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A Nonlinear Optimization Problem for Determining Safety Stocks in a Two-Stage Manufacturing System

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ABSTRACT

Safety stock is the inventory which is used to buffer against the uncertainties in business operations. Managers must decide how much safety stock of each raw material and each finished product should be maintained. Determining appropriate safety stock levels is an important decision. Too much safety stock would incur extra inventory carrying costs, whereas too less safety stock would increase the risk of having product stockouts and lost sales. In this paper, a nonlinear programming problem for determining safety stock levels in a two-stage manufacturing system, was presented. Instead of using the well-known search algorithms, simple decision rules for determining safety stock levels were derived from an analysis of the derivatives of cost functions, with respect to the delivery performances of suppliers and prior manufacturing process. Two algorithms based on those decision rules were proposed and tested on seventy-five problem instances. The results showed that the proposed algorithms provided, within 1 second, the solutions with less than 3% deviations, on average, from the known integer solutions or the best lower bounds. The algorithms also performed better than the pattern search algorithm, which was the method applied in the previous research.

Key words: safety stock, inventory, nonlinear programming problem, two-stage manufacturing systems

INTRODUCTION

Safety stock or buffer stock is the amount of inventory held in a short run to protect against demand and supply uncertainties and forecasting errors in business operations. When demands are underestimated, or supplies are insufficient or backordered, product stockouts may occur and cause the company some lost sales, especially when the degree of product substitutability is high. On the other hand, if too many safety stock quantities are held, high inventory costs would be charged to the company. The two types of costs: opportunity costs and inventory costs must be traded off to find the appropriate safety stock levels.

The classical approach for determining safety stock is to specify a desired service level or a stockout probability and use it to identify a safety factor, k. If the demand during lead time is assumed normally distributed, the safety factor is usually set to z and the safety stock is set to $z \cdot \sigma_L$, where z denotes the z-score to achieve the desired service level and σ_L denotes the standard deviation of the probability distribution of demand during lead time (Vollmann *et al.*, 1997). The other choices of safety factor, demand deviation, and safety stock calculations can be found in Krupp (1997); Silver

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et al. (1998); Zeng (2000); and Talluri *et al.* (2004).

Maia and Qassim (1999) derived optimum safety stocks for a one-stage manufacturing system, in which a finished product was produced from a number of raw materials. The problem was formulated as a nonlinear program (NLP), which minimized the total of inventory and opportunity costs. From the analysis, Maia and Qassim (1999) found that it was economical to either hold every safety stock at its maximum level or not hold it at all. A set of decision rules for finding optimum safety stocks was provided and illustrated through a small numerical example.

Siribanluoewut (2006) extended the work by Maia and Qassim (1999) to determine safety stocks for a two-stage manufacturing system. The problem was solved using three optimization heuristics, which were genetic algorithm, pattern search algorithm, and the hybrid genetic algorithm with pattern search. All the optimization heuristics performed efficiently on the test problems and the qualities of solutions reported were found not statistically different from each other. However, the pattern search algorithm provided good solutions in significantly shorter time than other heuristics did.

Inderfurth and Minner (1998) formulated an optimization problem of determining safety stocks in multi-stage manufacturing systems with normally distributed demands. The system was assumed to be under a periodic review, base-stock control policy, in which inventories were reviewed every fixed period of time and replenished up to a specified level. The safety factor in this study was found to be depending on service level, type of service level, and coverage time. The service level and coverage time for different types of multi-stage manufacturing systems were derived to establish the optimal policy for determining safety stocks in these multi-stage systems.

In this paper, the problem for determining safety stocks in the two-stage manufacturing system, as presented in Siribanluoewut (2006), was considered. Instead of using the optimization heuristics, which required the users to understand their mechanisms, a set of simple decision rules for finding optimum safety stocks was developed, and tested on the number of test instances as shown in the following sections.

MATERIALS AND METHODS

Problem description

A two-stage manufacturing system, as presented in Siribanluoewut (2006), was considered in this study. In such system, a manufacturer ordered *m* raw materials (RMs) for its stage-1 manufacturing process and *n* raw materials for its stage-2 manufacturing process. Each raw material was ordered from a single supplier. The stage-1 process produced a work-in-process (WIP) from those *m* raw materials. The WIP and the *n* other raw materials were then fed to stage 2 to produce a final product. Figure 1 illustrated this two-stage manufacturing system. The model formulation of this system was modified from that of the one-stage manufacturing system by Maia and Qassim (1999). The notations used in the formulation were as follows.

