Kasetsart J. (Nat. Sci.) 42 : 191 - 196 (2008)

Volatility Estimation of Straits Times Index Based on the Anh-Inoue Model

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ABSTRACT

This paper considers the new dynamic model, namely, the Anh-Inoue dynamic model of complete markets in which the prices of European calls and puts are given by the Black-Scholes formula. The model has memory and can distinguish between historical volatility (HV) and implied volatility (IV). A new method is provided to estimate the implied volatility. It is clear evidence that the historical volatility of Straits Times Index(STI) of Singapore Stock Exchange(SGX) is not constant while the volatility parameter ,σ, of the Black-Scholes model is assumed to be constant throughout the duration in time t. Furthermore, this model can capture some movement of Straits Times Index (STI) of Singapore Stock Exchange(SGX) reasonably well.

Key words: European calls and puts, memory, implied volatility, historical volatility, Anh-Inoue dynamic model

INTRODUCTION

Volatility is an important tool to investigate a risk in the valuation of options and other derivative securities. Hobson(1998; 2004) described the volatility of a financial asset as the variance per unit time of the logarithm of the price of the asset. Furthermore, volatility is a crucial parameter of the underlying price process in the Black-Scholes model. Empirical analyses of the volatility of many stock prices indicated that volatility is not constant (Blattberg and Gonedes 1974; Scott 1987; Anh *et al*. 2005). Therefore, this paper considers an assumption of the volatility of the Black-Scholes model in the pricing of a risky asset. It is widely known that the volatility parameter ,σ, is assumed to be constant throughout the duration in time t. But this paper shows that the assumption of the Black-Scholes model is not constant for the fixed period of Straits Times Index (STI) of Singapore Stock Exchange (SGX).

In the theory of option pricing of the Black-Scholes model, options are financial instruments designed to protect investors from the stock market randomness. A European option is a financial instrument giving to its owner the right but not an obligation to buy (European call) or to sell (European put) a share at the maturity time *T*. Therefore, the purchaser of a European call option on an asset with strike price *K* and expiry *T* has the right, but not an obligation, to buy one unit of the asset at time *T* for a price *K*. On the other hand, the seller of a European put option on an asset with strike price *K* and expiry *T* has the right, but not an obligation, to sell one unit of the asset at time *T* for a price *K*. However, this right will only

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Received date : 06/03/07 Accepted date : 06/08/07

be exercised if the price S_t of the asset at time *T* is above *K*; otherwise at expiry the option is worthless (Scott 1987, Hobson 1998).

The Black-Scholes pricing formula for stock options assumes that the price $(S_t)_{t \le T}$ of a stock is the solution to a stochastic differential equation

$$dS_t = S_t(mdt + \sigma dW_t) \quad (1)$$

where m is a constant drift,σ is the volatility parameter and W_t is a Brownian motion. The Black-Scholes price *C* of a call is given explicitly by

$$C(S_t,t;K,T,\sigma,m) \equiv Ke^{-m(T-t)}(M_t\phi(d_1)-(d_2)) \quad (2)$$

where $\phi(.)$ is the standard normal distribution, given by

$$\phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$$

and $M_t \equiv (S_t/Ke^{-m(T-t)})$ is the moneyness of the option, d_1 and d_2 are given by

$$d_1 = \frac{\ln(M_t) + \frac{1}{2}\sigma^2(T-t)}{\sigma(T-t)}$$

$$d_2 = d_1 - \sigma(T-t) \quad (3)$$

The term moneyness refers to the fact that if $M_t >$ 1 the option is said to be in-the-money and $M_t < 1$ is said to be out-of-the money. Moreover, the option price depends on the volatility only through the quantity σ^2(T-t), which is the integrated squared volatility over the remaining lifetime of the option (Hull and White 1987, Hobson 1998).

