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The Measurement of Elastic Constants of Solids and Liquid by Using Line-Focus Transducer

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ABSTRACT

This paper presents the determination of the elastic constants of solids and liquid by using line-focus transducer. Poisson's ratio ν, Young's modulus *E*, the shear modulus *G*, and the bulk modulus *K* of stainless steel, iron and aluminium were determined from the Rayleigh wave speed, measured by defocusing technique and the longitudinal wave speed, measured by pulse-echo technique. The bulk modulus of diesel and paraffin oil were determined from the longitudinal wave speed, measured by defocusing technique. The results obtained in this paper were in good agreement with the results in the standard references.

Key words: elastic constants, line-focus transducer

INTRODUCTION

A high frequency acoustic microscope operated at 100 MHz and above has been used for imaging of surface acoustic properties as well as quantitative measurement of leaky Rayleigh wave speed (Koichiro *et al.*, 1998). However, the penetration depth of the leaky Rayleigh wave is limited within a few ten microns for most engineering materials. This is the same size as a grain diameter of most metals, thus the accuracy of the wave speed measurement is extremely difficult for polycrystalline metals. Therefore, low frequency acoustic microscopy with short pulses has been proposed and applied for characterization of engineering materials. The advantage of PVDF focused transducer (Xiang *et al.*, 1996), in which an ultrasonic wave is emitted and received on the PVDF film, has made leaky Rayleigh wave speed measurement more accurate in the time domain.

This paper presents methods of determining the elastic constants of stainless steel, iron and aluminium from leaky Rayleigh wave speed measured by line-focus transducer and a longitudinal wave speed by pulse-echo technique. The obtained leaky Rayleigh wave speed was then substituted into the characteristic equation for leaky Rayleigh wave. The roots of this complex characteristics equation yield the transverse wave speed, the attenuation coefficient α, and subsequently Poisson's ratio ν, Young's modulus *E*, shear modulus *G*, and Bulk modulus *K* of the materials can be obtained. The bulk modulus of diesel and paraffin oil was determined from the longitudinal wave speed, measuring by defocusing technique with known the Rayleigh wave speed of soda-lime glass.

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MATERIALS AND METHODS

Principle of measuring leaky Rayleigh wave and longitudinal wave speed

Leaky Rayleigh waves provide a well known tool in acoustic microscopy. They are generated when ultrasound incident on the materials surface at critical Rayleigh angle. The generation of leaky Rayleigh waves on a liquid/solid interface by a line-focus transducer can be described by using a ray representation in Figure 1. When the wave from the transducer is focused on the sample surface, only a single echo pulse (a directly reflected wave) is received. If the transducer is moved toward the sample (defocusing) at distance z, the waveform will separate into two main pulses in the time domain. One pulse is the directly reflected echo wave (path #1) from the surface of the sample, and the other is the leaky wave (path #2) emanating from the surface as it propagates. Based on a geometrical interpretation of each wave propagation path in Figure 1, the directly reflected pulse is used as a reference, the arrival time of the leaky Rayleigh wave can be represented as

$$t = \frac{2(1-\cos\theta)}{V_W} z \qquad (1)$$

where z is the defocusing distance between the focal plane and the sample surface, V_W is the longitudinal wave speed of the coupling liquid, and θ is the Rayleigh critical angle given by

$$\theta = \sin^{-1}\left(\frac{V_W}{V_R}\right) \qquad (2)$$

where V_R is the Rayleigh wave speed in the sample.

The arrival time of the leaky Rayleigh wave is linear with the defocusing distance as in Eq. (1). The slope of this distance z as a function of the arrival time t between pulses provides a relationship in terms of the Rayleigh critical angle θ,

$$\frac{dz}{dt} = \frac{V_W}{2(1-\cos\theta)} \quad . \qquad (3)$$

Using the slope relationship and Eq.(2), the leaky Rayleigh wave speed can be expressed independently as

$$V_R = \left[\frac{1}{V_W\left(\frac{dz}{dt}\right)} - \frac{1}{4\left(\frac{dz}{dt}\right)^2}\right]^{-1/2} \quad . \qquad (4)$$

The leaky Rayleigh wave speed can be determined from Eq.(4). The values of z and t are easily accomplished by moving the transducer at known distance Δz and measuring the change in the time interval Δt between the two pulses. A schematic of the experimental set-up is shown in Figure 2. A pulser/receiver circuit generates narrow pulses to excite the transducer as a transmitter, then receives and amplifies the echo signals from the transducer as a sensor (Fernandez *et al.*, 1987). Waveforms were recorded by the digital oscilloscope at discrete transducer positions along the z-axis.

