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Original Article

Development and evaluation of teak (*Tectona grandis* L.f.) taper equations in northern Thailand

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ABSTRACT

Taper refers to the general decrease in the regular outline of a solid body from its base to its tip. Taper models are used to estimate the volume and value of wood products from harvesting trees. Teak (*Tectona grandis* L.f.) is highly valued as one of the world's most preferred timbers and a teak taper equation is required to inform optimal harvesting strategies given the limited plantation resource available in Thailand. Teak taper equations were developed and evaluated based on 331 sample trees collected in 2014 from eight plantations in northern Thailand aged from 10 to 46 yr using two taper model formulations—the Kozak variable-exponent taper model and the Goodwin cubic polynomial model comprising hyperbolic and parabolic terms. Variants based on both model types were fitted using nonlinear regression analysis with diameter at breast height, total tree height and height of girth measurement as the independent variables to estimate diameter underbark at the nominated height. Goodness-of-fit and leave-one-out cross validation with lack-of-fit statistical testing combined with extensive graphical analysis of residuals were used to select the best model. A Goodwin model variant (named FIO-teak1 as the first plantation teak taper model known to be published in Thailand) provided the best estimates of volume and diameter underbark. A simple case study confirmed that FIO-teak1 in combination with the Farm Forestry Toolbox software package could assist teak plantation managers in decision making associated with optimizing log grade value based on standing tree inventory data.

Introduction

Taper refers to the general decrease in the regular outline of a solid body from its base to its tip (Schreuder et al., 1993). Tree taper equations are important because reliable estimates of wood products and their value are essential to quantify expected commercial harvest returns (Salam and Pelkonen, 2012). Teak (*Tectona grandis* L.f.) is highly valued as one of the world's most preferred timbers (Thaiutsa, 2008; Ladrach, 2009).

Taper equations have been described for many species in almost every country where forest management has been administered, for example: more than 230 equations covering 50 species in Europe (Zianis et al., 2005); 25 species of eucalypts in Australia (Bi, 2000); 11 conifer species in the eastern USA and Canada (Li et al., 2012); 7 pine species in Swaziland (Crous et al., 2009); willow in Finland (Salam and Pelkonen, 2012), poplar in Sweden (Hjelm, 2013); radiata pine in Australia and New Zealand (Bi and Long, 2001; Goodwin, 2009); and *Styrax* sp. in Lao PDR (Ounekham, 2009). Many taper model forms and types have been developed and described; in addition to those above, see also Rojo et al. (2005), Hart (2009), Westfall and Scott (2010), Fonweban et al. (2011) and de-Miguel et al. (2012), as all these studies and their associated references provide extensive detail on taper model options and such discussion is beyond the scope of this paper.

The Forest Industry Organization (FIO) is a Thai government State enterprise whose role today includes managing more than 74,000 ha of government-owned, commercial, teak plantations throughout extensive areas of central and northern Thailand (Forest Industry Organization, 2014) with more than 80% located in northern Thailand (Thaiutsa, 2008).

There are no reports known of estate-level, teak taper equations available for use in Thailand, except for a simple trial example initiated by the first author and included in the Farm Forestry Toolbox (Goodwin, 2007; Warner, 2007). Therefore, the aim of the study was to develop a teak taper equation based on data collected from sample trees in available FIO plantations in northern Thailand.

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Materials and methods

Standard tree measuring equipment was used to collect sample tree data and consisted of: 1) a good quality fiberglass girth/diameter tape; 2) a fiberglass 25 m or 50 m length tape; 3) an altimeter (Haga Company; Nuremburg, Germany) for estimating the pre-felled, total tree height of each tree, in case the upper crown was destroyed during felling; 4) spray paint and chalk to mark reference details on each tree; 5) a hammer and chisel to extract bark chips and two small steel rulers with a scale in millimeters to measure the thickness of the bark; and 6) a global positioning unit to determine the easting and northing of each tree to facilitate any revisiting for data clarification. Chainsaw felling of each sample tree was carried out by FIO personnel. Field data were recorded on a customized paper sheet.

