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Analytical Study of AC Magnetic Susceptibility of (Bi, Pb) Sr-Ca-Cu-O Superconducting Systems Using Bean Critical State Model

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ABSTRACT

AC magnetic susceptibility, χ_{ac}, as a function of temperature, T, and applied field amplitude, H, of $(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ and $(Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_\gamma$ superconducting systems was analyzed using Bean critical state model. The critical temperature and grain volume fraction of specimens were utilized from the real part of AC susceptibility measurements, while the pinning force density was utilized from the imaginary part of the susceptibility measurements.

Calculations using Mathematica program indicated that $(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ superconductor of nominal composition (2234), prepared by solid reaction with an intermediate ground and pressed, provided the biggest hysteresis area, and the calculated χ_{ac}(H,T) results were comparable with the experimental χ_{ac}(H,T) data.

Key words: BPSCCO, χ_{ac} , critical state, Bean model

INTRODUCTION

The critical states of sintered high-temperature superconductors, containing weakly coupled superconductive grains have been studied by AC magnetic susceptibility, $\chi = \chi' + i\chi''$, measurements (Chen *et al.,* 1990; Ishida and Goldfarb, 1990; Celebi *et al.,* 1998). Measurement of the superconducting transition by means of complex AC susceptibility provides a sharp decrease in χ' due to diamagnetic shielding and a peak in χ'' representing losses. Both χ' and χ'' are sensitive to both the temperature,T, and the amplitude of the AC magnetic field,H, (Trivijitkasem and Sratongluan, 1999).

Ceramic superconductors are composed of an array of superconducting grains, which are interconnected by weak links. Consequently, the intergranular and granular pinning–depinning properties are used to define the irreversibility line in the H-T plane (Gonzalez *et al.,* 1995). Below this line, a finite critical current density exists, while above this line, it turns to zero. Some critical state models are employed to calculate χ_{ac} in ceramic superconductor, such as the Rollins-Silocox (R-S), Kimishima, Bean, and Anderson-Kim models. Ji *et al.* (1989) proposed a macroscopic critical state model to predict both odd and even harmonic susceptibility in high critical temperature (T_C) superconductors. Lee and

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Kao (1995) used the Anderson–Kim critical state model to analyze χ_{ac} in YBCO ceramic superconductors, which showed satisfactory agreement with the experimental results.

In the present work, Bean critical state model was used to analyze the fundamental susceptibility, χ_{ac}, as a function of temperature, T, and the AC magnetic field amplitude, H, of high T_c BSCCO superconductors. The calculated χ_{ac}(H,T) results were compared with the measured data on thin slab samples immersed in a pure AC field.

THEORY

When a superconductor is placed in an AC magnetic field, $H=H_{ac}\cos(\omega t)$, the Fourier expansion of time dependent magnetization M(t) is given by Equation (1):

$$M(t) = H_{ac}\sum_{n=1}^{\infty}\left[\chi'_{n-1}\cos(n\omega t) + \chi''_{n-1}\sin(n\omega t)\right], \quad (1)$$

where, n = 1, 2, 3, ..., χ'_{n-1} and χ''_{n-1} are the n^{th} order components of the real and imaginary part of complex AC susceptibility, respectively.

The real and imaginary fundamental susceptibility, χ'_0 and χ''_0 (n equals 1), can be calculated from the following Equation (2a) and (2b) respectively:

$$\chi'_0 = \frac{1}{\pi}\int_0^{2\pi} m(t)\cos(\omega t)\, d(\omega t) \quad (2a)$$

and $$\chi''_0 = -\frac{1}{\pi}\int_0^{2\pi} m(t)\sin(\omega t)\, d(\omega t) \quad (2b)$$

where, reduced magnetization $m(t) = M(t)/H_{ac}$.

The critical state model proposed by Bean assumed that the critical current density, J_c, is constant for a small total field, H. Then the pinning force density ($\alpha = J_C B$), which is Lorentz force per volume, holds for the equilibrium state of the local flux density, B, in the sample.

In order to calculate the hysteresis loop of reduced magnetization, m(t), and the reduced AC field, $h(t) = H/H_{ac} = \cos(\omega t)$, the virgin magnetization, $m_{vir} = (\langle B\rangle/\mu_0)-h$, is introduced, where $\langle B\rangle$ is the average local field for the inner space of the sample.

