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Improvement of Cornering Characteristic Using VariableSteering Ratio

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ABSTRACT

A variable steering ratio (VSR) is introduced for a purpose of improving cornering characteristic of a vehicle. Two degree-of-freedom vehicle models are used for derivation and calculation of the VSR gain, and simulating the vehicles cornering characteristic, with and without adjustable gain. Approximated bounds of validity for a specific case of a car using VSR is justified using Dugoff's tire model. A sample of two dimensional look-up table is generated within the bound. Understeer, neutral steer, and oversteer vehicles are tested within the bound of validity of VSR to see effects of this variable gain. A constant steer angle testing is also used with an understeer vehicle model with and without VSR to confirm the effects of the gain. Results show that, within the bound of validity, understeer and oversteer vehicles with VSR units have neutral steer characteristic. The improvement of cornering characteristic is also evident from the numerical simulations.

Key words: steady-state turning, variable steering ratio, vehicle handling, vehicle simulation, vehicle steering

INTRODUCTION

In steady-state cornering, oversteer is not a desirable characteristic since the lateral acceleration at the center of mass causes the rear wheels to slip sideways more than that of the front, thus diminishing the radius of turn. Therefore the steer angle is needed to be reduced to maintain the turning radius (Gillespie, 1992). Also, even though understeer is a more desirable characteristic, neutral steer would be a preferred one. The purpose of this paper is to use variable steering ratio (VSR) to make a vehicle be neutral steer. The basic idea is that, for a neutral steer vehicle, the steer angle to follow the curve at any speed is simply the Ackerman angle (Gillespie, 1992; Siwakosit, 2005). In other words, the "input" steer angle will be equal to the "output" steer angle (calculated Ackerman angle) for this case. But for an oversteer vehicle, the "input" steer angle is smaller than the "output" steer angle. The converse is true for an understeer vehicle. Therefore, in this case, to make the steer angle from a driver be equal to the Ackerman angle, adjustable gain must be included in the steering system to adjust the steer angle. Of course, this gain is not a constant, but the function of vehicle parameters and dynamic variables of the vehicle.

For simplicity, a 2 degree-of-freedom vehicle model (Siwakosit, 2005) will be used to derive and calculate the VSR and simulate the vehicle's cornering characteristic, with and without

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adjustable gain. Also, approximated bounds of validity for a specific case of a car using VSR will be justified using Dugoff's tire model (Dugoff *et al*., 1969; Guntur and Sankar, 1980). A sample of two dimensional look-up table will be generated within the bound. Three types of vehicles, BMW 320i (Heydinger, 1991), Suzuki Samurai (Heydinger, 1991), and Pontiac Fiero (Garrott *et al*., 1988), which are understeer, neutral steer, and oversteer, respectively, will be tested within bound of validity of VSR to see effects of this variable gain. Constant steer angle testing (Gillespie, 1992) will be used with BMW 320i with and without VSR to confirm the effects of the gain.

ANALYSIS

From a 2 degrees-of-freedom vehicle model (bicycle model) with linear non-dynamic tires (Gillespie, 1992; Siwakosit, 2005), Ackerman angle can be calculated as following (Siwakosit, 2005),

$$\delta_{Ack} = \frac{L}{\rho_{ss}} \quad (1)\ , \quad \frac{1}{\rho_{ss}} = \frac{r_{ss}}{\sqrt{v_{ss}^2 + u^2}} \quad (2)$$

where ρ_{ss} = steady-value of radius of turn (ft)
δ_{Ack} = Ackerman angle (rad)
r_{ss} = steady-state value of yaw rate during pure cornering (rad/sec)
v_{ss} = steady-state value of lateral velocity during pure cornering (ft/sec)
u = forward velocity (ft/sec)
L = wheel base of a vehicle (ft)

Let, $\delta_{desired} = \delta_{Ack}$, where $\delta_{desired} = \frac{\delta_{ST}}{n}$, δ_{ST} = steering wheel angle (rad), and n = steering gear ratio.