The dataset was stored in a customized Access database and some preliminary analysis and data checking used Excel, with both these software packages being components of the Office software package (2007; Microsoft Corp.; Redmond, WA, USA). The main data analysis was carried out using the R language and environment for statistical computing (R Core Team, 2015) linked with the RStudio software (version 0.98.1062; www.rstudio.com).

Sample tree selection, measurement and taper modeling

Stands in eight FIO plantations in four northern Thai provinces were sampled (see Table 1 for statistics). A sampling procedure selecting sample trees based on area stratified by age using a specially designed recording sheet was developed, then tested and revised with the FIO data measurement teams, emphasizing strict procedural consistency and accuracy. Additional field checks of the teams and some data checking were undertaken during the sample tree measurement phase (January–May, 2014).

Accurate modeling of taper to determine different high-value products was required in the lower bole, so girth measurements were taken above ground level at 0.3 m, 0.5 m, 0.8 m and breast height (1.3 m above ground on the uphill side of the tree) to also provide sufficient detail to allow sectional area to be corrected if necessary for pronounced buttressing in the lower bole. Digital photographs of chainsawn cross sections including a metric scale measure were taken at these lower sampling heights where there appeared to be buttressing, so that image analysis could be carried out post sampling if required. Sampling occurred usually at 2 m intervals above breast height at a representative point (no obvious defect or exceptional girth) until the main stem was no longer apparent. Total height (to the nearest centimeter) was measured to the tallest green shoot. At each representative sample point, measurements were recorded of the overbark circumference (recorded as the girth to the nearest millimeter) and of bark thickness (to the nearest millimeter, in the holes formed by the removal of bark chips down to the cambium at three equidistant points around the girth at each measurement height, to derive an average bark thickness) and height from the ground (to the nearest centimeter, based on the reference line marked at breast height before felling).

Table 1
Summary statistics for the 331 teak sample trees by plantation location.

Location[a]	KMK	WGC	MMS	MMJ	MMM	TGK	MHP	MML
Tree count (total = 331)								
Number	54	51	35	52	42	33	19	45
Diameter at breast height overbark (cm)								
Mean	35.0	25.2	28.3	24.2	25.1	35.9	33.6	24.0
SD	8.58	5.37	5.98	5.55	4.50	7.88	5.22	4.83
Minimum	18.2	16.1	19.6	15.8	16.9	24.0	26.7	15.9
Maximum	53.2	39.9	45.0	36.5	38.3	52.6	45.4	36.1
Total height (m)								
Mean	24.8	22.6	23.5	19.2	22.0	25.4	24.0	18.5
SD	4.19	3.61	2.60	3.81	2.60	3.62	2.91	2.75
Minimum	15.2	14.1	17.8	12.4	16.3	18.1	18.5	14.1
Maximum	31.2	31.5	27.8	26.7	27.1	32.9	29.0	28.2
Tree age (yr)								
Age range	35	18	10	18	14	16	7	16
Minimum	10	18	26	28	28	30	34	27
Maximum	43	35	35	45	41	45	40	42
Number of record heights per tree								
Mean	11	12	12	10	11	13	12	10

[a] Phrae province (KMK = Kunmaekammee; WGC = Wangchin; MMS = Maesaroi); Lampang province (MMJ = Maejang; MMM = Maemai; TGK = Tungkwean); Chiang Mai province (MHP = Maehopha); Lamphun province (MML = Maelee).

Two taper model formulations were chosen based on a literature review and also on their different approaches, so that they could be tested for their suitability to model teak taper as described below. Variants of both models were appraised by removing terms.