For the slab sample, the virgin magnetization, m_{vir}, can be expressed by Equation (3):

$$\begin{aligned} m_{vir} &= -\frac{\alpha}{2} + \frac{(h-\alpha)^2}{2\alpha}, && \text{for } 0\le h\le\alpha, \\ &= -\frac{\alpha}{2}, && \text{for } \alpha\le h\le 1. \end{aligned} \quad (3)$$

For $0\le\alpha\le 1$, the time dependent reduced magnetization m(t) is given by Equation (3a):

$$\begin{aligned} m(t) &= \frac{\alpha}{2} - \frac{1}{4\alpha}\{\cos(\omega t) - \cos\theta\}^2, && \text{for } 0\le\omega t\le\theta, \\ &= \frac{\alpha}{2}, && \text{for } \theta\le\omega t\le\pi, \\ &= -\frac{\alpha}{2} + \frac{1}{4\alpha}\{\cos(\omega t) + \cos\theta\}^2, && \text{for } \pi\le\omega t\le\pi+\theta, \\ &= -\frac{\alpha}{2}, && \text{for } \pi+\theta\le\omega t\le 2\pi. \end{aligned} \quad (3a)$$

where, $\theta = \cos^{-1}(1-2\alpha)$.

For $1\le\alpha$, the reduced magnetization m(t) is expressed by Equation (3b)::

$$\begin{aligned} m(t) &= \frac{1}{2\alpha}\cos\theta\cos(\omega t) - \frac{1}{4\alpha}\{\cos^2(\omega t) - 1\}, && \text{for } 0\le\omega t\le\pi, \\ &= \frac{1}{2\alpha}\cos\theta\cos(\omega t) + \frac{1}{4\alpha}\{\cos^2(\omega t) - 1\}, && \text{for } \pi\le\omega t\le 2\pi. \end{aligned} \quad (3b)$$

In order to calculate χ_0 (H, T), the following expressions, Equation (4), are introduced:

$$\begin{aligned} \chi'_0 &= (1-f_g)\chi'_{0m} + f_g\chi'_{0g}, \\ \chi''_0 &= (1-f_g)\chi''_{0m} + f_g\chi''_{0g} \end{aligned} \quad (4)$$

where, f_g is the effective volume fraction of superconduction grain, χ'_{0g} and χ''_{0g} are the components of grain susceptibility.

The theoretical matrix susceptibilities, χ'_{0m}, χ''_{0m} and χ'_{0g}, χ''_{0g}, are derived by performing the following analytical integrations, only the expressions for intergranular susceptibilities are shown in Equation (5).

$$\chi'_{0m} = \frac{1}{\pi\alpha_m}\left[(1-2\alpha_m)\sin^{-1}\sqrt{\alpha_m}\right.$$
$$2$$
$$\left.\left\{1-\frac{4}{3}\alpha_m(1-\alpha_m)\right\}\sqrt{\alpha_m(1-\alpha_m)}\right], \quad \text{fo r} 0\le\alpha_m\le 1,$$

$$\chi''_{0m} = \frac{2}{3\pi}\alpha_m(3-2\alpha_m), \quad \text{for } 0\le\alpha_m\le 1,$$

$$\chi'_{0m} = \frac{1}{2\alpha_m}-1, \quad \text{for } 1\le\alpha_m,$$

and
$$\chi''_{0m} = \frac{2}{3\pi\alpha_m}, \quad \text{for } 1\le\alpha_m \qquad (5)$$

The pinning force density, α, as a function of temperature consists of the intergranular matrix pinning force density, $\alpha_m(T)$, and the granular pinning force density, $\alpha_g(T)$. For (B,P)SCCO superconduction system, the pinning force densities are assumed as Equation (6) and (7):

$$\alpha_m(T) = \alpha_m(0)\left(1-\frac{T}{T_{Cm}}\right)^{1.9} \qquad (6)$$

$$\alpha_g(T) = \alpha_g(0)\left(1-\left(\frac{T}{T_{Cg}}\right)^2\right)^2 \qquad (7)$$

where, $\alpha_g(0)$ is the granular pinning force density at 0 K, and T_{Cm}, T_{Gg} are the Josephson intergranular and granular critical temperature, respectively.

According to Bean model, $\alpha(T)$ at χ''_{0max} equals 0.75, so $\alpha(0)$ can be determined from Equation (8) and (9):

$$\alpha_m(0) = 0.75\Big/\left[1-(T_{pm}/T_{Cm})\right]^{1.9} \qquad (8)$$

and
$$\alpha_g(0) = 0.75\Big/\left[1-(T_{pg}/T_{Cg})^2\right]^2 \qquad (9)$$

where, T_p is the temperature at χ'' (maximum).