Thus, from (1) and (2),

$$\frac{r_{ss}}{\delta_{desired}} = \frac{\sqrt{u^2 + v_{ss}^2}}{L} \qquad (3)$$

But steady-state value of r could be written as (Gillespie, 1992),

$$\frac{r_{ss}}{\delta} = \frac{uLC_{\alpha f}C_{\alpha r}}{\frac{m}{2}u^2\left(bC_{\alpha r} - aC_{\alpha f}\right) + L^2C_{\alpha f}C_{\alpha r}} \qquad (4)$$

where, a = distance from front axle to CG of vehicle (ft)
b = distance from rear axle to CG of vehicle (ft)
m = total mass of vehicle (slug)
$C_{\alpha f}$, $C_{\alpha r}$ = cornering stiffness of front and rear tires, respectively (lb/rad)
δ = steer angle to a vehicle (rad)

Let $\frac{r_{ss}}{\delta} = C_1$ = constant, then, from (3) and (4),

$$\frac{\delta}{\delta_{desired}} = \frac{\sqrt{u^2 + v_{ss}^2}}{C_1 L} \qquad (5)$$

But, steady-state value of v can be written as,

$$\frac{v_{ss}}{\delta} = \frac{\left(R_2\frac{2aC_{\alpha f}}{I_z} - R_4\frac{2C_{\alpha f}}{m}\right)}{\left(R_4R_1 - R_2R_3\right)} \qquad (6)$$

where, $R_1 = -\frac{2(C_{\alpha f} + C_{\alpha r})}{mu}$,

$$R_2 = \frac{2(C_{\alpha r}b - C_{\alpha f}a)}{mu} - u$$

$$R_3 = \frac{2(C_{\alpha r}b - C_{\alpha f}a)}{I_z u},$$

$$R_4 = -\frac{2(C_{\alpha f}a^2 + C_{\alpha r}b^2)}{I_z u}$$

I_Z = yaw axis moment of inertia of vehicle (slug-ft^2)

Let $\frac{v_{ss}}{\delta} = C_2$ = other constant, then from (5) and (6),

$$\frac{\delta^2}{\delta^2_{desired}} = \frac{u^2 + C_2^2\delta^2}{C_1^2L^2} \qquad (7)$$

Solve for δ, thus

$$\delta^2 = \frac{u^2\delta^2_{desired}}{(C_1^2L^2 - C_2^2\delta^2_{desired})} \qquad (8)$$

or, $$\delta = \text{sign}(\delta_{desired})\sqrt{\frac{u^2\delta^2_{desired}}{(C_1^2L^2 - C_2^2\delta^2_{desired})}} \qquad (9)$$

δ is an input to a vehicle which will produce an Ackerman angle for each particular speed, and since $\delta_{desired} = \delta_{Ack}$, a vehicle will have a neutral steer characteristic.

Stability of δ depends on C_1, C_2, L, and $\delta_{desired}$. Also, C_1 and C_2 are functions of u and vehicle parameters as in (4) and (5) ,respectively. Conditions for stability of δ are,

1. $\delta^2_{desired} < \frac{C_1^2L^2}{C_2^2}$, for $\delta \in \Re$

where, $\Re$ is the set of real number.

2. $C_1 \neq 0$, or ,$C_2 \neq \infty$, for $\delta_{desired} \neq 0$

This implies that $u \neq 0$ (from (4) and(5)) to make a vehicle be able to steer.

Bound of validity of (9), called variable steering ratio (VSR) equation, is dependent on linearity of a vehicle and tire models. For a particular vehicle, if a range of validity of linear tire model is known, bound of validity of VSR equation can be calculated as following. By running simulation at several forward velocities, find $\delta_{desired}$ at each one that gives the maximum value of slip angle at that run as close to the upper or lower limit of range of linearity of slip angles as possible. Then, at each forward velocity, the maximum (or minimum) value of $\delta_{desired}$ is calculated by iterative manners, and plotted to indicate approximated bound. Of course, bound of validity of a vehicle model without VSR unit could also be found approximately by the same means. Two dimensional look-up table or VSR input/output envelope is then generated within bound of validity of VSR equation to give information that how δ varies with $\delta_{desired}$.

For conciseness, a reader is referred to Siwakosit (2005) and Guntur and Sankar (1980) for equations of motion and equations of Dugoff's tire model, respectively. Then, a vehicle model based on data of BMW 320i with Dugoff's and linear non-dynamic tire models will be built with VSR unit to be tested and compared with the same models without VSR unit. The models will be tested for both in and out of bound of validity of VSR. Three car models with different cornering characteristics, BMW 320i, Suzuki Samurai, and Pontiac Fiero, will be tested with and without VSR unit within bound of validity. Trajectories of all cars will be shown and compared. In addition, Ackerman angles for all cars will be calculated by using (1) and (2), and then compared with $\delta_{desired}$.