Kozak's variable-exponent taper model was chosen as it has been successfully applied to many species globally including in North America, Europe, Scandinavia and Asia (Kozak, 2004; Heidarsson and Pukkala, 2011; Fonweban et al., 2012). Model “02” was the last in a series of models developed by Kozak and associated researchers; this model was chosen because it was reported to be consistently the best for estimating diameter underbark and tree and log volumes (Kozak, 2004). Notably, it includes an implied taper and bark thickness model because the diameter at breast height overbark is an input (Equation (1)):

$$d_{ub} = a_0 D_{ob}^{a_1} H^{a_2} X^M \tag{1}$$

where

$$M = b_1(h/H)^4 + b_2(1/e^{(D_{ob}/H)}) + b_3 X^{0.1} + b_4(1/D_{ob}) + b_5 H^Q + b_6 X$$

$$X = 1 - (h/H)^{1/3}/1 - (1.3/H)^{1/3}$$
$$Q = (1 - (h/H)^{1/3})$$

and a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5 and b_6 are coefficients, d_{ub} is the diameter underbark (centimeters), measured at height h (meters) above ground, D_{ob} is the diameter overbark (centimeters) at breast height and H is the total tree height (meters).

The second taper model tested was described by Goodwin (2009) as a cubic polynomial comprising hyperbolic and parabolic terms. It has been generally used in Australia where Wang and Baker (2005) found it to be better than the Kozak model for plantation *Eucalyptus globulus* in Victoria. Second-stage models (β_1, β_2 and β_3) suggested by Goodwin (2007, 2009) as applicable to many species were used to develop the starting point in the current study (Equation (2)):

$$d_{ub} = (H - h)(S + \beta_3(h - h_1) + D_{ub}/[H - h_1]) \tag{2}$$

where $S = \beta_1 \beta_2^2 (h_1 - h)/[(1 + \beta_2 h)(1 + \beta_2 h_1)(1 + \beta_2 H)]$

$$\beta_1 = c_0 + c_1 H + c_2 H^2 + c_3 (D_{ub}/10)^2$$
$$\beta_2 = d_0 + d_1 H + d_2/H$$
$$\beta_3 = f_0 + f_1 H + f_2/H + f_3(D_{ub}/10) + f_4(D_{ub}/10)^2$$

and $c_0, c_1, c_2, c_3, d_0, d_1, d_2, f_0, f_1, f_2, f_3$ and f_4 are second stage candidate coefficients, d_{ub} is the diameter underbark (centimeters), measured at height h (meters) above ground, D_{ub} is the diameter underbark (centimeters) at breast height (h_1, meters) and H is total tree height (meters).

Statistical analysis

Taper models were developed using nonlinear regression (using the *nls* and *nlme* modules in R) with extensive use made of graphical analysis, several goodness-of-fit (GOF) statistics and an index derived from lack-of-fit (LOF) analysis statistics based on cross validation to provide comparative information regarding models based on the same dataset. While mixed effects models containing both fixed and random model parameters that can be estimated simultaneously have been reported to improve the precision of taper functions, Fonweban et al. (2012) also noted that the improved performance from mixed-effects models over fixed-effects models was dependent on additional measurements or observations, while de-Miguel et al. (2012) considered that fixed-effects models are more accurate when the aim is prediction, as in the current study. Thus, mixed effects were not considered in this study but deserve future investigation. Preliminary modeling with both model types found no benefit from applying weights, which was consistent with the approach reported by Goodwin (2009) and Kozak (2004) in their major studies of their respective models.

Recognizing the potential correlation among data points taken from the same tree, the model analyses avoided using any confidence limits or hypothesis tests even though the predictive effect of a model would be unaffected as the estimates of the regression coefficients are still unbiased (see for example, West et al., 1984; Tasissa and Burkhart, 1998; Kozak, 2004; Rojo et al., 2005).

