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Modified Product Estimators for Population Mean in Simple Random Sampling Using Coefficients of Kurtosis and Skewness and Function of Quartiles of Auxiliary Variable

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ABSTRACT

This paper deals with four new product estimators for estimation of the population mean in simple random sampling using the coefficient of kurtosis, the coefficient of skewness and the function of quartiles of the auxiliary variable. Mean square errors of the four proposed estimators are derived up to the first degree approximation and are compared with the modified product estimators proposed by Pandey and Dubey (1988); Upadhyaya and Singh (1999); Singh (2003); Singh and Tailor (2003); and Singh *et al*. (2004). The two proposed estimators performed better than those previous works. An empirical study has been carried out in support of the results.

Keywords: auxiliary variable, coefficient of kurtosis, coefficient of skewness, quartiles, product estimator

INTRODUCTION

The conventional product estimator (Murthy, 1964) for the population mean $\bar{Y}$ with one auxiliary variable x is defined by Equation 1:

$$\bar{y}_p = \bar{y}\frac{\bar{x}}{\bar{X}} \quad (1)$$

where $\bar{y}$ is the sample mean of the study variable y, $\bar{x}$ is the sample mean of the auxiliary variable x and $\bar{X}$ is the population mean of the auxiliary variable x. The Mean Square Error (MSE) of the conventional product estimator is obtained by Equation 2:

$$\text{MSE}\left(\bar{y}_P\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2 + C_x^2 + 2\rho C_x C_y\right) \quad (2)$$

where $f = \dfrac{n}{N}$ and N is the population size and n is the sample size,

C_x is a population coefficient of variation of auxiliary variable,

C_y is a population coefficient of variation of the study variable,

ρ is the correlation coefficient between the study variable and the auxiliary variable.

The product estimator is often used to estimate the population mean when the correlation between the auxiliary and the study variables is negative. Moreover, when the correlation between the auxiliary and the study variables is positive, the ratio estimator is often used to estimate the population mean (Sisodia and Dwivedi, 1981; Upadhyaya and Singh, 1999; Al-Omari *et al.,* 2009; Banerjie and Tiwari, 2011; Adewara *et al.,* 2012; Subramani and Kumarapandiyan, 2012; Jeelani *et al.,* 2013). The known value of the parameters of the auxiliary variable such as the coefficient of

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variation, the coefficient of kurtosis, the coefficient of skewness and the standard deviation have been used to propose product estimators for the population. The product estimators have already been proposed by many of the earlier authors. The product estimators of $\bar{Y}$ based on C_x (Pandey and Dubey, 1988) is proposed as shown in Equation 3:

$$\bar{y}_{PD} = \bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right) \quad (3)$$

The MSE of this product estimator is shown by Equation 4:

$$\text{MSE}\left(\bar{y}_{PD}\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2+\theta_{PD}^2C_x^2+2\theta_{PD}\rho C_xC_y\right) \quad (4)$$

where $\theta_{PD} = \dfrac{\bar{X}}{\bar{X}+C_x}$.

The product estimators for $\bar{Y}$ based on C_x and $\beta_2(x)$ (Upadhyaya and Singh, 1999) are presented as Equations 5 and 6:

$$\bar{y}_{US_1} = \bar{y}\left(\frac{\beta_2(x)\bar{x}+C_x}{\beta_2(x)\bar{X}+C_x}\right) \quad (5)$$

$$\bar{y}_{US_2} = \bar{y}\left(\frac{C_x\bar{x}+\beta_2(x)}{C_x\bar{X}+\beta_2(x)}\right) \quad (6)$$

where $\beta_2(x)$ is the population coefficient of kurtosis of auxiliary variable.
The MSEs of these estimators are given by Equations 7 and 8:

$$\text{MSE}\left(\bar{y}_{US_1}\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2+\theta_1^2C_x^2+2\theta_1\rho C_xC_y\right) \quad (7)$$

$$\text{MSE}\left(\bar{y}_{US_2}\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2+\theta_2^2C_x^2+2\theta_2\rho C_xC_y\right) \quad (8)$$

where $\theta_1 = \dfrac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+C_x}$ and $\theta_2 = \dfrac{C_x\bar{X}}{C_x\bar{X}+\beta_2(x)}$.