Lastly, a constant steer angle test (Gillespie, 1992) will be performed on an understeer car model, BMW 320i, using Dugoff's tire model with and without valid VSR. From Gillespie (1992),

$$\left(\frac{r_{ss}}{u}\right) = \frac{\delta_{desired}}{L} - \frac{K}{Lg}(ur_{ss}) \qquad (10)$$

where g = gravitational acceleration = 32.2 ft/sec^2

K = understeer gradient (rad)

Data will be collected from various values of u and r_{ss} obtained from a constant input value of δ =2 deg. K is determined from slope of $\left(\frac{r_{ss}}{u}\right)$ versus (ur_{ss}). If K has positive value, the vehicle is understeer. For oversteer, K will be negative. Neutral steer vehicle has $K \approx 0$.

RESULTS

Figure 1 shows approximated bounds of validity of BMW 320i with and without VSR unit resulting from multiple simulations of the car model using Dugoff's tire model. The maximum slip angle is limited to approximately 5.4 deg, which is a limitation of a linear tire model for a condition used here (Siwakosit, 2005). VSR input/output relationship at various speeds for positive value of $\delta_{desired}$ is shown in figure 2.

Figure 3 shows trajectories of BMW 320i with and without valid VSR unit. Figure 4 shows respectively time histories of slip angles, yaw rates, and lateral velocities of this case. Figure 5 shows trajectories of a case when the vehicle with linear tire model and Dugoff's tire model so equipped with VSR unit receive inputs, which are out of bound of validity of VSR.

Trajectories of 3 different types of car with and without VSR unit within the bound of validity are shown in figure 6. Calculated Ackerman angles using (1) and (2) for all cars with and without VSR are shown and compared in figure 7. Figure 8 shows effects of valid VSR unit to an understeer vehicle indicated by constant steer angle testing.

DISCUSSIONS

From figure 1, it is obvious that bound of validity of linear model of this vehicle without VSR is broader than that of the same car with VSR. This is because of the compensation by VSR unit. Since BMW 320i is an understeer vehicle, an output steer angle from VSR unit will be larger

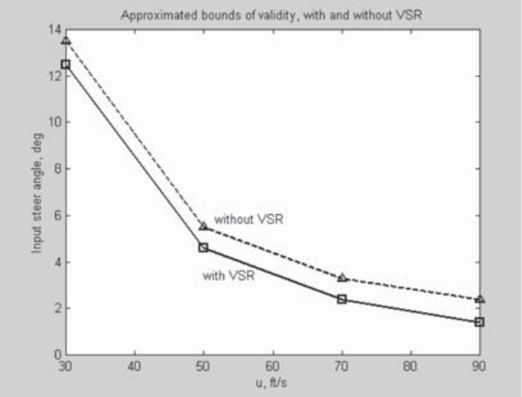

Figure 1 Approximated bound of validity from multiple simulations.

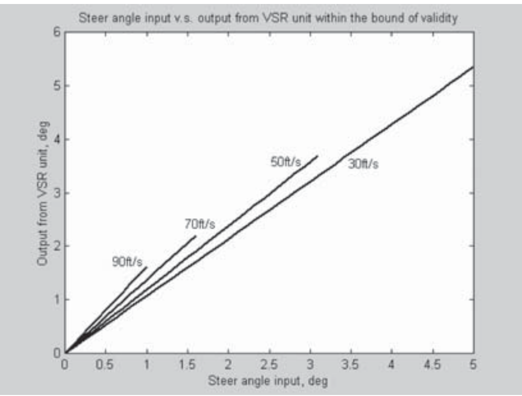

Figure 2 2-D look-up table for VSR of BMW 320i at various speeds.