MATERIALS AND METHODS

$(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ and $(Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_\gamma$ superconducting system were prepared by the conventional solid state reaction technique with an intermediate ground and either pressed (P) or not pressed (N). The preparation method was described in Trivijitkasem and Sratongluan (2000). The AC magnetic susceptibility measurement was performed using an AC magnetometer (Lake Shore model 7130). The samples were cooled down to 55 K by the zero field cooled (ZFC) method. Various AC magnetic fields of 125 Hz fundamental frequency were gradually applied to the sample, following by increasing the temperature to 110 K at a heating rate of 0.86°C/min.

Using Bean critical state model and Mathematica program, the hysteresis loops of reduced magnetization, m(t), and the reduced AC magnetic field, h(t), were calculated from Equation (3). The theoretical real and imaginary AC magnetic susceptibility as a function of temperature and magnetic field were also calculated from Equation (4) and (5).

RESULTS AND DISCUSSION

Figure 1 shows the measured χ' and χ'' as a function of temperature, T, from 55-115 K and applied field amplitude, H = 0.1, 1.0, 10, 100, 200, 300, 400 and 500 A/m of $(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ and $(Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_\gamma$ superconductors. Critical temperature T_c and granular volume fraction f_g of the samples are listed in Table 1 and were determined from the real part of the AC magnetic susceptibility χ'

measurement at H = 0.1 A/m. The intergranular matrix pinning force density, $\alpha_m(0)$, and the granular pinning force density, $\alpha_g(0)$, at absolute temperature were calculated from Equation (8) and (9) respectively, where T_p is the peak temperature determined from the imaginary part of the AC magnetic susceptibility χ'' measurement.

The hysteresis loop of reduced magnetization, m(t), and the reduced magnetic field, h(t), were calculated from Equation (3) using the χ experimental data at 101 K and 0.1 A/m. The results are presented in Figure 2, which shows that the biggest hysteretic area belonged to the $(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_b$ (P) superconductor prepared by an intermediate ground and pressed. In contrast, the sample prepared without an intermediate ground and pressed had a smaller hysteretic area.

The real and imaginary magnetic susceptibility (χ' and χ'') as a function of temperature, T, from 50–115 K and the AC field amplitude, H = 0.1, 1.0, 10, 100, 200, 300, 400 and 500 A/m, of the $(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ and $(Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_\gamma$ superconductors were calculated from Equation (4) and (5) using the data from Table 1.

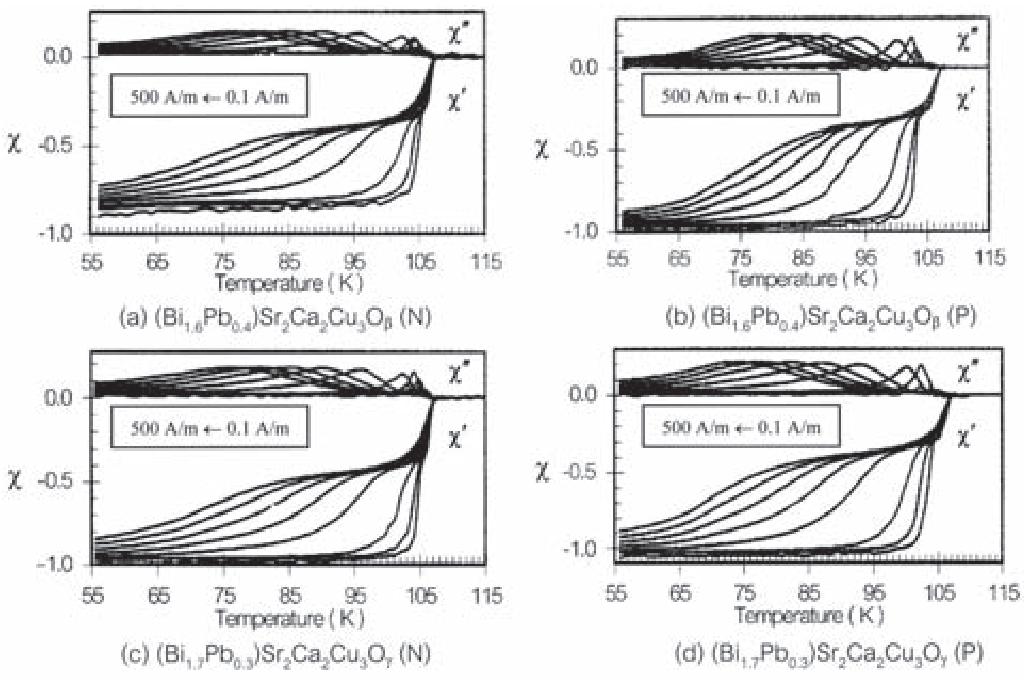

Figure 1 The observed AC susceptibility, χ, at 125 Hz and field amplitude, from right to left, H = 0.1, 1, 10, 100, 200, 300, 400 and 500 A/m, respectively.