<u>Stage 1</u>

i	index of raw materials in stage 1; $i = \{1, 2,\ldots, m\}$
$p_{1,i}$	the on-time delivery performance of supplier i
$q^*_{1,i}$	the quantity of stage-1 raw material i that is delivered on time
$q_{1,i}$	the quantity of stage-1 raw material i that is ordered
$x_{1,i}$	the safety stock of stage-1 raw material i
$k_{1,i}$	the delivery performance to manufacture of stage-1 raw material i
$c_{1,i}$	the unit inventory cost of stage-1 raw material i
p_{s_1}	the stage-1 manufacturing performance
q_w	the quantity of WIP that is required
x_w	the safety stock of WIP

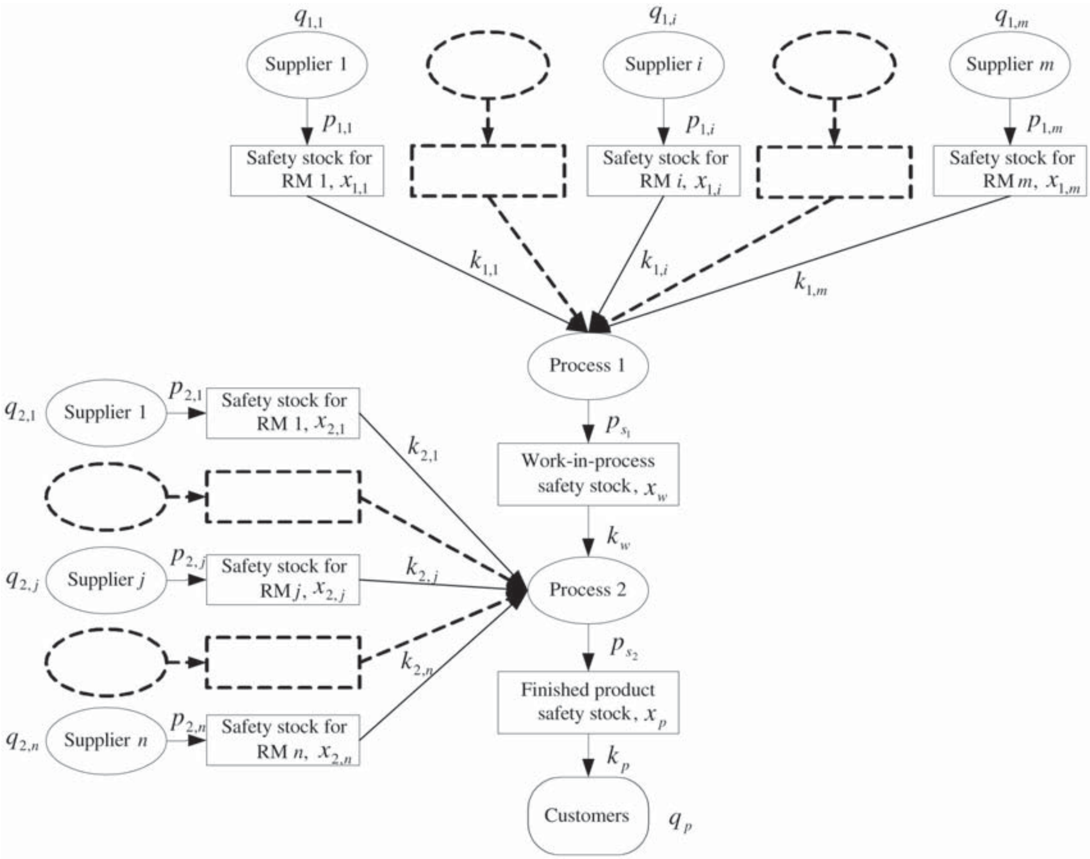

Figure 1 The two-stage manufacturing system.

k_w the WIP delivery performance to stage-2 manufacturing process

c_w the unit inventory cost of WIP

Stage 2

j index of raw materials in stage 2; $j = \{1, 2,\ldots, n\}$

$p_{2,j}$ the on-time delivery performance of supplier j

$q^*_{2,j}$ the quantity of stage-2 raw material j that is delivered on time

$q_{2,j}$ the quantity of stage-2 raw material j that is ordered

$x_{2,j}$ the safety stock of stage-2 raw material j

$k_{2,j}$ the delivery performance to manufacture of stage-2 raw material j

$c_{2,j}$ the unit inventory cost of stage-2 raw material j

p_{s_2} the stage-2 manufacturing performance

q_p the quantity of finished product that is required

x_p the safety stock of finished product

k_p The finished product delivery performance to customer

c_p the unit inventory cost of finished product

c_o the unit opportunity cost of finished product that is not delivered on time

As in Maia and Qassim (1999), the on-time delivery performance of supplier i and the on-time delivery performance of supplier j could be calculated from the past data records, using Equations (1) and (2), respectively.

$$p_{1,i} = \frac{q_{1,i}^{*}}{q_{1,i}} \tag{1}$$

$$p_{2,i} = \frac{q_{2,i}^{*}}{q_{2,i}} \tag{2}$$

If the manufacturer held safety stocks for every raw material, the delivery performances to manufacture of stage-1 raw material i and stage-2 raw material j could be defined as in Equations (3) and (4), respectively.