MATERIALS AND METHODS

The Anh-Inoue model

The Black-Scholes model does not explain the difference between historical volatility HV and implied volatility IV. In Anh and Inoue (2005), a dynamic model of complete financial markets was introduced, in which the prices of European calls and puts are given by the Black-Scholes formula but HV and IV may be different. The price process $(S(t) : t \in R)$ of this new model, the Anh-Inoue model, is defined via an AR(∞)-type equation for the log-price process Z(t) :=logS(t). This equation takes the form

$$\frac{dZ}{dt}(t) - m = -\int_{-\infty}^{t} pe^{-q(t-s)} \left\{\frac{dZ}{dt}(s) - m\right\} ds + \sigma\frac{dW}{dt}(t) \quad (4)$$

where $m \in R$, σ, $q \in (0,\infty)$, $q \in (-q,\infty)$ and (W(t) : $t \in R$) is a one-dimensional standard Brownian motion on a probability space (Ω,F,P) . Therefore, S(t) is a solution of (4), for $t \in R$,

$$S(t) = S(0)\exp\left\{mt - \sigma\int_{0}^{t}\left(\int_{-\infty}^{s} pe^{-(p+q)(s-u)} dW(u)\right)ds + \sigma W(t)\right\} \quad (5)$$

Compared with the Black-Scholes model, the model defined by (4) has two additional parameters p and q, which describe the *memory of the market*. When p = 0, Eq. (5) produces the Black-Scholes price process given by

$$S(t) = S(0)\exp(mt+\sigma W(t)) \quad (6)$$

The log-price process $(Z(t) : t \in R)$ of (4) is a Gaussian process with stationary increments which has memory. However, Anh and Inoue (2005) neglected such a consideration. Therefore, the Black-Scholes model, which is log-normal, is still dominant among a large number of market models used by practitioners. The intention was to incorporate a crucial aspect, namely, memory, into the Black-Scholes model, without losing its usefulness and simplicity, particularly, the Black-Scholes formula.(also see Bibby and Sorensen 1997). This paper will provide clear evidence that financial markets have memory and that the model defined by Bibby and Sorensen (1997) can capture some movement of Straits Times Index reasonably well.

By Anh *et al*. (2005), the constant σ of (4) is equal to the implied volatility of the model, defined via the Black-Scholes formula in (7). Notice that, in the model defined by (4), the prices of European calls and puts are given by the Black-Scholes formula as in the Black-Scholes model.

$$HV(t-s) := \sqrt{\frac{Var\left[\log(S(t)/S(s))\right]}{t-s}} (t > S \ge o) \quad (7)$$

If (S(t)) is Black-Scholes, then HV(t) = σ for every t > 0. However, in the present model, HV(t) = f(t) where the function f(t) is given (see Anh and Inoue 2005)

$$f(t) = \sigma\sqrt{\frac{q^2}{(p+q)^2} + \frac{p(2q+p)}{(p+q)^2}\cdot\frac{\left(1-e^{-(p+q)t}\right)}{t}} \quad (t > o) \qquad (8)$$

If p>0, then f(t) is decreasing, while if p<0, then f(t) is increasing. Thus,

$$\lim_{t\to 0+} f(t) = \sigma, \lim_{t\to\infty} f(t) = \frac{\sigma q}{p+q} \qquad (9)$$

This model suggests a new method for historical estimation of implied volatility. In fact, in the traditional method, it is HV(1) that is regarded as the historical estimate of volatility (see, *e.g.*, Hull 1997). The choice of time lag (1 day) has been adopted because it can be conveniently computed from closing data. In the Black-Scholes model, HV(t) is constant whence the choice of t is not so relevant. Thus, in this traditional method, only one value of HV(t) is used to estimate volatility. However the observed market data shows that HV(t) is not constant. The new method is introduced to estimate of implied volatility based on this model (4) by nonlinear least squares. The estimated function hv(t) of HV(t) is fitted by the function f(t).Thus the estimate values are σ, p and q. Since σ is equal to the implied volatility in the model defined by (4), this estimated value of σ is the historically estimated value of implied volatility.

The estimated value of σ for real data such as Straits Times Index is almost always larger than the traditional value of HV, that is, HV(1). Since IV>HV(1) much more often than IV<HV(1) in real markets, this has the effect of narrowing the gap between HV and IV, and making the Black-Merton-Scholes theory more consistent with empirical data.

Estimating HV(t) from historical volatility

Let *N* be the number of trading days of the stock prices observed at a fixed time t ∈{1,2,...}, which is reasonably smaller than *N*. For i =1,2,...,*N*, let S_i be the closing price on the i-th day. Let $u_i = u_{i,t}$ by

$$u_i = \log(S_{i+t}/S_i) \quad (i=1,2,...,N-t) \qquad (10)$$

Then the estimate of HV(t) is given by

$$hv(t) = 100\sqrt{\frac{252}{t(N-t-1)}\sum_{i=1}^{N-t}\left(u_i - \bar{u}\right)^2} \qquad (11)$$

where $\bar{u}$ is the sample mean of defined by

$$\bar{u} := \frac{1}{N-t}\sum_{i=1}^{N-t} u_i \qquad (12)$$

The number 252 in (11) is the number of trading days in one year, and has the effect of converting the return into that per annum. On the other hand, the number 100 in (11) gives the return in terms of percentage.