Table 1 Critical temperature, T_C, granular volume fraction, f_g, pinning force density of grain, α_g (0) and intergrain, α_m (0) at 0 K of (Bi, Pb)Sr-Ca-Cu-O superconductors.

Sample	T_C (K)	f_g	α_g (0)	α_m (0)
$(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ (N)	106.2	0.26	616.6	146.7
$(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ (P)	105.0	0.28	1285.8	148.1
$(Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_\gamma$ (N)	107.0	0.18	557.0	377.7
$(Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_\gamma$ (P)	105.2	0.26	2166.9	182.7

The calculated values of $\chi'(H, T)$ and $\chi''(H, T)$ are shown in Figure 3, which indicates that the calculated χ' and χ'' curves are similar to the experimental χ' and χ'' curves in Figure 1.

A comparison of the observed $\chi_0(H, T)$ behavior with the calculated results, at H = 0.1 A/m, for the superconducting system are presented in Figure 4, which shows that the superconducting system prepared by an intermediate ground and pressed provided a more consistent calculated and observed AC susceptibility $\chi_{ac}(H, T)$. This confirms that Bean critical state model can be used

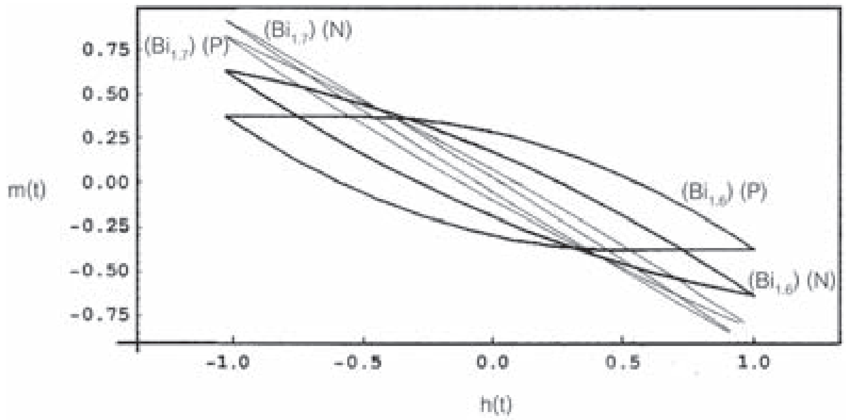

Figure 2 Calculated hysteresis loops of the thin slab superconductors at 101 K and field amplitude 0.1 A/m.