$$k_{1,i} = \frac{q_{1,i}^{*} + x_{1,i}}{q_{1,i}} = p_{1,i} + \frac{x_{1,i}}{q_{1,i}} \qquad \forall i = \{1, 2, \ldots, m\} \tag{3}$$

$$k_{2,j} = \frac{q_{2,j}^{*} + x_{2,j}}{q_{2,j}} = p_{2,j} + \frac{x_{2,j}}{q_{2,j}} \qquad \forall j = \{1, 2, \ldots, n\} \tag{4}$$

The inventory cost of the safety stock of each raw material could be computed from Equations (5) or (6) as follows.

$$C_{1,i} = c_{1,i}x_{1,i} = c_{1,i}q_{1,i}(k_{1,i} - p_{1,i}) \qquad \forall i = \{1, 2, \ldots, m\} \tag{5}$$

$$C_{2,j} = c_{2,j}x_{2,j} = c_{2,j}q_{2,j}(k_{2,j} - p_{2,j}) \qquad \forall j = \{1, 2, \ldots, n\} \tag{6}$$

The manufacturing performance of stage-1 process, p_{s_1}, was defined as the ratio between on-time and planned production, accounting for all delays that may occur, but excluding those caused by material stockouts. The p_{s_1} could be found from Equation (7). The WIP delivery performance to stage-2 manufacturing process, k_w, was given in Equation (8).

$$p_{s_1} = \frac{q_w^{*}}{q_w} \tag{7}$$

$$k_w = p_{s_1} \prod_{i=1}^{m} k_{1,i} + \frac{x_w}{q_w} \tag{8}$$

The inventory cost of the WIP safety stock could be calculated from Equation (9).

$$C_w = c_w x_w = c_w q_w (k_w - p_{s_1} \prod_{i=1}^{m} k_{1,i}) \tag{9}$$

Similarly, the manufacturing performance of stage-2 process, p_{s_2}, the product delivery performance, k_p, and the product inventory cost could be calculated as follows.

$$p_{s_2} = \frac{q_p^{*}}{q_p} \tag{10}$$

$$k_p = p_{s_2} k_w \prod_{j=1}^{n} k_{2,j} + \frac{x_p}{q_p} \tag{11}$$

$$C_p = c_p x_p = c_p q_p (k_p - p_{s_2} k_w \prod_{j=1}^{n} k_{2,j}) \tag{12}$$

Finally, the opportunity cost, defined as the cost incurring whenever the finished product failed to be delivered to the customers on time, was given in Equation (13).

$$C_o = c_o q_p (1 - k_p) \tag{13}$$

Mathematical model

A nonlinear programming (NLP) model, for determining the delivery performances $k_{1,i}$, k_w, $k_{2,i}$, and k_p was formulated in this section. The objective of this NLP model was to minimize the total of the inventory costs charged for holding all the safety stocks and the opportunity costs, subject to the bounds on the delivery performances. The model was formulated as follows.

$$\text{Min } C = c_o q_p \left(1 - k_p\right) + \sum_{i=1}^{m} c_{1,i} q_{1,i} \left(k_{1,i} - p_{1,i}\right) + c_w p_w \left(k_w - p_{s_1} \prod_{i=1}^{m} k_{1,i} \right)$$
$$+ \sum_{j=1}^{n} c_{2,j} q_{2,j} \left(k_{2,j} - p_{2,j}\right) + c_p q_p \left(k_p - p_{s_2} k_w \prod_{j=1}^{n} k_{2,j} \right) \tag{14}$$

Subject to

$$p_{1,i} \le k_{1,i} \le 1 \qquad \forall i = \{1, 2, \ldots, m\} \tag{15}$$

$$p_{2,j} \le k_{2,j} \le 1 \qquad \forall j = \{1, 2, \ldots, n\} \tag{16}$$

$$p_{s_1} \prod_{i=1}^{m} k_{1,i} \le k_w \le 1 \tag{17}$$

$$p_{s_2} k_w \prod_{j=1}^{n} k_{2,j} \le k_p \le 1 \tag{18}$$

Solution analysis

It was known that the optimal solution of the NLP is necessarily on the border of the feasible region, if the Hessian matrix of the objective function is indefinite, as in this problem (see Marsden and Tromba (1981), for example). Therefore, the optimal delivery performances $k_{1,i}$, k_w, $k_{2,j}$, and k_p in the presented NLP must be either on their lower bounds or upper bounds. In this paper, the analysis followed the method in Maia and Qassim (1999) by defining reference costs, $c_{1,i}^*$ for the stage-1 raw material i, c_w^* for the WIP, and $c_{2,j}^*$ for the stage-2 raw material j, as shown in Equations (19) - (21). The upper bounds of these reference costs were found from the derivatives of the cost function with respect to the delivery performances $k_{1,i}$, k_w, $k_{2,j}$, and $k_{2,j}$.