The residual standard error (the square root of the sum of squares divided by the respective degrees of freedom), the adjusted coefficient of determination (R^2_{adj}) and the Bayesian information criterion (BIC) were used for GOF analysis to select the better models for further LOF analysis and validation testing. These statistics have been widely reported as suitable for comparison between models based on the same dataset, for example, by Ritz and Streibig (2008), Maindonald and Braun (2010), Fonweban et al. (2011) and Tahar et al. (2012), from which Equations (3) and (4) were sourced:

$$R^2_{adj} = 1 - \left(\frac{(n-1)\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{(n-p)\sum_{i=1}^{n}(y_i - \bar{y})^2} \right) \quad (3)$$

where $y_i, \hat{y}_i$ and $\bar{y}$ are the measured, predicted and average values of the dependent variable, respectively, n is the total number of observations used to fit the model and p is the number of model parameters.

$$\begin{aligned} \text{BIC} = &-2(\text{maximized log likelihood}) \\ &+ \ln(n)(\text{number of parameters}) \end{aligned} \quad (4)$$

where n is the number of observations and the BIC tends to penalize more complex models, with lower values usually resulting for simpler models (Hastie et al., 2013) and was considered suitable for GOF appraisal (Shmeuli, 2010).

The best of both the Goodwin and Kozak model variants based on their GOF statistics were then chosen for further analysis using LOF procedures.

The best test of an equation to indicate how well it predicts is to consider the accuracy of its predictions which can be done using cross validation—testing the model on data not used in the model fitting—and evaluating LOF statistics (Maindonald and Braun, 2010). Leave one out (LOO) cross validation is a well known statistical approach (Venables and Ripley, 2002; Maindonald and Braun, 2010; Hastie et al., 2013) that has been used in forestry and reported to be reliable in the evaluation of the predictive performance of models (for example, Tarp-Johansen et al., 1997; Bi and Long, 2001; Kozak and Kozak, 2003; Rojo et al., 2005). LOO cross validation was applied to each of the 331 trees in turn to produce estimates for each excluded tree based on the model fit using the remaining 330 trees. These data were then subjected to LOF analysis, using the percentage error ($\bar{e}\%$) as a measure of the overall prediction accuracy and also to indicate positive and negative bias (Fonweban et al., 2011) and the relative error in prediction (RE%) to indicate the precision of the estimates (Huang et al., 2003); these terms are defined in Equations (5) and (6):

$$\bar{e}\% = 100 \times \left(\sum_{i=1}^{n} \frac{\left(y_i - \hat{y}_i\right)}{n} \right) \Big/ \bar{y} \quad (5)$$

$$RE\% = 100 \times \sqrt{\left(\sum_{i=1}^{n} \frac{\left(y_i - \hat{y}_i\right)^2}{n} \right)} \Big/ \bar{y} \quad (6)$$

where y_i is an observed value and $\hat{y}_i$ is its predicted value, n is the number of observations, $\bar{y}$ is the mean of the observed values and the closer the terms are to zero, the better.

LOF analysis investigated three different aspects of the models using the LOO procedure: 1) prediction of d_{ub} given h; 2) prediction of h given d_{ub}; and 3) prediction of the volume underbark of a log in each sample tree with the upper and lower log heights selected at random. The sample tree measurements were divided into roughly equal classes so that the LOF could be appraised at different diameter and relative height ranges in the sample trees. The results were combined into an unweighted index using the LOF statistics from the three tests with the lowest combined index determining