The product estimators based on σ_x, $\beta_1(x)$ and $\beta_2(x)$ (Singh, 2003) are defined using Equations 9–11:

$$\bar{y}_{GN_1} = \bar{y}\left(\frac{\bar{x}+\sigma_x}{\bar{X}+\sigma_x}\right) \quad (9)$$

$$\bar{y}_{GN_2} = \bar{y}\left(\frac{\beta_1(x)\bar{x}+\sigma_x}{\beta_1(x)\bar{X}+\sigma_x}\right) \quad (10)$$

$$\bar{y}_{GN_3} = \bar{y}\left(\frac{\beta_2(x)\bar{x}+\sigma_x}{\beta_2(x)\bar{X}+\sigma_x}\right) \quad (11)$$

where $\beta_1(x)$ and $\beta_2(x)$ are the population coefficients of skewness and kurtosis of the auxiliary variable. The MSEs of these estimators are given by Equation 12:

$$\text{MSE}\left(\bar{y}_{GN_k}\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2+\varphi_k^2C_x^2+2\varphi_k\rho C_xC_y\right);\ k=1,2,3 \quad (12)$$

where $\varphi_1 = \dfrac{\bar{X}}{\bar{X}+\sigma_x}$, $\varphi_2 = \dfrac{\beta_1(x)\bar{X}}{\beta_1(x)\bar{X}+\sigma_x}$ and $\varphi_3 = \dfrac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+\sigma_x}$, respectively.

The product estimator for $\bar{Y}$ based on ρ (Singh and Tailor, 2003) is proposed as shown in Equation 13:

$$\bar{y}_{ST} = \bar{y}\frac{\left(\bar{x}+\rho\right)}{\left(\bar{X}+\rho\right)} \quad (13)$$

The MSE of the estimator is given by Equation 14:

$$\text{MSE}(\bar{y}_{ST}) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2+\theta_{ST}^2C_x^2+2\theta_{ST}\rho C_xC_y\right) \quad (14)$$

where $\theta_{ST} = \dfrac{\bar{X}}{\bar{X}+\rho}$.

The product estimator for $\bar{Y}$ based on $\beta_2(x)$ (Singh *et al.*, 2004) is proposed by Equation 15:

$$\bar{y}_{pr} = \bar{y}\left(\frac{\bar{x}+\beta_2(x)}{\bar{X}+\beta_2(x)}\right) \quad (15)$$

The MSE of the estimator is given by Equation 16:

$$\text{MSE}\left(\bar{y}_{pr}\right) = \frac{1-f}{n}\bar{Y}^2\left(C_y^2+\theta_{pr}^2C_x^2+2\theta_{pr}\rho C_xC_y\right) \quad (16)$$

where $\theta_{pr} = \dfrac{\bar{X}}{\bar{X}+\beta_2(x)}$.

This paper proposes new product estimators based on the function of quartiles of the auxiliary variable such as the inter-quartile range and the quartile average (Subramani and Kumarapandiyan, 2012). In addition, to improve the product estimators of $\bar{Y}$, the $\beta_1(x)$ and $\beta_2(x)$ of the auxiliary variable will be used.

MATERIALS AND METHODS

Proposed estimators

For estimating the population mean $\bar{Y}$ of y, a simple random sample of size n is drawn without replacement from the population. It is assumed that the information on the inter-quartile range, the quartile average, the coefficient of kurtosis and the coefficient of skewness of the auxiliary variable x are known and then the proposed estimators are as shown in Equations 17–20:

$$\bar{y}_{AS_1} = \bar{y}\left(\frac{\beta_1(x)\bar{x}+Q_{IR}}{\beta_1(x)\bar{X}+Q_{IR}}\right) \quad (17)$$

$$\bar{y}_{AS_2} = \bar{y}\left(\frac{\beta_2(x)\bar{x}+Q_{IR}}{\beta_2(x)\bar{X}+Q_{IR}}\right) \quad (18)$$