$$c_{1,i}^* \le \frac{c_{1,i} q_{1,i}}{q_w p_{s_1} \prod_{i2=i+1}^{m} p_{1,i_2}} \qquad \forall i = \{1, 2, \ldots, m\} \tag{19}$$

$$c_w^* \le \frac{c_w q_w}{q_p p_{s_2} \prod_{j=1}^{n} p_{2,j}} \tag{20}$$

$$c_{2,j}^{*} \le \frac{c_{2,j}q_{2,j}}{q_p p_{s_2} k_w \prod_{j_2=j+1}^{n} p_{2,j_2}} \qquad \forall j = \{1,2,\ldots,n\} \tag{21}$$

The reference costs, $c_{1,i}^{*}$ c_{w}^{*} and $c_{2,j}^{*}$ were then analyzed against all the unit costs in the model to identify when the corresponding delivery performances and safety stocks should be set to their lower or upper bounds. If the opportunity cost was high, the manufacturer should hold safety stocks to prevent the products shortages. In contrary, it would not be economical to stock the materials, when the inventory costs (and hence the reference costs) were costly. The optimal solution of the presented optimization model could be derived as follows:

Stage-1 raw materials:

(i) If $c_{1,i}^{*} \le \min(c_w, c_p)$ and $\begin{cases} c_o \le c_{1,i}^{*} \text{ then } k_{1,i} = p_{1,i} \text{ and } x_{1,i} = 0 \\ c_o > c_{1,i}^{*} \text{ then } k_{1,i} = 1 \text{ and } x_{1,i} = q_{1,i}\left(1 - p_{1,i}\right) \end{cases}$

(ii) If $c_{1,i}^{*} > \min(c_w, c_p)$ then $k_{1,i} = p_{1,i}$ and $x_{1,i} = 0$

Work-in-process:

(iii) If $c_{w}^{*} \le c_p$ and $\begin{cases} c_o \le c_{w}^{*} \text{ then } k_w = p_{s_1}\prod_{i=1}^{m} k_{1,i} \text{ and } x_w = 0 \\ c_o > c_{w}^{*} \text{ then } k_w = 1 \text{ and } x_w = q_w\left(1 - p_{s_1}\prod_{i=1}^{m} k_{1,i}\right) \end{cases}$

(iv) If $c_{w}^{*} > c_p$ then $k_w = p_{s_1}\prod_{i=1}^{m} k_{1,i}$ and $x_w = 0$

Stage-2 raw materials:

(v) If $c_{2,j}^{*} \le c_p$ and $\begin{cases} c_o \le c_{2,j}^{*} \text{ then } k_{2,j} = p_{2,j} \text{ and } x_{2,j} = 0 \\ c_o > c_{2,j}^{*} \text{ then } k_{2,j} = 1 \text{ and } x_{2,j} = q_{2,j}\left(1 - p_{2,j}\right) \end{cases}$

(vi) If $c_{2,j}^{*} > c_p$ then $k_{2,j} = p_{2,j}$ and $x_{2,j} = 0$

Finished product:

(vii) $c_o > c_p$ then $k_p = p_{s_2} k_w \prod_{j=1}^{n} k_{2,j}$ and $x_p = 0$

(viii) $c_o > c_p$ then $k_p = 1$ and $x_p = q_p\left(1 - p_{s_2} k_w \prod_{j=1}^{n} k_{2,j}\right)$

Proposed algorithms

Since the exact values of the reference costs were not known, they could be initially set to their upper bounds in which all other raw-material delivery performances, besides the one corresponding to the considered raw material, ($k_{1,i}$: $\forall i \neq i'$ and $k_{2,j}$: $\forall j \neq j'$) were set at their lower bounds. The estimated reference costs of the raw materials in every stage were sorted in a non-decreasing order and the values were recalculated as in Equations (19) and (21). This solution finding algorithm was specified as Algorithm 1.

From the preliminary testing, it was found that when the estimated values of reference costs were not much different from each other or from the opportunity cost, Algorithm 1 may not always provide the optimal solutions. Algorithm 2 was then proposed. Again, the reference costs of the raw materials in every stage were sorted as in Algorithm 1. At the initial step, the delivery performances and safety stocks of all raw materials were set to their lower bounds. The delivery performances and safety stocks of WIP and finished product were found from the decision rules presented in the previous section, accordingly. The total cost was calculated and recorded. Then, the delivery performance and safety stock of each raw material in each stage were increased to their upper bounds, one by one, corresponding to the non-decreasing order of the raw-material reference costs. The delivery performances and safety stocks of WIP and finished product, including the total costs, were recalculated and recorded at every step. Finally, the minimum total cost and the best solution were identified.

