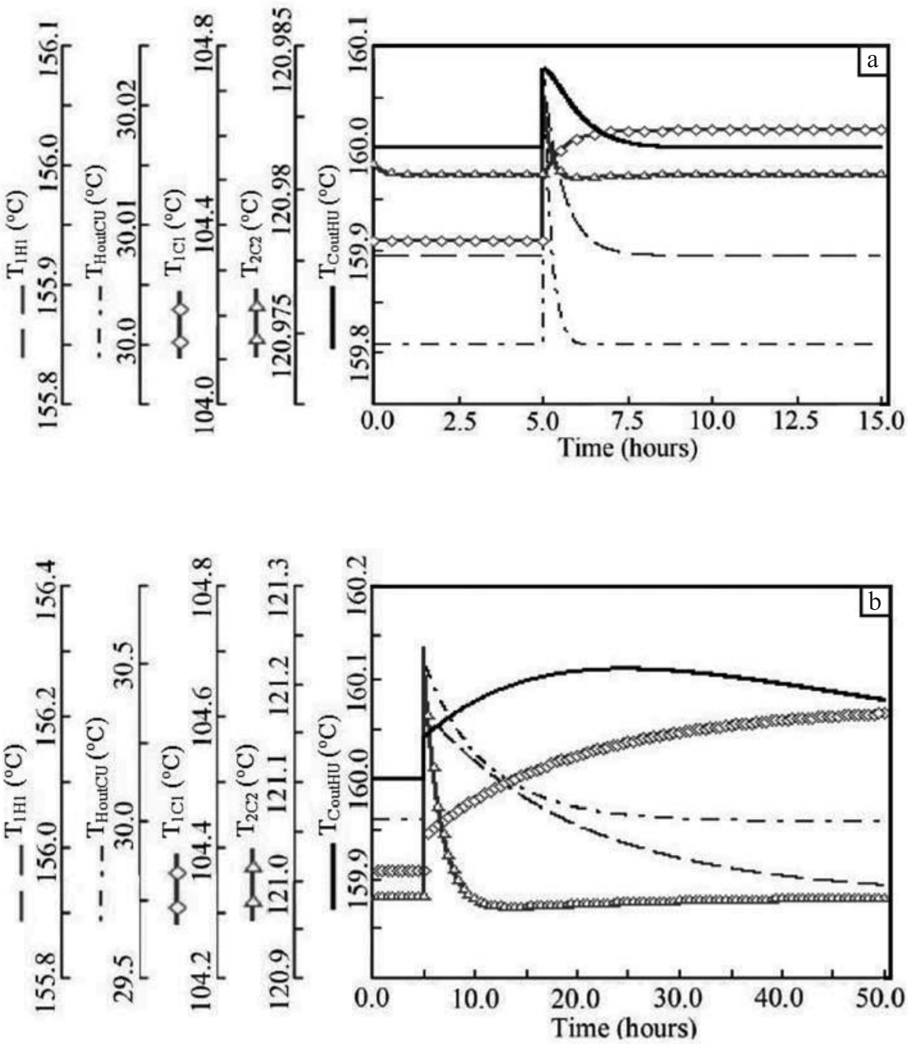


Figure 6 Temperature responses of controlled variables disturbed by inlet flow rate of hot stream with failure of PI controller TCC1: (a) Passivity-based tuning parameters; and (b) Defaulted tuning parameters from ASPEN Dynamics. (See equations in the text for definition of the terms).

CONCLUSION

The passivity theorem is one of the cornerstones of nonlinear control theory. This work has applied it to a simple heat exchanger network (HEN) by following the passivity-based decentralized controller synthesis procedure with the aim of tuning the PI controllers.

The controller's performance was tested by increasing the inlet flow rate of the hot stream to a HEN and comparing the results with a conventional controller based on the Ziegler-Nichols method. The results showed that the passivity-based PI controller had a faster and better dynamic response than the conventional controller.

In addition, when some parts of the system failed (for example where controller TCC1 failed), a HEN with the remaining passivity based PI controllers can operate and control the outlet temperatures to their setpoints. The system as a whole did not fail due to the problem in controller TCC1. On the other hand, a HEN with the remaining conventional controllers could not operate and rejected disturbance because all the responses were too slow.

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