$$\bar{y}_{AS_3} = \bar{y}\left(\frac{\beta_1(x)\bar{x}+Q_{A}}{\beta_1(x)\bar{X}+Q_{A}}\right) \quad (19)$$

$$\bar{y}_{AS_4} = \bar{y}\left(\frac{\beta_2(x)\bar{x}+Q_{A}}{\beta_2(x)\bar{X}+Q_{A}}\right) \quad (20)$$

where Q_{IR} is the inter-quartile range; $Q_{IR} = Q_3 - Q_1$, Q_3 is the third quartile, Q_1 is the first quartile of the auxiliary variable and Q_A is the quartile average; $Q_A = (Q_3 + Q_1)/2$.

To obtain the bias and the MSE of the proposed product estimators, it is assumed that $\bar{y} = \bar{Y} + e_0$ and $\bar{x} = \bar{X} + e_1$ such that $E(e_0) = 0$, $E(e_1) = 0$, $E\left(e_0^2\right) = \frac{1-f}{n}\bar{Y}^2C_y^2$, $E\left(e_1^2\right) = \frac{1-f}{n}\bar{X}^2C_x^2$, and $E\left(e_0e_1\right) = \frac{1-f}{n}\bar{X}\bar{Y}\rho C_xC_y$.

Theorem 1. The bias, $B\left(\bar{y}_{AS_i}\right)$, and the MSE, $\mathrm{MSE}\left(\bar{y}_{AS_i}\right)$, of proposed product estimators $\bar{y}_{AS_i}$ for $i = 1, 2, 3, 4$ on the first degree approximation are given by

$$B\left(\bar{y}_{AS_i}\right) = \frac{1-f}{n}\bar{Y}\left(\theta_{AS_i}\rho C_xC_y\right)$$

and

$$\mathrm{MSE}\left(\bar{y}_{AS_i}\right) = \frac{1-f}{n}\bar{Y}^2\left(\theta_{AS_i}^2C_x^2 + 2\theta_{AS_i}\rho C_xC_y + C_y^2\right).$$

Proof.

Expressing $\bar{y}_{AS_i}$ in terms of e's, results in Equation 21:

$$\bar{y}_{AS_1} = \left(\bar{Y}+e_0\right)\frac{\left[\beta_1(x)\left(\bar{X}+e_1\right)+Q_{IR}\right]}{\left[\beta_1(x)\bar{X}+Q_{IR}\right]}$$

$$\bar{y}_{AS_1} - \bar{Y} = \frac{\beta_1(x)\bar{Y}e_1}{\beta_1(x)\bar{X}+Q_{IR}} + e_0 + \frac{\beta_1(x)e_0e_1}{\beta_1(x)\bar{X}+Q_{IR}} \quad (21)$$

Taking expectation on both sides of Equation 21,

$$E\left(\bar{y}_{AS_1} - \bar{Y}\right) = \frac{\beta_1(x)\bar{Y}E\left(e_1\right)}{\beta_1(x)\bar{X}+Q_{IR}} + E\left(e_0\right) + \frac{\beta_1(x)E\left(e_0e_1\right)}{\beta_1(x)\bar{X}+Q_{IR}}.$$

Substituting the values of $E(e_0)$, $E(e_1)$ and $E(e_0\,e_1)$ finally obtains the bias of $\bar{y}_{AS_i}$ as Equation 22:

$$B\left(\bar{y}_{AS_1}\right) = \frac{1-f}{n}\bar{Y}\left(\theta_{AS_1}\rho C_xC_y\right) \quad (22)$$

where $\theta_{AS_1} = \frac{\beta_1(x)\bar{X}}{\beta_1(x)\bar{X}+Q_{IR}}$.