The implied volatility σ is estimated by using HV(t) defined by (7). In Examples 3.1 and 4.1 of Anh and Inoue (2005) and Anh *et al.* (2005), HV(t) = f(t), where the function f(t) = f(t;σ,p,q) is given by (8). Then using the nonlinear least squares to obtain the estimated values of σ, p and q, by fitting f(t) to the estimate hv(t) of HV*(t)*.

By numerically calculation of the triple (σ,p,q) that minimizes

$$\sum_{t=1}^{25}\left\{hv(t) - f(t;\sigma,p,q)\right\}^2 \qquad (13)$$

Let ARN be the *average residue norm* given by

$$ARN = \sqrt{\frac{1}{D_{max}}\sum_{t=1}^{D_{max}}\left\{hv(t) - f(t;\sigma_0,p_0,q_0)\right\}^2} \qquad (14)$$

The value of ARN describes how close f(.) is to hv(.) on average.

RESULTS AND DISCUSSION

Figure 1 shows the plotting of (t,hv(t))(t=1,2,...,25) for Straits Times closing indices from 3 December 2004 through 12 April 2005, for which N=90 trading days. Figure 2 shows the plotting of the fitted function f(t) = f(t;9.50,0.1717,0.00003) and hv(t). Table 1 provides the resulting values of σ, p, q and ARN for the 90-day data sets of σ, p, q and ARN for the 90-day data sets of Straits Times closing indices ending in April 2005.

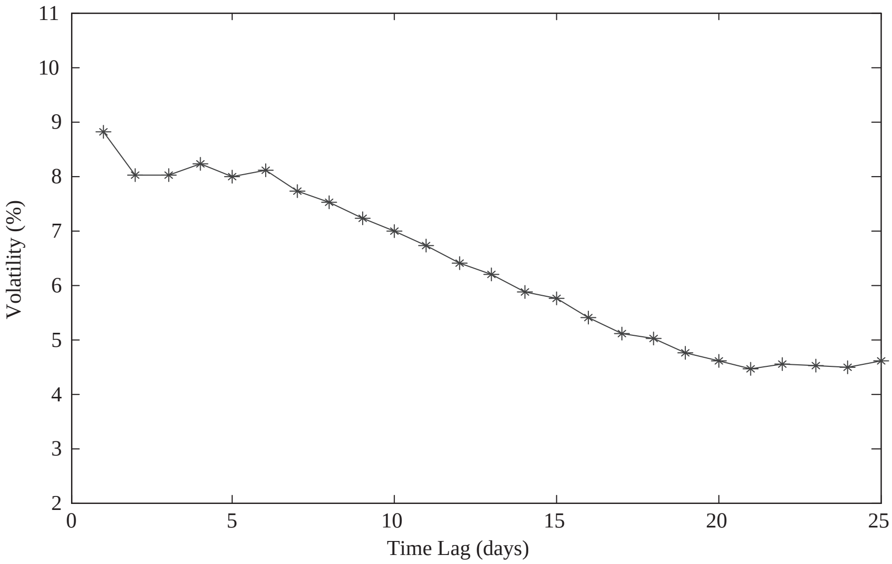

Figure 1 Historical Volatility of Straits Times Index, 3 December 04- 12 April 05, 90 trading days, D_{max} = 25.