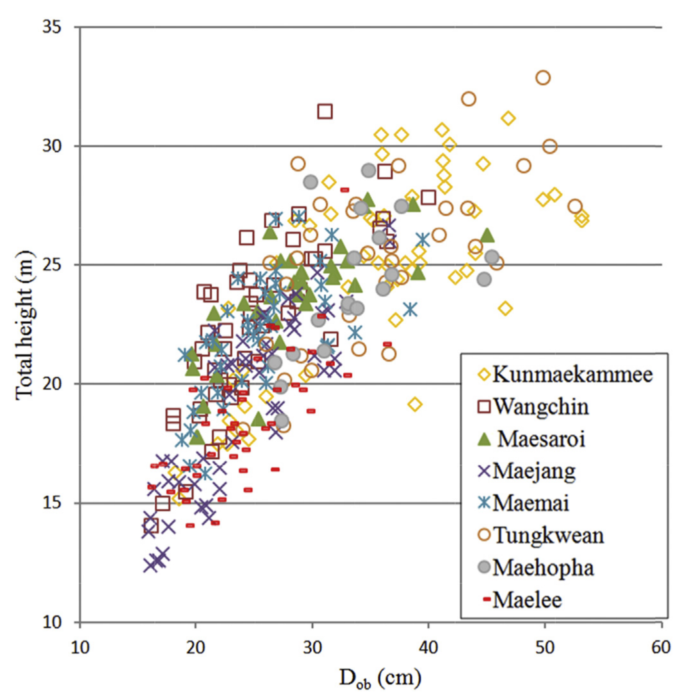

Fig. 1. Sample tree height and diameter at breast height overbark (D_{ob}) by plantation.

Table 2
Summary of goodness-of-fit statistics for models selected for lack-of-fit analysis (bold numbers indicate best model for each statistic).

Model	Coeffs[a]	Adjusted[b]	Residual SE	ΔBIC[c]
Diameter at breast height underbark input data				
Goodwin 5a	6	0.98454	1.03018	80
Goodwin 6a	5	0.98453	1.03046	75
Goodwin 7	4	0.98342	1.06680	328
Goodwin X3	6	0.98482	1.02090	12
Goodwin X3A	5	0.98480	1.02160	10
Goodwin X4	5	**0.98484**	**1.02027**	**0**
Kozak02	9	0.98264	1.04590	216
Kozak021	8	0.98406	1.04615	210
Kozak024	8	0.98398	1.04875	229
Diameter at breast height overbark input data				
Kozak02	9	0.98264	1.09181	539
Kozak021	8	0.98263	1.09203	533
Kozak024	8	0.98258	1.09348	543

[a] Number of coefficients in model.
[b] Adjusted coefficient of determination, with values shown to 5 decimal places to highlight the small differences between some models.
[c] Bayesian information criterion (BIC) difference from the model with the lowest BIC = 0 (actual value = 10,874); differences >10 provide strong evidence to reject the null hypothesis (Kass and Raftery, 1995).

Table 3
Index values for three separate and for overall lack-of-fit statistics for models based on diameter at breast height overbark and underbark input data (bold numbers indicate best model for each statistic).

	d_{ub}[a] given h[b]	h given d_{ub}	V[c] given d_{ub}	Overall[d]
Diameter at breast height overbark input data				
Goodwin 5a	1.065	0.957	0.896	0.973
Goodwin 6a	1.041	0.977	0.901	0.973
Goodwin 7	1.204	1.154	**0.625**	0.994
Goodwin X3	1.025	0.794	0.806	0.875
Goodwin X3A	1.009	0.761	0.681	**0.817**
Goodwin X4	1.205	**0.729**	0.852	0.929
Kozak 02	1.215	1.040	1.324	1.193
Kozak 021	1.215	1.092	1.316	1.208
Kozak 024	1.291	1.146	1.337	1.258
Diameter at breast height underbark input data				
Goodwin 5a	0.639	1.078	0.997	0.905
Goodwin 6a	**0.629**	1.107	1.017	0.918
Goodwin 7	0.696	1.346	0.810	0.951
Goodwin X3	0.755	1.021	1.048	0.941
Goodwin X3A	0.650	0.988	1.014	0.884
Goodwin X4	0.710	0.911	0.923	0.848
Kozak 02	1.186	0.897	1.147	1.077
Kozak 021	1.186	0.948	1.140	1.091
Kozak 024	1.278	1.053	1.167	1.166

[a] Diameter underbark.
[b] Height.
[c] Log volume given two log end diameters underbark.
[d] Combined 3 statistics using equal weighting.

the best model (Oswalt and Saunders, 2006; Goodwin, 2009; de-Miguel et al., 2012). The records for h = 1.3 m were omitted in the LOF analysis, as the residuals for such records were already constrained to zero by the Goodwin model formulation. Furthermore, to reduce potential correlation between measurements in the same tree, only one randomly chosen value from each tree in each subclass of the tree stem was used in each of the LOF procedures.