A numerical example

In this section, a small example, consisting of three raw materials in stage 1 and two raw materials in stage 2, was presented. The data for this example was given in Table 1. The opportunity cost was assumed to be 8.44 baht.

Algorithm 1:

The initial reference costs for stage-1 raw materials 1, 2 and 3 were found to be 2.1778, 5.4652 and 9.2014 baht, respectively. Thus, the stage-1 raw material order followed the natural order. The reference costs for RM 1, RM 2 and RM 3 were recalculated and their values became 2.1778, 5.1868 and 8.1043 baht, respectively. Following the proposed decision rules, the safety stocks of RM 1 and RM 2 should be set to their upper bounds, which were 8 and 10 units, respectively. The safety stock for RM 3 and WIP were found unnecessary.

Next, the initial reference costs for stage-2 raw materials 4 and 5 were found to be 7.5490 and 5.5337 baht, respectively. Hence, the

Table 1 Data for a small example with three raw materials in stage 1 and two raw materials in stage 2.

Materials	*q*	*q**	*p*	*c*	Initial ref. cost (baht)	Ref. cost (baht)	Algorithm 1 *k*	Algorithm 1 *x*	Algorithm 2 *k*	Algorithm 2 *x*
RM 1	157	149	0.9490	2.86	2.1778	2.1778	1.0000	8	1.0000	8
RM 2	139	129	0.9281	8.29	5.4652	5.1868	1.0000	10	0.9281	0
RM 3	244	242	0.9918	7.44	9.2014	8.1043	0.9918	0	0.9918	0
WIP	232	224	0.9655	9.40	-	16.5918	0.9576	0	0.8887	0
RM 4	117	107	0.9145	8.88	7.5490	6.9549	1.0000	10	0.9145	0
RM 5	216	199	0.9213	3.50	5.5337	5.5337	1.0000	17	1.0000	17
Product	173	156	0.9017	7.20	-	-	1.0000	23.61	1.0000	46.21
Total cost (baht)							424.1001		415.0962	

algorithm would consider RM 5, prior to RM 4. The reference costs of RM 4 and RM 5 were recalculated and found to be 6.9549 and 5.5337 baht. Thus, the safety stocks of RM 4 and RM 5 were set to their upper bounds, which are 10 and 17 units, respectively. Finally, the product safety stock was computed and set to 23.61 units. The corresponding total cost is 424.10 baht.

Algorithm 2:

Following the initial reference costs found in Algorithm 1, the priority for increasing raw-material safety stock levels would be in the orders of RM 1 – RM 2 – RM 3 and RM 5 – RM 4. Twenty-four solutions were evaluated and shown in Table 2. From the Table, the sixth solution provided the minimum total cost of 415.10 baht, with the safety stock levels set to 8 units for RM 1, 17 units for RM 5, and 46.21 units for the finished product. Algorithm 2 provided a superior solution to Algorithm 1 for this test instance.

RESULTS

To facilitate the implementation, Algorithms 1 and 2 were coded in MATLAB® 6.5. Both algorithms were tested on 75 test instances (from 5 test problem sets, each with 15 instances) in Siribanluoewut (2006). Table 3 presented structures of the test instances and the

Table 2 The twenty-four solutions evaluated by Algorithm 2.

No.	Safety Stocks (units)							Total cost
	RM 1	RM 2	RM 3	WIP	RM 4	RM 5	Product	(baht)
1	0	0	0	0	0	0	62.14	447.42
2	8	0	0	0	0	0	56.19	427.44
3	8	10	0	0	0	0	47.13	445.15
4	8	10	2	0	0	0	46.09	452.54
5	0	0	0	0	0	17	52.67	438.73
6	**8**	**0**	**0**	**0**	**0**	**17**	**46.21**	**415.10**
7	8	10	0	0	0	17	36.38	427.23
8	8	10	2	0	0	17	35.25	433.98
9	0	0	0	0	10	17	41.43	446.56
10	8	0	0	0	10	17	34.36	418.58
11	8	10	0	0	10	17	23.61	424.10
12	8	10	2	0	10	17	22.38	430.09
13	0	0	0	36.33	0	0	41.56	640.70
14	8	0	0	25.82	0	0	41.56	564.82
15	8	10	0	9.84	0	0	41.56	497.48
16	8	10	2	8.00	0	0	41.56	495.10
17	0	0	0	36.33	0	17	30.33	619.36
18	8	0	0	25.82	0	17	30.33	543.48
19	8	10	0	9.84	0	17	30.33	476.14
20	8	10	2	8.00	0	17	30.33	473.76
21	0	0	0	36.33	10	17	17.00	612.16
22	8	0	0	25.82	10	17	17.00	536.28
23	8	10	0	9.84	10	17	17.00	468.94
24	8	10	2	8.00	10	17	17.00	466.56

average percentage of deviations from the optimal NLP total costs, including the solution times, by Algorithms 1 and 2. The result showed that Algorithm 2 did outperform Algorithm 1.