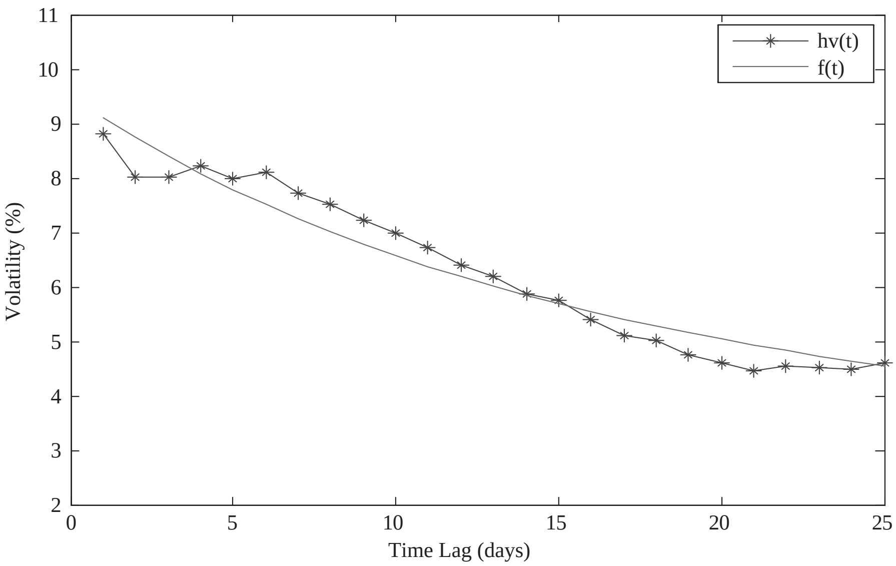

Figure 2 Fitting of f(t) to hv(t) for Straits Times Index, 3 December 04- 12 April 05, 90 trading days with (σ_0,p_0,q_0) = (9.50,0.1717,0.00003) and D_{max} = 25.

If the market followed the Black-Scholes model (6), then hv(t) would be approximately a constant, that is the implied volatility. However, Figure 1 clearly shows that hv(t) is not constant here.

Table 1 shows the resulting values of σ,p,q and ARN for the 90-day data sets. For example, the period 24 Nov 04 - 01 Apr 05 in Table 1 means the closing indices of the 90 trading days from 24 November 2004 through 01 April 2005. Each value of ARN in the Table is given by (14) with D_{max} = 25, where σ_0, p_0 and q_0 are the corresponding estimates of σ,p and q, respectively.

For the Black-Scholes framework, f(t)=σ, the historical estimate of implied volatility is 9.50%. While the traditional one based on the Black-Scholes model is hv(1) = 8.80% for the period 3 Dec 04-12 Apr 05 in Table 1. Thus the former is larger than the latter by 9.50 – 8.80 = 0.70%. It is seen that f(t) approximates hv(t) very well as can be seen in Figure 2. In this case, ARN=0.3566

CONCLUSION

This paper considers a new dynamic model, namely, the Anh-Inoue dynamic model, for historical estimation of implied volatility of Straits Times Index(STI) of the Singapore Stock Exchange(SGX) for 90 trading days from 24 November 2004 through 14 April 2005. The result shows that the implied volatility (f(t)=σ) approximates the estimated function hv(t) of HV(t) very well when the market is stable during the observed period. The historical volatility of Straits Times Index is not constant while the volatility parameter σ of the Black-Scholes model is assumed to be constant throughout the duration in time t. Moreover, the implied volatility (IV) of Straits Times Index can be estimated by the Anh-Inoue dynamic model which is almost always larger than the value obtained from traditional methods, namely HV(1) due to the memory of the financial market model or long range dependence(LRD). Furthermore, it is clear evidence that the financial market has memory. Finally, the Anh-Inoue dynamic model of complete financial market can capture some movement of Straits Times Index reasonably well.

Table 1 The results for Straits Times Index data sets of 90 trading days ending in April 2005, D_{max} = 25

Period	σ(%)	hv(1)%	p	q	ARN
24 Nov 04-01 Apr 05	10.00	9.03	0.1983	0.00002	0.4408
25 Nov 04-04 Apr 05	10.03	9.07	0.1993	0.00013	0.4133
26 Nov 04-05 Apr 05	10.01	9.04	0.1967	0.00002	0.3863
29 Nov 04-06 Apr 05	9.99	9.03	0.1937	0.00008	0.3694
30 Nov 04-07 Apr 05	9.99	8.99	0.1938	2×10(-14)	0.3612
01 Dec 04-08 Apr 05	9.92	8.96	0.1890	0.00001	0.3835
02 Dec 04-11 Apr 05	9.80	8.77	0.1821	0.00008	0.3860
03 Dec 04-12 Apr 05	9.50	8.80	0.1717	0.00003	0.3566
06 Dec 04-13 Apr 05	9.00	8.43	0.1506	0.00004	0.3880
07 Dec 04-14 Apr 05	8.89	8.32	0.1427	0.00011	0.3898
Average	9.71	8.84	0.1818	0.00005	0.3875

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