The longitudinal wave speed can be measured by pulse-echo technique. The measurement can be performed using the same set-up as in Figure 2. From the transit time Δt (time between the front-echo and back-echo surface of sample), the wave speed is given by

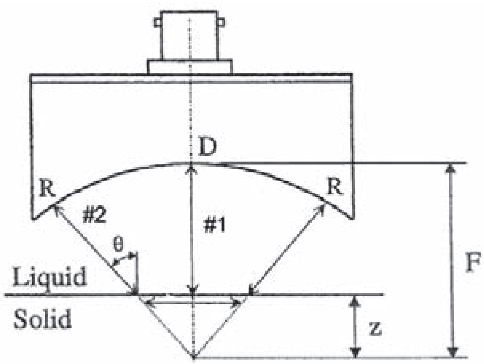

Figure 1 Ray representation of the lens-less line-focus transducer.

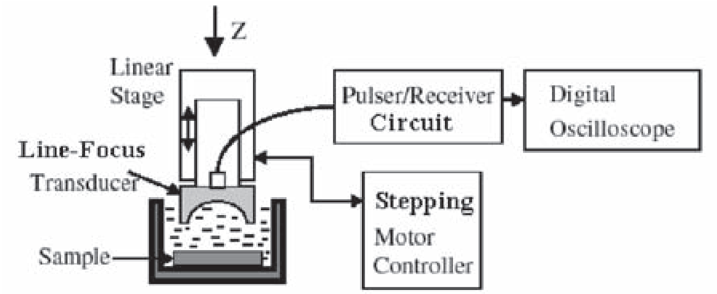

Figure 2 Schematic of the experimental set-up for the measurement of leaky Rayleigh waves.

$$V_L = 2d / \Delta t \tag{5}$$

where d is the thickness of the sample.

Estimation of Elastic Constants

Since a leaky Rayleigh wave at a liquid/ solid interface loses energy in the liquid, the wave number k is a complex quantity, expressed in terms of the wave speed V_R and the attenuation coefficient α as

$$k = \omega / V_R + i\alpha \tag{6}$$

where ω is the angular frequency.

Consider a leaky Rayleigh wave propagates on an isotropic solid half-space in contact with a liquid half-space, the complex-valued wave number k satisfies the following characteristic equation given by Viktorov (1967):

$$4k^2qs - (k^2 + s^2)^2 - i\frac{\rho_W q k_T^4}{\rho q_W} = 0 \tag{7}$$

where $q = \sqrt{k^2 - k_L^2}$, $s = \sqrt{k^2 - k_T^2}$, $q_W = \sqrt{k_W^2 - k^2}$

and $k_j = \dfrac{\omega}{V_j}$: j = L, T, W,

ρ is the density of solid, ρ_W is the density of liquid and its wave speed is V_W, V_L and V_T are the longitudinal and transverse wave speed, respectively.

Using the measured speed V_R and V_L and the density ρ with the speed V_W and density ρ_W of the liquid are known, we can obtain the attenuation coefficient α and the transverse wave speed V_T from the condition that the real and imaginary parts of Eq.(7) are zero (Lee *et al.*, 1995). The Poisson's ratio v, Young's modulus E, the shear modulus G, and the bulk modulus K of the solid are given by (Ristic, 1983)

$$\left.\begin{aligned} v &= \frac{\lambda}{2(\lambda + \mu)}, \\ E &= \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}, \\ G &= \mu, \\ K &= \lambda + \frac{2}{3}\mu, \end{aligned}\right\} \tag{8}$$

where $\lambda = \rho\left(V_L^2 - 2V_T^2\right)$ and $\mu = \rho V_T^2$.

The longitudinal wave speed of oil V_L was obtained from defocusing technique, with known Rayleigh wave speed of soda-lime glass (reference material). The bulk modulus of diesel and paraffin oil is given by (Ristic, 1983)

$$K = \rho V_L^2. \tag{9}$$

where ρ is the density of oil measured by mass per volume.

RESULTS AND DISCUSSION

Measurements had been carried out for three solids, stainless steel, iron, and aluminium. Using the directly reflected surface echo as the scope trigger, a typical series of waveform traces as a function of position z were obtained as shown in Figure 3. The traveling time between a peak position of directly reflected wave and Rayleigh wave for each position z were plotted as a function of z as shown in Figure 4 for stainless steel, iron and aluminium samples respectively. The leaky Rayleigh wave speed was determined from these plots.

The materials density was determined using V_T Archimedes's principle. The transverse wave speed was calculated using Eq. (2). Subsequently, the elastic constants, Young's modulus *E*, the shear modulus *G*, the bulk modulus *K* and Poisson's ratio *V* were determined through Eq. (8). The results were listed in Table 1.