The MSE of $\bar{y}_{AS_i}$ up to the first order approximation is obtained by squaring and taking the expectation of Equation 21,

$$\mathrm{MSE}\left(\bar{y}_{AS_1}\right) = E\left(\bar{y}_{AS_1} - \bar{Y}\right)^2 = E\left[\frac{\beta_1(x)\bar{Y}e_1}{\beta_1(x)\bar{X}+Q_{IR}} + e_0\right]^2$$

$$= \left[\frac{\beta_1(x)\bar{Y}}{\beta_1(x)\bar{X}+Q_{IR}}\right]^2 E\left(e_1^2\right) + 2\left[\frac{\beta_1(x)\bar{Y}}{\beta_1(x)\bar{X}+Q_{IR}}\right]E\left(e_0e_1\right) + E\left(e_0^2\right).$$

Substitution of the values of $E\left(e_0^2\right)$, $E\left(e_1^2\right)$ and $E(e_0\,e_1)$ produces the MSE of $\bar{y}_{AS_i}$ as Equation 23:

$$\mathrm{MSE}\left(\bar{y}_{AS_1}\right) = \frac{1-f}{n}\bar{Y}^2\left(\theta_{AS_1}^2C_x^2 + 2\theta_{AS_1}\rho C_xC_y + C_y^2\right) \quad (23)$$

Similarly, the bias and the MSE of $\bar{y}_{AS_2}$, $\bar{y}_{AS_3}$ and $\bar{y}_{AS_4}$ would be as shown in Equations 24–29:

$$B\left(\bar{y}_{AS_2}\right) = \frac{1-f}{n}\bar{Y}\left(\theta_{AS_2}\rho C_xC_y\right) \quad (24)$$

$$\mathrm{MSE}\left(\bar{y}_{AS_2}\right) = \frac{1-f}{n}\bar{Y}^2\left(\theta_{AS_2}^2C_x^2 + 2\theta_{AS_2}\rho C_xC_y + C_y^2\right) \quad (25)$$

$$B\left(\bar{y}_{AS_3}\right) = \frac{1-f}{n}\bar{Y}\left(\theta_{AS_3}\rho C_xC_y\right) \quad (26)$$

$$\mathrm{MSE}\left(\bar{y}_{AS_3}\right) = \frac{1-f}{n}\bar{Y}^2\left(\theta_{AS_3}^2C_x^2 + 2\theta_{AS_3}\rho C_xC_y + C_y^2\right) \quad (27)$$

$$B\left(\bar{y}_{AS_4}\right)=\frac{1-f}{n}\bar{Y}\left(\theta_{AS_4}\rho C_x C_y\right) \quad (28)$$

$$\mathrm{MSE}\left(\bar{y}_{AS_4}\right)=\frac{1-f}{n}\bar{Y}^2\left(\theta_{AS_4}^2 C_x^2+2\theta_{AS_4}\rho C_x C_y+C_y^2\right) \quad (29)$$

where $\theta_{AS2}=\dfrac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+Q_{IR}}$, $\theta_{AS3}=\dfrac{\beta_1(x)\bar{X}}{\beta_1(x)\bar{X}+Q_A}$

and $\theta_{AS4}=\dfrac{\beta_2(x)\bar{X}}{\beta_2(x)\bar{X}+Q_A}$.

Notice that the proposed estimators $\bar{y}_{AS_1}$, $\bar{y}_{AS_2}$, $\bar{y}_{AS_3}$ and $\bar{y}_{AS_4}$ are biased.

RESULTS AND DISCUSSION

Efficiency comparison

This section shows that the proposed product estimators $\bar{y}_{AS_i}$ (i = 1, 2, 3, 4) are more efficient than $\bar{y}_{PD}$, $\bar{y}_{US_j}$ (j = 1, 2), $\bar{y}_{GN_k}$ (k = 1, 2, 3), $\bar{y}_{ST}$, and $\bar{y}_{pr}$. In a comparison of the proposed estimator $\bar{y}_{AS_i}$ with $\bar{y}_{PD}$, the condition is obtained by Equation 30:

$$\mathrm{MSE}\left(\bar{y}_{AS_i}\right)<\mathrm{MSE}\left(\bar{y}_{PD}\right)$$