In the LOF comparisons, D_{ob} was converted to D_{ub} for input to the Goodwin model using a bark thickness model derived from the sample tree data to ensure a fair comparison with the Kozak model, since the Kozak taper model (using D_{ob} as an input) also included an implied bark thickness model. The Kozak models using D_{ub} as an input were also compared with the Goodwin models using D_{ub} to remove any confounding effect of bark thickness.

Results and discussion

Measurements from 331 sample trees were checked and compiled in a database (Fig. 1 and Table 1 present some of the data). Some d_{ub} data affected by pronounced buttressing (defined here as a difference between inferred tape sectional area and actual cross sectional area of greater than 3%) in the lower bole of larger trees were adjusted using cross sectional area analysis from the digital images.

Twenty-six models (18 Goodwin and 8 Kozak variants) were fitted using unweighted nonlinear regression and evaluated in the first instance with the GOF statistics.

Of the 26 models tested, Table 2 summarizes the GOF results for the better-performing models that were then subjected to LOF analysis. The high adjusted R^2 values (0.9825–0.9848) indicated that these models provided a good fit to the data. The original formulation (Kozak 02) was the best of the Kozak models for both D_{ob} and D_{ub} as input; the b_5 term was significant, in contrast to the results reported by Rojo et al. (2005). Generally, it was possible to reduce the Goodwin model variants to 4–6 terms without any excessive adverse effect on the GOF statistics.

The standardized residuals plotted against the residual values were also checked for each plantation and none indicated any major trends away from a balanced distribution around zero, supporting the use of the model in all eight plantations sampled (data not shown).

Graphical analysis of standardized residuals did not find any indications of serious bias or trends (for example, see Fig. 2). Scrutiny of a few outlying records provided no practical justification for

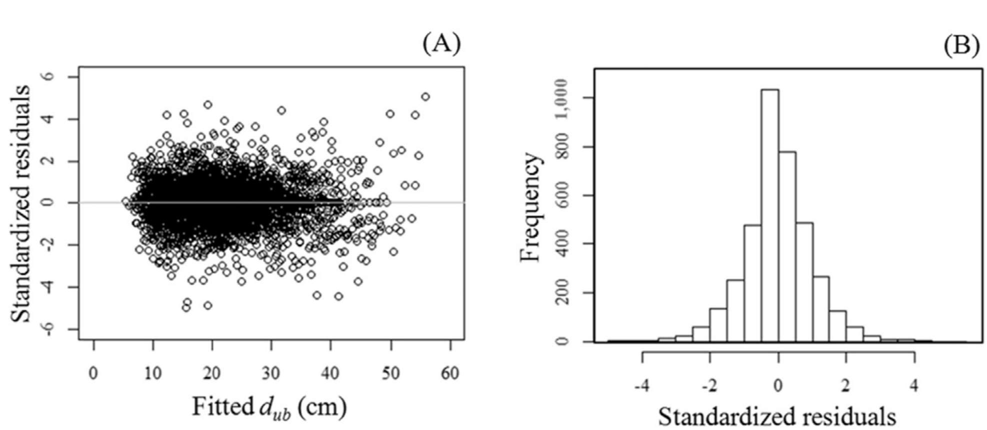

Fig. 2. Residual analysis for Goodwin X3A model: (A) standardized residuals versus fitted diameter underbark (d_{ub}) values; (B) histogram of standardized residuals.

their removal, and it was concluded that they simply reflected some of the variability inherent in a dataset collected from plantation teak trees.

Of several linear and nonlinear bark models described in the literature, the double bark thickness (*BT2*) power model developed using the nonbuttressed sample tree data ($BT2 = 3.03489$ $(d_{ob}^{0.62856})/10$, all units in centimeters) was the most suitable because importantly the standardized residuals were evenly distributed and heteroscedasticity was not apparent, unlike in the linear bark thickness models considered (data not shown).