Measurements had been carried out for two kinds of liquid, diesel and paraffin oil. Using the defocusing technique as carried out in three solids but replace water by diesel and paraffin oil and using soda-lime glass as a sample. The Rayleigh wave speed of soda-lime glass was known from Kaye and Laby (1986) and using Eq. (4), we obtain the longitudinal wave speed of diesel and paraffin oil. The materials density was determined using mass per volume. Subsequently, the Bulk modulus *K* was determined from Eq. (9). The results of measurement of longitudinal wave speed of the liquid are shown in Figure 5. The bulk modulus was listed in the Table 1. The obtained results and the values published in Kaye and Laby (1986) and Tat and Gerpen (2002) also listed in Table 1 are in good agreement.

CONCLUSIONS

Preliminary results on measurements of the leaky Rayleigh waves using lens-less line-focus PVDF transducer of the solids and liquid in this paper, demonstrates that the method can be useful for characterization the mechanical properties of materials. It shows that the line-focus PVDF transducer measurement system is an effective tool for the determining of elastic constants of materials.

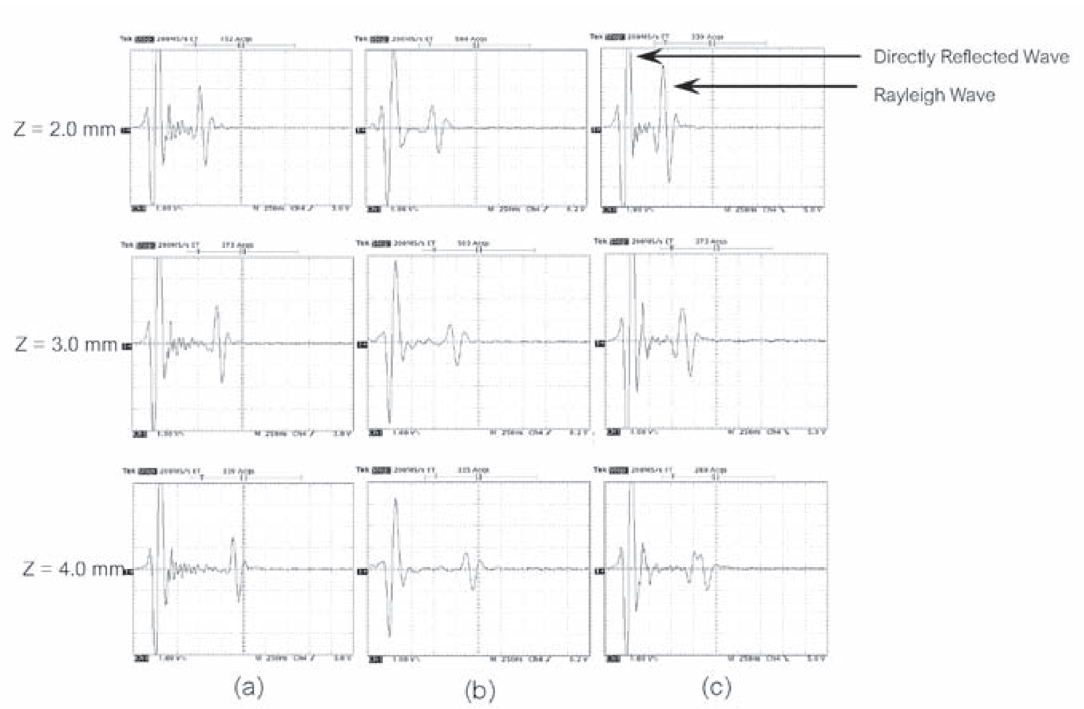

Figure 3 Typical experiment waveforms for (a) stainless steel, (b) iron and (c) aluminium.