As aforementioned, the optimization model presented in this paper was an NLP model. Therefore, the levels of safety stocks in the final solution may be reported as non-integers. This safety stock determination problem could also be modeled as a mixed integer nonlinear program (MINLP) for minimizing the total of the opportunity costs and the inventory costs charged for holding all the safety stocks, subject to the bounds on the safety stock levels. The safety stocks $x_{1,i}$, x_w, $x_{2,j}$, and x_p, which were required to be integers, would be sought from the MINLP, in lieu of the delivery performances $k_{1,i}$, k_w, $k_{2,j}$, and k_p in the NLP. However, the MINLP was a much more complex problem. It may not be solved in reasonable computation times with regular optimization methods, even for small-size problem instances. Thus, it was suggested that the safety stock levels should be found by applying Algorithm 2 and then rounding down the non-integer safety stocks to their nearest integers. The rounded solutions were compared with true optimal integer solutions found from the enumeration method, in which all possible integer solutions were enumerated and evaluated. However, the enumeration method could not be implemented on the large problem instances, due to their long computation times. Therefore, only the true optimal integer solutions of test problem sets 1 and 2 could be identified. The qualities of these rounded solutions were presented in Tables 4 and 5. For test problem sets 3, 4 and 5, the rounded solutions were compared with the corresponding MINLP lower bounds (i.e. the optimal NLP solutions) instead. The qualities of these solutions were given in Tables 6-8. Furthermore, the pattern search algorithm (using a complete search, a mesh expansion factor of 1.0 and a mesh contraction factor of 0.5) was also investigated. The details of this algorithm can be found in Kolda *et al.* (2003). The qualities of the integer solutions found from the pattern search algorithm were also presented in Tables 4-8, for comparison purpose.

From Tables 4 and 5, Algorithm 2 with solution rounding provided high-quality results. The rounded solutions were 2.10% deviating from the known MINLP optimum on average (with a maximum deviation of 10.20%) for problem set 1, and 3.86% deviating from the known MINLP optimum on average (with a maximum deviation of 20.23%) for problem set 2. Algorithm 2 with solution rounding provided the good solutions in much shorter times (i.e. less than 1 second) than the enumeration method did (i.e. more than 7 minutes for problem set 1 and more than 35 minutes for problem set 2, on average). For larger test problem sets, the average deviation of the Algorithm-2 solutions from the corresponding MINLP lower bounds were less than 2.5%, with

Table 3 Structures of the test instances and the average percentage of deviations from the true optimal total costs of Algorithms 1 and 2.

Set	No. of RMs		No. of	% Deviation from true optimum		Average solution time (sec.)	
	Stage 1	Stage 2	instances	Algorithm 1	Algorithm 2	Algorithm 1	Algorithm 2
1	3	1	15	0.00%	0.00%	0.0013	0.0047
2	3	2	15	0.47%	0.00%	0.0013	0.0033
3	7	2	15	0.24%	0.00%	0.0020	0.0047
4	12	2	15	0.98%	0.00%	0.0033	0.0073
5	15	2	15	2.79%	0.00%	0.0013	0.0087
		Average		0.90%	0.00%	0.0019	0.0057

Table 4 The quality of the rounded solutions for test problem set 1.

No.	Enumeration		Algorithm 2			Pattern search		
	Total costs (baht)	Time (seconds)	Total costs (baht)	Time (sec)	% Dev. from opt.	Total costs (baht)	Time (sec)	% Dev. from opt.
1	226.2863	557.00	226.2863	0.03	0.00	226.2863	0.656	0.00
2	223.0773	454.98	245.4778	0.00	10.04	223.3209	0.547	0.11
3	188.9893	574.17	191.7372	0.00	1.45	190.3275	0.89	0.71
4	174.6790	233.69	179.7564	0.00	2.91	174.6790	0.453	0.00
5	422.4267	616.30	434.7873	0.00	2.93	442.0500	0.656	4.65
6	538.7574	805.59	538.7574	0.00	0.00	538.7574	0.343	0.00
7	85.4226	753.52	86.9119	0.00	1.74	89.8888	0.484	5.23
8	194.1343	40.50	196.3468	0.00	1.14	199.1684	0.578	2.59
9	53.3469	9.66	58.7888	0.00	10.20	58.6819	0.344	10.00
10	240.7639	16.64	240.7639	0.00	0.00	240.7639	0.391	0.00
11	396.7343	419.02	398.7735	0.00	0.51	453.2415	0.453	14.24
12	173.3962	692.55	173.4065	0.00	0.01	176.3493	0.61	1.70
13	359.3905	51.55	359.3905	0.00	0.00	359.3905	0.562	0.00
14	225.1412	171.69	226.3883	0.02	0.55	244.0558	0.422	8.40
15	399.7456	1182.19	399.7456	0.00	0.00	399.7456	0.484	0.00
Average		438.6033		0.0033	2.10		0.5249	3.18

Table 5 The quality of the rounded solutions for test problem set 2.