The LOF analysis undertaken on the better performing models from the GOF analysis indicated that no taper model performed best in all tests nor did one test provide the overall best ranking, with the GOF rankings changing in the LOF analysis, which all emphasized the importance of using a range of tests (Kozak and Kozak, 2003). The LOF statistics were used to decide which model was best for the intended use in log volume prediction. Generally, estimation of diameter given height produced the most consistent predictions followed by volume given diameter and then diameter given height. Based on the combined LOF index, underbark models were generally more consistent than overbark, perhaps because the variation associated with a bark thickness model was removed, which improved the taper model consistency. However, if the bark thickness model was able to compensate for some overestimation or underestimation in the taper model, then the combined effect could provide an enhanced model. Goodwin models were better than Kozak models for comparable overbark or underbark input data (Table 3). Interestingly, Kozak models using D_{ub} as input were better than the same model using D_{ob} as originally proposed by Kozak (2004), suggesting that developing a separate *BT2* model from the taper model could be advantageous.

Based on the GOF and LOF analyses and the intended use of the model for log product optimization, Goodwin X3A was the best taper model using the measured D_{ob} (with the bark thickness model applied to the D_{ob} to determine D_{ub} as the model input; nominal reference data for the bark thickness and taper models: $D_{ob} = 35$ cm predicts *BT2* at h_1 of 2.8 cm; so $D_{ub} = 32.2$ cm; $D_{ob} = 35$ cm, $H = 27$ m; $h = 3.1$ m, predicts d_{ub} of 28.8 cm). Furthermore, using the actual D_{ub} as input was only slightly behind Goodwin X4 which was the best model overall using this input; however, D_{ob} is usually measured based on the taper equations referenced earlier. The LOF indices for the best 18 models are summarized in Table 3. The LOF histograms for Goodwin X3A (d_{ub} output determined from D_{ob} less double bark thickness to give D_{ub} as an input) for d_{ub} given h are shown in Fig. 3.

The estimation of d_{ub} given h based on the LOO cross validation for the Goodwin X3A model using D_{ob} corrected for bark thickness resulted in a mean value in each class that was within 1–2% of the measured value, with a generally non skewed distribution about the mean and no evidence of pronounced irregularity given the relatively small sample size in some classes (Fig. 3). The Goodwin X3A model predicted most values of d_{ub} in the lower most-valuable part of the bole to within 10%. A few extreme values were associated with smaller diameters, where a small difference produced a large ratio of estimated to actual, unlike in the relative height class at the bottom of the bole.

For h given d_{ub}, the Goodwin X3A model resulted in a mean estimate in the lower part of the bole that was within 4% and generally within 10%. Similarly the mean value was within 2% of the volume of a random log in the more valuable section of the bole (below 6 m) and more generally to within 10%, which was well in the range noted by Huang et al. (2003) as "realistic and reasonable", while some studies reported a deterioration in their models' predictions toward the top of the tree (for example, Tarp-Johansen et al., 1997; Bi and Long, 2001). Overall, the log volume was underestimated except for a slight tendency to overestimate in the lower 6 m (data not shown).

Case study: maximizing log value using the taper model

The log pricing options for teak plantation sawlogs in Thailand involve a complex trade-off between log center girth and log length,