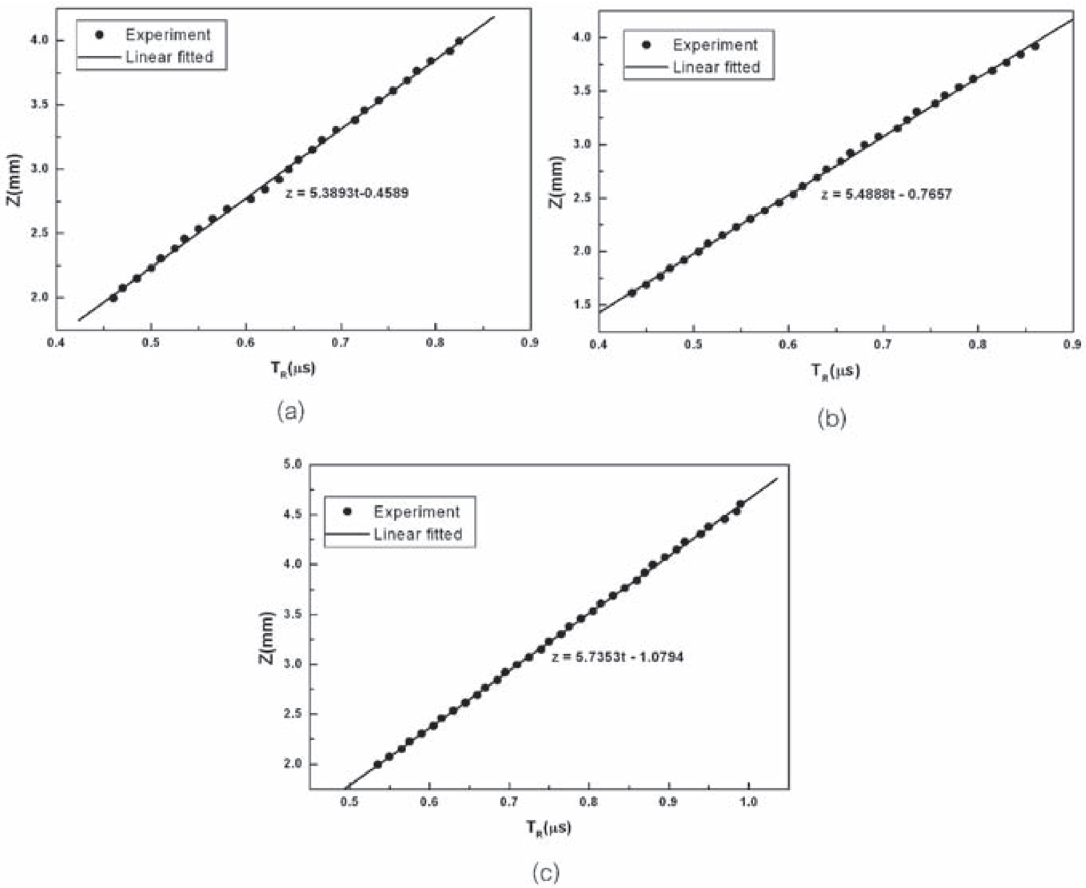

Figure 4 The defocusing distance as a function of travel time for (a) aluminium, (b) iron and (c) stainless steel.

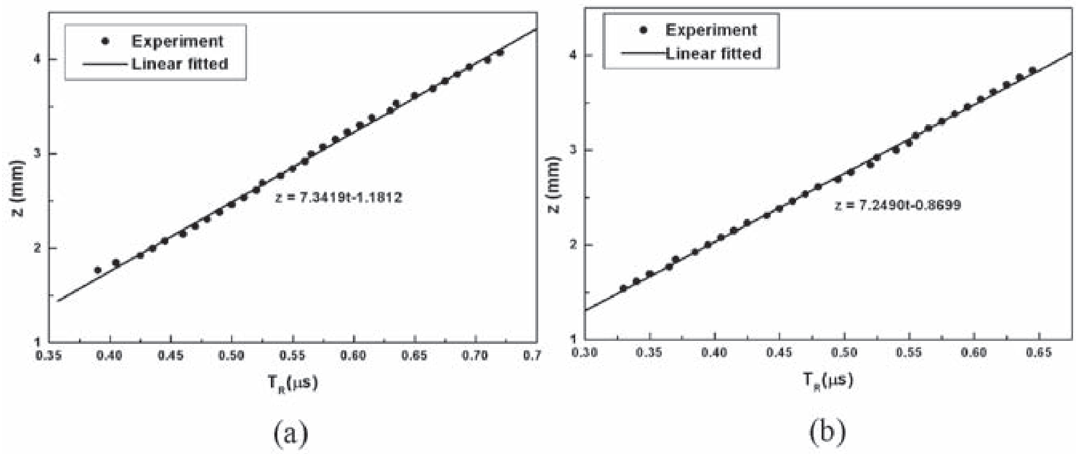

Figure 5 The measurement of longitudinal wave speed by using defocusing technique of (a) diesel (b) paraffin oil.

Table 1 The elastic constants, Poisson's ratio and density of stainless steel, iron, aluminium, diesel oil and paraffin oil.

Material	Elastic constants (GPa)			Poisson's Ratio, ν	Density, ρ (kg/m^3)
	E	*G*	*K*		
Stainless	214.9	84.0	162.7	0.280	7890
Kaye and Laby (1986)	215.3	83.9	166.0	0.293	7800
Iron	209.6	80.6	175.8	0.301	7690
Kaye and Laby (1986)	211.4	81.6	169.8	0.293	7852
Aluminium	71.5	26.6	75.9	0.343	2580
Kaye and Laby (1986)	70.3	26.1	75.5	0.345	2698
Diesel oil	-	-	1.69	-	820
Tat and Gerpen (2002)	-	-	1.64	-	853
Paraffin oil	-	-	1.67	-	830
Kaye and Laby (1986)	-	-	1.62	-	800

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