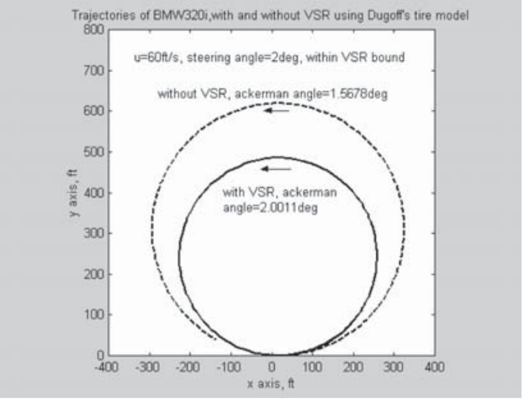

Figure 3 Trajectories of BMW320i with and without VSR unit.

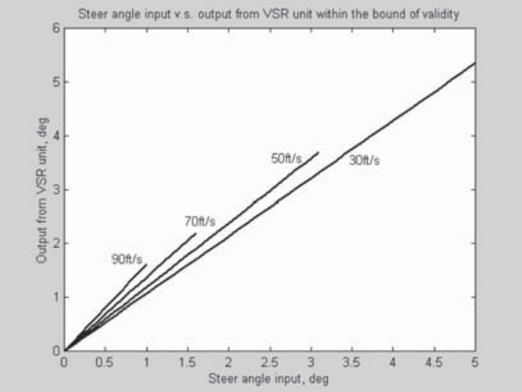

Figure 4 Slip angles, yaw rates, and lateral velocities of Figure 3.

than its input to match the calculated Ackerman angle with $\delta_{desired}$. For this reason, an understeer vehicle with VSR unit will reach the limit of linearity sooner than the original one since it is forced to put more slip angles, caused by larger steer angle, to the tire. Figure 2 shows VSR input/output envelope of the same case, an understeer vehicle with valid VSR unit. This envelope is generated within bound of validity of VSR unit for this particular case. At each forward velocity, maximum value of $\delta_{desired}$ is decreasing when forward velocity is increasing. Also for this case, a slope of each curve at particular forward velocity is increasing with forward velocity. For this particular case, it means that the higher a forward velocity, the larger the output to input ratio produce by VSR unit, the narrower the range of $\delta_{desired}$ that can be used. This approximated envelope is limited by, of course, bound of validity of VSR.

Trajectories of an understeer vehicle, BMW 320i, using Dugoff's tire model with and without VSR, are shown in figure 3. A forward speed is 60 ft/s, and a steering angle is 2 degrees, which are within the bound of validity of VSR. It is obvious that a radius of turn of a vehicle with VSR unit is smaller. A calculated Ackerman angle

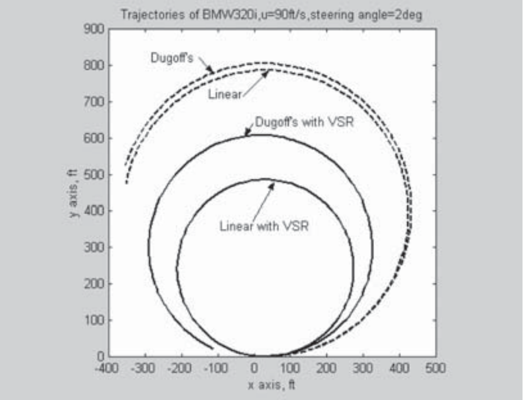

Figure 5 Trajectories of BMW320i when driven outside VSR bound.

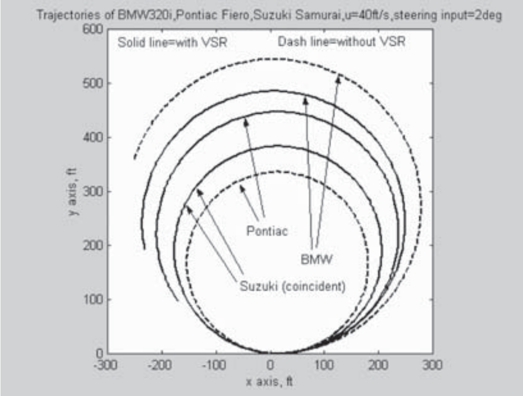

Figure 6 Trajectories of 3 linear vehicles with and without VSR units.