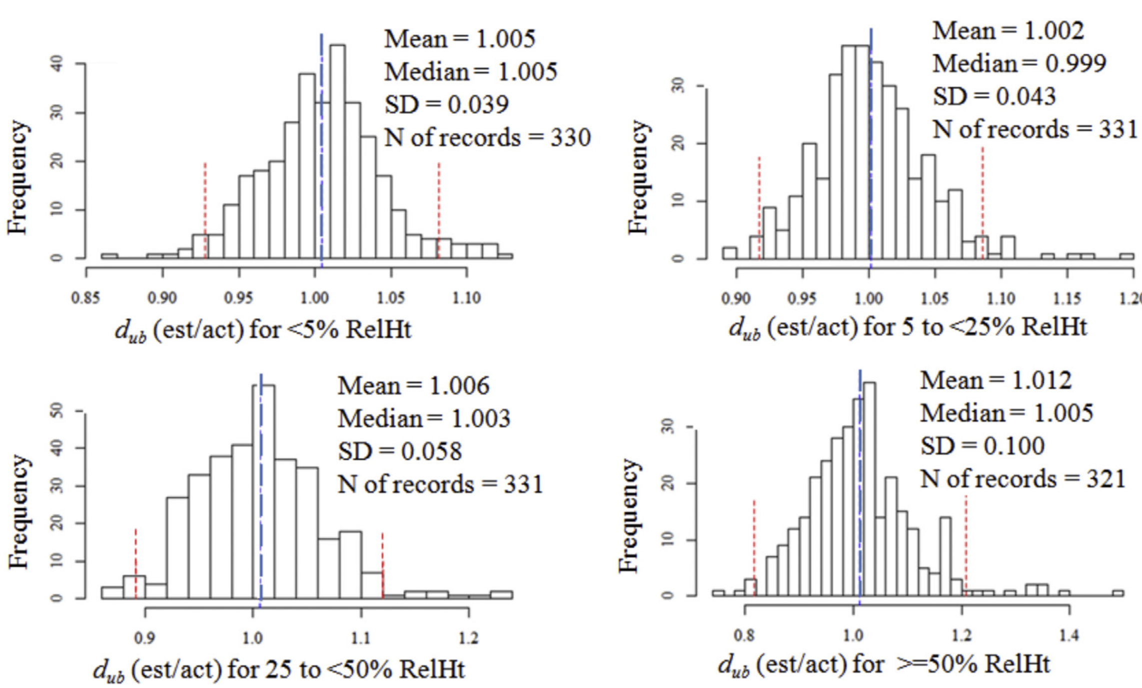

Fig. 3. Goodwin X3A model lack-of-fit statistics for diameter underbark (d_{ub}) estimated/actual (est/act) given height using breast height overbark adjusted by bark thickness as input for four relative height (RelHt) classes for one record per class per tree (N = number; large dashed line = mean; small dashed lines = mean ± 1.96 SD).

with more than 70 potential grades (Forest Industry Organization, 2015). The Goodwin X3A model was encoded in the Farm Forestry Toolbox software package (the Toolbox) which is capable of using a taper model and an optimization algorithm to determine the maximum log product value according to given log grade specifications (Warner, 2007). In a simple case study in comparison with one manual approach (which addresses the complexity of options by focusing on producing the longest possible logs of the highest grade from the bole upwards based on the log center girth), Toolbox optimization using Goodwin X3A enabled a more sophisticated analysis of log cutting options from a nominal tree ($D_{ob} = 50$ cm, $H = 30$ m, total bole volume $= 1.849$ m^3) and resulted in 14.8% more value from the same tree with exactly the same total volume of log products (1.826 m^3) than if it had been processed using the manual approach for the same log grading options. Further evaluation using extensive inventory data is planned.

Practical application

The Goodwin model X3A using D_{ob} corrected to D_{ub} as an input (named “FIO-teak1” as the first, known, published plantation teak taper model in Thailand) provided the best estimates based on graphical, goodness-of-fit and lack-of-fit analysis with nonzero coefficient values of $c_1 = 0.59256$, $d_0 = 0.63308$, $f_2 = 0.77715$, $f_3 = 0.012398$ and $f_4 = -0.0027653$ in Equation (2).

The FIO-teak1 model integrated in the Farm Forestry Toolbox produced sawlog products with a value nearly 15% higher than one manual method in a simple case study indicating its potential to increase financial returns and to empower forest managers in their evaluation of complex market options, log demands and silvicultural opportunities, thus increasing the productivity and profitability of commercial teak plantations in Thailand.

Conflict of interest

There is no conflict of interest.

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