No.	Enumeration		Algorithm 2			Pattern search		
	Total costs (baht)	Time (seconds)	Total costs (baht)	Time (sec)	% Dev. from opt.	Total costs (baht)	Time (sec)	% Dev. from opt.
1	231.96	388.13	231.96	0.05	0.00	231.96	0.70	0.00
2	231.15	310.00	267.04	0.00	15.52	232.33	0.58	0.51
3	195.83	843.94	196.89	0.00	0.54	198.17	0.55	1.20
4	286.68	1640.67	291.76	0.00	1.77	286.68	0.63	0.00
5	448.69	3548.72	461.05	0.00	2.75	468.31	0.84	4.37
6	623.79	3839.44	623.79	0.00	0.00	623.79	0.48	0.00
7	95.46	3736.39	99.51	0.00	4.24	97.66	0.61	2.30
8	233.65	252.59	235.87	0.00	0.95	245.51	0.53	5.08
9	63.30	22.25	76.10	0.00	20.23	73.49	0.42	16.11
10	353.81	161.06	353.81	0.00	0.00	353.81	0.59	0.00
11	415.36	1184.09	415.36	0.00	0.00	415.36	0.72	0.00
12	137.41	1197.42	150.03	0.00	9.18	142.33	0.64	3.58
13	110.35	9400.22	111.73	0.00	1.25	110.35	0.63	0.00
14	171.36	5738.92	173.87	0.00	1.47	238.33	0.69	39.08
15	210.21	930.83	210.21	0.02	0.00	210.21	0.64	0.00
Average	2212.98	0.00	3.86	0.62	4.82			

Table 6 The quality of the rounded solutions for test problem set 3.

No.	LB of Total costs (baht)	Algorithm 2 Total costs (baht)	Algorithm 2 Time (sec)	Algorithm 2 % Dev. from LB	Pattern search Total costs (baht)	Pattern search Time (sec)	Pattern search % Dev. from LB
1	341.4262	349.9014	0.05	2.48	381.6719	0.532	11.79
2	698.4675	699.5436	0.00	0.15	698.7817	0.641	0.04
3	306.6030	313.5858	0.00	2.28	327.0063	0.469	6.65
4	782.9640	789.0657	0.00	0.78	791.1078	0.516	1.04
5	141.2319	145.3478	0.00	2.91	147.0369	0.625	4.11
6	248.9744	252.0126	0.00	1.22	250.6189	0.5	0.6
7	418.7751	418.7751	0.00	0.00	418.7751	0.313	0.00
8	870.7950	871.3350	0.00	0.06	871.3350	0.516	0.06
9	188.2021	189.3901	0.00	0.63	189.3901	0.563	0.63
10	607.2085	611.1003	0.02	0.64	618.0523	0.766	1.79
11	310.3231	317.2456	0.00	2.23	313.7932	0.578	1.12
12	767.5441	779.7637	0.00	1.59	775.4064	0.687	1.02
13	796.4144	811.3657	0.00	1.88	872.1025	0.765	9.50
14	216.6816	220.6952	0.00	1.85	226.5848	0.563	4.57
15	329.8299	331.9408	0.00	0.64	348.9675	0.453	5.80
Average			0.0047	1.29		0.5658	3.25

Table 7 The quality of the rounded solutions for test problem set 4.

No.	LB of Total costs (baht)	Algorithm 2 Total costs (baht)	Algorithm 2 Time (sec)	Algorithm 2 % Dev. from LB	Pattern search Total costs (baht)	Pattern search Time (sec)	Pattern search % Dev. from LB
1	781.7861	781.7861	0.05	0.00	937.4422	0.875	19.91
2	116.0982	128.7529	0.00	10.90	118.4608	1.125	2.04
3	250.6759	253.0172	0.00	0.93	276.9489	1.281	10.48
4	863.7852	866.8405	0.02	0.35	932.2620	0.844	7.93
5	416.318	425.6585	0.00	2.24	434.4976	1.282	4.37
6	769.3648	784.0514	0.00	1.91	775.9314	0.906	0.85
7	831.0751	834.5863	0.00	0.42	861.3861	0.875	3.65
8	732.6523	737.9136	0.00	0.72	769.4482	1.157	5.02
9	171.9773	180.0650	0.02	4.70	175.0647	0.765	1.80
10	356.2244	366.1220	0.00	2.78	359.5120	1.062	0.92
11	883.9080	883.9080	0.02	0.00	883.9080	0.39	0.00
12	197.3155	217.3515	0.00	10.15	206.1805	0.672	4.49
13	532.1986	533.0392	0.00	0.16	632.9300	0.703	18.93
14	905.1188	910.7604	0.00	0.62	921.6216	1.125	1.82
15	805.6426	807.9736	0.00	0.29	845.8867	0.781	5.00
Average			0.0073	2.41		0.9229	5.81

the maximum deviation of about 10%. The solving times were still less than 1 second for all test instances.