$$\frac{1-f}{n}\bar{Y}^2\left(\theta_{AS_i}^2 C_x^2+2\theta_{AS_i}\rho C_x C_y+C_y^2\right)<\frac{1-f}{n}\bar{Y}^2\left(C_y^2+\theta_{PD}^2 C_x^2+2\theta_{PD}\rho C_x C_y\right)$$

$$\theta_{AS_i}^2 C_x^2+2\theta_{AS_i}\rho C_x C_y+C_y^2<C_y^2+\theta_{PD}^2 C_x^2+2\theta_{PD}\rho C_x C_y$$

$$\theta_{AS_i}^2 C_x^2+2\theta_{AS_i}\rho C_x C_y<\theta_{PD}^2 C_x^2+2\theta_{PD}\rho C_x C_y$$

$$C_x^2\left(\theta_{AS_i}^2-\theta_{PD}^2\right)+2\rho C_x C_y\left(\theta_{AS_i}-\theta_{PD}\right)<0\,;\, i=1,2,3,4 \quad (30)$$

If the condition in Equation 30 is satisfied, the proposed estimator $\bar{y}_{AS_i}$ (i = 1, 2, 3, 4) is more efficient than $\bar{y}_{PD}$.

Similarly, the conditions for which the proposed product estimators $\bar{y}_{AS_i}$ are more efficient than $\bar{y}_{US_j}$, $\bar{y}_{GN_k}$, $\bar{y}_{ST}$, and $\bar{y}_{pr}$ can be obtained as follows from Equations 31–34:

$$\mathrm{MSE}\left(\bar{y}_{AS_i}\right)<\mathrm{MSE}\left(\bar{y}_{US_j}\right) \text{ if } C_x^2\left(\theta_{AS_i}^2-\theta_j^2\right)+2\rho C_x C_y\left(\theta_{AS_i}-\theta_j\right)<0\,;\, i=1,2,3,4 \quad (31)$$

$$\mathrm{MSE}\left(\bar{y}_{AS_i}\right)<\mathrm{MSE}\left(\bar{y}_{GN_k}\right) \text{ if } C_x^2\left(\theta_{AS_i}^2-\varphi_k^2\right)+2\rho C_x C_y\left(\theta_{AS_i}-\varphi_k\right)<0\,;\, i=1,2,3,4 \quad (32)$$

$$\mathrm{MSE}\left(\bar{y}_{AS_i}\right)<\mathrm{MSE}\left(\bar{y}_{ST}\right) \text{ if } C_x^2\left(\theta_{AS_i}^2-\theta_{ST}^2\right)+2\rho C_x C_y\left(\theta_{AS_i}-\theta_{ST}\right)<0\,;\, i=1,2,3,4 \quad (33)$$

$$\mathrm{MSE}\left(\bar{y}_{AS_i}\right)<\mathrm{MSE}\left(\bar{y}_{pr}\right) \text{ if } C_x^2\left(\theta_{AS_i}^2-\theta_{pr}^2\right)+2\rho C_x C_y\left(\theta_{AS_i}-\theta_{pr}\right)<0\,;\, i=1,2,3,4 \quad (34)$$

If the conditions in Equations 31–34 are satisfied, the proposed estimator $\bar{y}_{AS_i}$ (i = 1, 2, 3, 4) is more efficient than $\bar{y}_{US_j}$ (j = 1, 2), $\bar{y}_{GN_k}$ (k = 1, 2, 3), $\bar{y}_{ST}$, and $\bar{y}_{pr}$.

Empirical study

A motor fuel example is used to illustrate the performance of the proposed product estimators using a dataset from Weisberg (1980). The following data were considered in this analysis: a motor fuel tax in cents per gallon (y) and the length of Federal-aid primary highways in miles (x). The study population data are summarized as follows:

$N=48, n=15, \bar{Y}=7.6700, \bar{X}=5565.4170$, C_x = 0.6208, C_y = 0.1277, S_x = 3454.9459, S_y = 0.9408, ρ = –0.5221, $\beta_1(x)$ = 1.1895, $\beta_2(x)$ = 5.0453, Q_{IR} = 4825.2500, Q_A = 5195.3700.