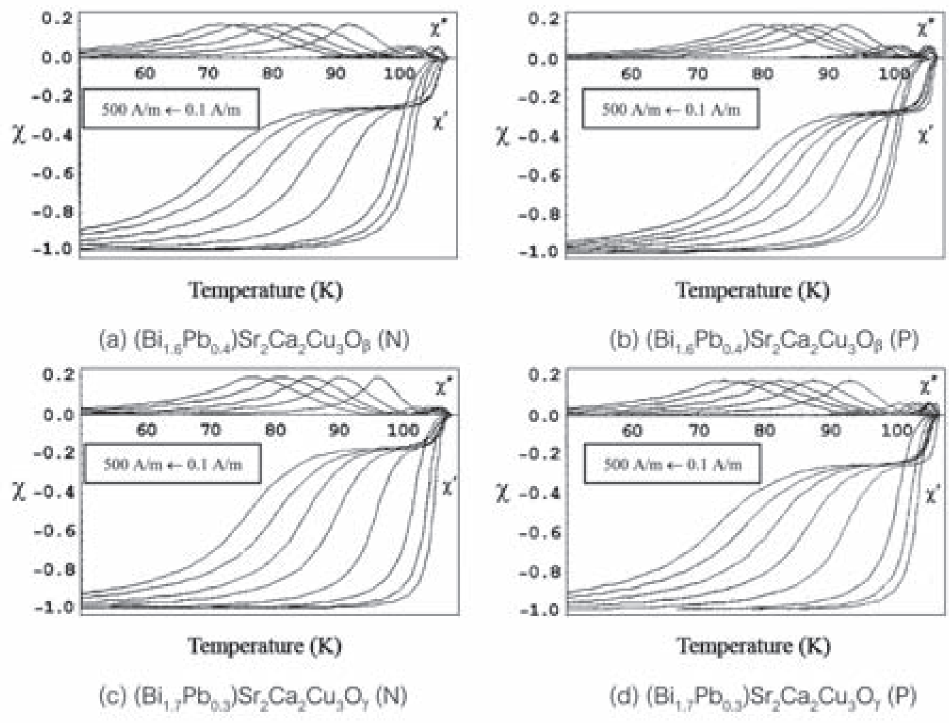

Figure 3 The calculated AC susceptibility χ at 125 Hz and field amplitude, from right to left, H = 0.1, 1, 10, 100, 200, 300, 400 and 500 A/m, respectively.

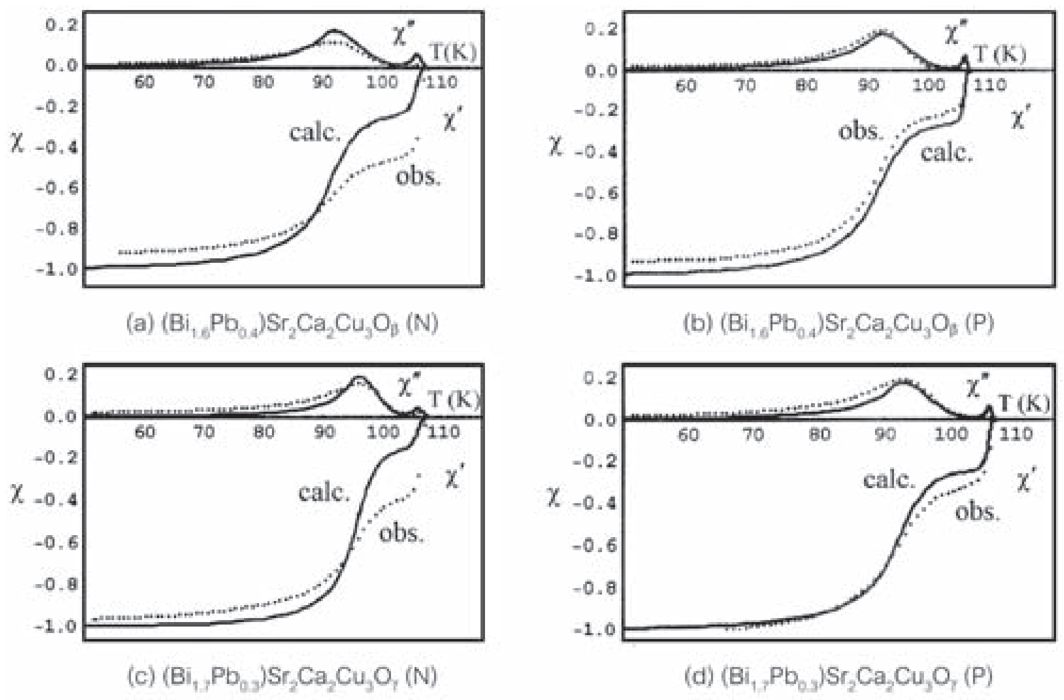

Figure 4 A comparison of the observed and calculated χ (H, T) at 125 Hz and H = 0.1 A/m for P=pressed and N=not pressed.

for reproducing the observed χ_{ac} data, especially for a lower H-field of the $(Bi_{1.6}Pb_{0.4})$ and $(Bi_{1.7}Pb_{0.3})$ superconductors prepared by an intermediate ground and pressed.

CONCLUSION

Bean critical state model was used to analyze the fundamental AC susceptibility as a function of temperature, T, and AC magnetic field amplitude, H, of high T_C $(Bi_{1.6}Pb_{0.4})Sr_2Ca_2Cu_3O_\beta$ and $(Bi_{1.7}Pb_{0.3})Sr_2Ca_2Cu_3O_\gamma$ superconducting system. The pinning force density was determined from the susceptibility measurement data using the zero field-cooled method. Calculations from Mathematica program indicated that the biggest hysteresis area belonged to the $(Bi_{1.6}Pb_{0.4})$ superconductor of nominal composition (2234) prepared by solid reaction with an intermediate ground and pressed. Bean critical state model was used to reproduce the observed χ_{ac} data. The results showed a consistent calculated and observed AC susceptibility χ_{ac}, especially at 0.1 A/m field amplitude, in the (Bi, Pb) superconducting system prepared by an intermediate ground and pressed.

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