The qualities of solutions and the computation times from Algorithm 2 and from pattern search seemed to be competitive, especially for the small-size test problems. The differences between the total costs found from Algorithm 2 and from pattern search were compared using the paired t-test and the signed rank test (Montgomery and Runger, 2004). The former was tested whether or not the average of the differences in total costs equaled zero. The latter was a non-parametric hypothesis test on the median of the differences in total costs. Under the normality assumption of data, the paired t-test was more powerful than the signed rank test. However, the signed rank test was less sensitive to the outliers. Herein, the signed rank test was applied because the distributions of the total costs showed significant departures from normal distributions. The summary of the statistical tests was presented in Table 9.

Table 8 The quality of the rounded solutions for test problem set 5.

No.	LB of Total costs (baht)	Algorithm 2			Pattern search		
		Total costs (baht)	Time (sec)	% Dev. from LB	Total costs (baht)	Time (sec)	% Dev. from LB
1	335.2583	341.7339	0.05	1.93	338.9744	1.204	1.11
2	448.8165	457.5385	0.02	1.94	468.6784	1.078	4.43
3	241.1376	243.6205	0.00	1.03	243.0759	1.313	0.80
4	689.6022	692.9453	0.00	0.48	708.7569	1.641	2.78
5	977.388	980.5293	0.02	0.32	1010.1633	1.391	3.35
6	326.5053	328.5352	0.00	0.62	328.1805	1.078	0.51
7	750.4167	751.3261	0.00	0.12	751.2409	1.000	0.11
8	810.929	813.4720	0.00	0.31	858.0251	1.766	5.81
9	763.4033	764.7848	0.00	0.18	777.3780	1.250	1.83
10	1047.4942	1047.4942	0.02	0.00	1047.4942	1.734	0.00
11	964.0926	972.7866	0.00	0.90	1626.5400	1.532	68.71
12	600.9621	612.3057	0.00	1.89	685.6725	0.656	14.10
13	819.0913	820.9975	0.00	0.23	1098.4323	1.265	34.10
14	966.8934	968.9883	0.00	0.22	1042.7315	1.360	7.84
15	730.5132	730.5132	0.02	0.00	730.5132	1.172	0.00
Average			0.0087	0.68		1.2960	9.70

Table 9 The statistical results from the paired t-test and the signed rank test.

Problem set	Average of the difference in total costs	Median of the difference in total costs	*p*-value	
			Paired t-test	Signed rank test
1	3.9592	0.0000	0.3578	0.7109
2	1.9540	0.0000	0.7095	< 1.0000
3	8.6375	1.6891	0.3865	0.1180
4	27.9770	10.8612	0.0372*	0.0438*
5	79.2191	12.5932	0.0999	0.0287*

* indicates a significant difference in total costs found from both methods

From the statistical tests, the qualities of solutions found from both methods were not significantly different for test problem sets 1, 2 and 3. However, Algorithm 2 became superior to the pattern search for larger problem sets. Notice on the test results, the total cost obtained from the pattern search could be as poor as 68% deviating from the MINLP lower bounds in large problem instances, while those from Algorithm 2 would not be worse than 20% from the lower bounds. Algorithm 2 was hence the most efficient method for solving this safety stock determination problem, in terms of both solution quality and computation time.

DISCUSSION

It had been shown in the previous section that Algorithm 2, which was based on a basic NLP theorem, could provide high quality solutions in short computation times for the safety stock level determination problem in the considered two-stage manufacturing system. The algorithm utilized only a set of simple decision rules, in contrast to the pattern search heuristic, which required the users to comprehend its mechanisms. The decision rules for finding optimum safety stocks also matched the common managerial logics that when the opportunity cost was high, the safety stocks should be held to prevent the product deficiency, but they should not be stocked when the inventory costs were high. Moreover, the search heuristic such as pattern search would terminate the search after some stopping criteria had been satisfied. Therefore, in some cases, it might not thoroughly search the solution space for the solutions.

CONCLUSION

In this research, two algorithms for determining safety stocks in a two-stage manufacturing system were proposed by analyzing the derivatives of cost function and the cost comparisons. The algorithms were found to work very efficiently on the test problems. They could provide high-quality solutions for every test instance in less than 1 second. The deviations from the known integer solutions or the lower bounds were less than 3% on average. The algorithms also outperformed the pattern search algorithm, which was presented in the previous research.

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