To consider the performance of the various product estimators, the percent relative efficiency of all proposed estimators is calculated with respect to $\bar{y}_{PD}$, $\bar{y}_{US_j}$ (j = 1, 2), $\bar{y}_{GN_k}$ (k = 1, 2, 3), $\bar{y}_{ST}$, and $\bar{y}_{pr}$. The percent relative efficiency of $\bar{y}_{AS_1}$ with respect to $\bar{y}_{PD}$ can be computed as

$$\mathrm{PRE}\left(\bar{y}_{AS_1},\bar{y}_{PD}\right)=\frac{\mathrm{MSE}\left(\bar{y}_{PD}\right)}{\mathrm{MSE}\left(\bar{y}_{AS_1}\right)}\times 100$$

$$=\frac{C_y^2+\theta_{PD}^2 C_x^2+2\theta_{PD}\rho C_x C_y}{\theta_{AS_1}^2 C_x^2+2\theta_{AS_1}\rho C_x C_y+C_y^2}\times 100.$$

Similarly, the percent relative efficiencies of various estimators are presented in Table 1.

From Table 1, the proposed product estimators $\bar{y}_{AS_1}$ and $\bar{y}_{AS_3}$ are more efficient than the product estimators proposed by Pandey and Dubey (1988), Upadhyaya and Singh (1999), Singh (2003), Singh and Tailor (2003) and Singh *et al*. (2004) by considering the percent relative

Table 1 Percent relative efficiencies (PREs) of $\bar{y}_{AS_1}$, $\bar{y}_{AS_2}$, $\bar{y}_{AS_3}$ and $\bar{y}_{AS_4}$ which are defined by Equations 17–20.

PREs(.,.)	$\bar{y}_{PD}$	$\bar{y}_{US_1}$	$\bar{y}_{US_2}$	$\bar{y}_{GN_1}$	$\bar{y}_{GN_2}$	$\bar{y}_{GN_3}$	$\bar{y}_{ST}$	$\bar{y}_{pr}$
$\bar{y}_{AS_1}$	327.41	327.45	326.43	114.99	131.84	254.90	327.53	326.85
$\bar{y}_{AS_2}$	140.83	140.85	140.41	49.46	56.71	109.64	140.88	140.59
$\bar{y}_{AS_3}$	350.79	350.83	349.74	123.20	141.26	273.10	350.91	350.18
$\bar{y}_{AS_4}$	144.28	144.30	143.85	50.68	58.10	112.33	144.34	144.03

$\bar{y}_{PD}$ is defined in Equation 3; $\bar{y}_{US_1}$ is defined in Equation 5; $\bar{y}_{US_2}$ is defined inEquation6; $\bar{y}_{GN_1}$ is defined in Equation 9; $\bar{y}_{GN_2}$ is defined in Equation 10; $\bar{y}_{GN_3}$ is defined in Equation 11; $\bar{y}_{ST}$ is defined in Equation 13; $\bar{y}_{pr}$is defined in Equation 15.

efficiency. In addition, the proposed product estimators $\bar{y}_{AS_2}$ and $\bar{y}_{AS_4}$ were more efficient except for $\bar{y}_{GN_1}$ and $\bar{y}_{GN_2}$. Thus, if the inter-quartile range, the quartile average, the coefficient of kurtosis and the coefficient of skewness of the auxiliary variable *x* are known, $\bar{y}_{AS_1}$ and $\bar{y}_{AS_3}$ are recommended for use in practical applications.

CONCLUSION

The product estimators were proposed by using the known values of the inter-quartile range, the quartile average coefficient of kurtosis and the coefficient of skewness of the auxiliary variable. The bias and mean squared error of the proposed estimators were obtained and compared with the existing product estimators. Furthermore, the conditions of the proposed estimators are more efficient than other existing estimators proposed by many authors mentioned earlier. In addition, the numerical example revealed that the efficiency of the proposed product estimators was better than some others. Moreover, using the inter-quartile range, the quartile average coefficient of kurtosis and the coefficient of skewness of the auxiliary variable can reduce the MSE of the estimator of the population mean.

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