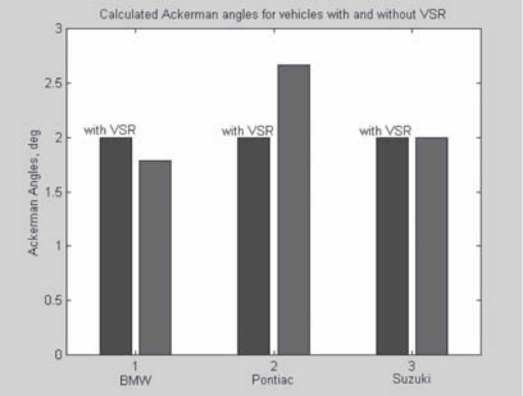

Figure 7 Ackerman angles of 3 vehicles with and without VSR units.

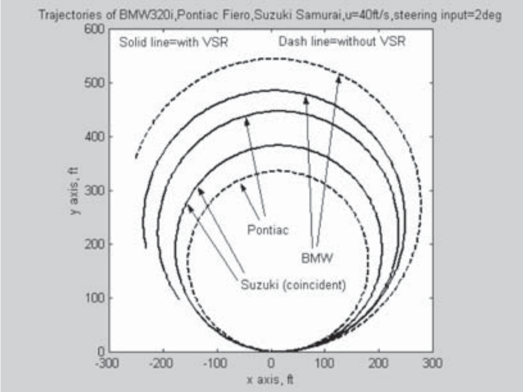

Figure 8 A constant steering angle test.

from equations (1) and (2) for the case of a vehicle with VSR matches with $\delta_{desired}$ applied. For the case without VSR unit, a calculated Ackerman angle is smaller than $\delta_{desired}$. Figure 4 shows that with valid VSR unit, a vehicle generates more slip angles than the vehicle without one. The reason for this case is already discussed above. Notice that, from this figure, the maximum value of slip angle is less than 5.4 degs. The vehicle with VSR unit produces more yaw rate and magnitude of lateral velocity than the vehicle without VSR since the vehicle is forced to turn more by VSR unit. In figure 5, trajectories of a vehicle with linear and Dugoff's tire models with input outside bound of validity of VSR (u = 90 ft/s and $\delta_{desired}$ = 2 degrees), are presented. For a vehicle using linear model, VSR unit is working without problem, since it is derived from linear concepts. It could reduce a radius of turn of an understeer vehicle with linear tire model effectively. But this is not a case for a vehicle with Dugoff's tire model. A vehicle with invalid VSR unit could not effectively reduce a radius of turn. Instead, a vehicle without VSR unit has a radius of turn more closely resemble to one with linear tire model since its linear behavior is still preserved according to figure 1. Although VSR unit is still working outside bound of validity, this shows that VSR unit could only operate well within the bound.

Trajectories of BMW 320i, Suzuki Samurai, and Pontiac Fiero using linear non-dynamic tire model, without and with valid VSR are shown in figure 6. With VSR unit, it is obvious that the radius of turn of Pontiac Fiero, an oversteer vehicle, is larger than the same car without VSR unit. Also for BMW 320i with VSR unit, radius of turn is smaller than that of the same car without one. For Suzuki Samurai, a neutral steer vehicle, the trajectories are not different for without and with valid VSR unit. Figure 7 shows calculated Ackerman angles for all cars. With valid VSR unit, all cars have neutral steer handling characteristic since calculated Ackerman angles are all equal to desired steer input. It is obvious that, with valid VSR, cornering characteristic of Pontiac Fiero is changed from oversteer (Ackerman angle larger than steer angle input) to neutral steer. Again, BMW 320i with VSR unit is neutral steer. Constant steer angle test has been used to find the handling characteristic of BMW 320i with Dugoff's tire model, without and with VSR unit. Figure 8 shows that, with valid VSR, the car has much less understeer behavior, almost neutral steer, because the slope of the curve corresponding to a car with VSR unit is nearly zero. For the case without VSR unit, the vehicle is understeer.

CONCLUSIONS

Concept of variable steering ratio has been derived based on understanding of Ackerman angle. Limitation and conditions of VSR have been proposed and discussed. Bound of validity of VSR equation and linearity of a particular case of BMW 320i is approximately constructed based upon limitation of linear tire model. Effects of VSR unit to an understeer vehicle are discussed in details. Input/output envelope of VSR unit is also shown and described. Results show that within bound of validity, understeer, and oversteer cars with VSR will have neutral steer characteristic due to compensation by VSR unit. However, effects of VSR are certainly limited within its validity bound which depends on the linearity of the system.

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