Kasetsart J. (Nat. Sci.) 43 : 385 - 391 (2009)

Minimizing The Cost of an Integrated Model by EWMA Control Chart

Pramote Charongrattanasakul* and Adisak Pongpullponsak

ABSTRACT

Two key tools for process management are statistical process control (SPC) and maintenance management (MM). An economic model can be created when they are coordinated. This paper studied the exponentially weighted moving average (EWMA) control chart of SPC and planned maintenance, as well comparing the optimal values of an X-bar control chart and an EWMA control chart. A mathematical model was developed to analyze the cost computed from the EWMA control chart. This was used to find the optimal values of variable parameters (n^*,h^*,L^*,k^*) that minimized the hourly cost.

Key words: Integrated model, EWMA control chart, SPC, PM

INTRODUCTION

Quality control is becoming a business strategy leading to success, growth and enhanced competitive position. Organizations with successful quality improvement processes have two key tools for the control of the production process: statistical process control (SPC) and maintenance management (MM).

Lorenzen and Vance (1986) proposed a general method for determining the economic design of control charts. This method can be applied regardless of employment statistics. It is necessary only to calculate the average run-length of the statistics, assuming that the process is both under control and out-of-control in some specified manner. Alexander *et al*. (1995) combined Duncan's cost model (Duncan, 1956) with the Taguchi loss function to develop a loss model for determining three test parameters. This loss model explicitly considered quality. Rahim and Banerjee (1993) determined jointly the optimal design parameters using an X-bar control chart and the preventive maintenance (PM) time for a production system with an increasing failure rate. Montgomery (1980) and Ho and Case (1994a) considered that the economic design of control charts for monitoring the process mean had been investigated extensively in the literature. Montgomery *et al*. (1995) and Ho and Case (1994b) presented literature on control charts employing an EWMA-type (exponentially weighted moving average) statistic. Several authors have explored the economic design of EWMA control charts to monitor the process mean. Park *et al*. (2004) extended the traditional economic design of an EWMA chart to the case where the sampling interval and sample size could vary depending on the current chart statistic. Serel and Moskowitz (2008) considered that when the

Department of Mathematic, Faculty of Science, King Mongkut's University of Technology Thonburi, Bangkok 10140, Thailand.
* Corresponding author, e-mail: motecooler@hotmail.com

assignable causes led to changes in both the process mean and variance, simultaneous use of mean and dispersion charts was important to detect the changes quickly. Joint economic design of EWMA charts involving the process mean and dispersion have been explored.

The current paper developed an integrated model of a control chart and planned maintenance with reference to the integration of three scenarios first proposed by Linderman, and Kathleen E. McKone-Sweet (1986) and four scenarios by Zhou and Zhu (2008). In the current model, a control chart was used to monitor the equipment and to provide signals indicating equipment deterioration, while planned maintenance was scheduled at regular intervals to pre-empt equipment failure. Based on Alexander's cost model, the economic behavior of the integrated model was investigated and an optimal design was developed to determine the four policy variables (*n,h,L,k*) that minimize hourly cost.

METHODOLOGY

Nomenclature

The following nomenclature has been used:

Cycle Time (E[*T*])

(T_0) the expected time searching for a false alarm,

(T_P) the expected time to identify maintenance requirement and to perform planned maintenance,

(T_A) the expected time to determine occurrence of assignable causes,

(T_R) the expected time to identify maintenance requirement and to perform reactive maintenance,

(T_C) the expected time to perform a compensatory maintenance,

(τ) the mean elapse time from the last sample before the assignable cause to the occurrence of the assignable cause,

(ARL_I) the average run length during the in-control (out-of-control (ARL_O)) period,

(*E*) the expected time to sample and chart one item.

Cycle Cost (*E*[*C*])

(C_I) the cost of quality loss per unit time (the process is in an in-control state (out-of-control state (C_O)) often estimated by a Taguchi Loss function,

(C_P) the cost of performing planned maintenance (reactive maintenance (C_R), compensatory maintenance (C_C)),

(C_F) the fixed cost of sampling,

(C_V) the variable cost of sampling,

(C_f) the cost to investigate a false alarm,

the indicator variable (if it equals 1 production is continuous during planned maintenance (γ_P) (reactive maintenance (γ_R), compensatory maintenance (γ_C), validate assignable cause(γ_A)) or 0 otherwise),

(p_i^I) the probability that run length of control chart equals *i* during in-control (out-of-control (p_i^O)) period,

(*n*) the sampling size (*n** for optimal),

(*h*) the interval between sampling (*h** for optimal),

L the width of control limit in units of standard deviation (*L** for optimal),

(*k*) the number of samples taken before planned maintenance (*k** for optimal).

Problem statement and assumption

The integrated model espouses the framework as shown in Figure 1. The process begins with an in-control state with a process failure mechanism that follows a Weibull distribution with probability density function shown by Equation 1:

$$f(t) = \lambda^{v} v t^{v-1} e^{-(\lambda t)^{v}} \tag{1}$$

where, $\lambda, v, t \geq 0$

and a cumulative distribution function shown by Equation 2:

$$F(t) = 1 - e^{-(\lambda t)^{v}} \tag{2}$$

Monitoring Scenarios

As shown in Figure 1, the integrated

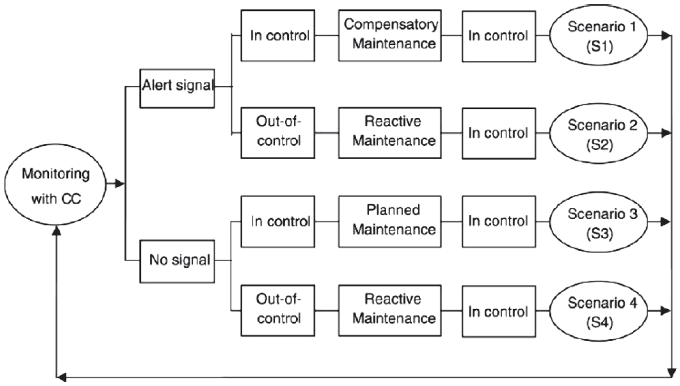

Figure 1 Integrated monitoring model

model can result in four different scenarios. In S1, the process begins with an "in-control" state and inspections occur after h hours of monitoring whether the process has changed from an "in-control" to an "out-of-control" state. However, there is an alert signal in the control chart before the scheduled time when maintenance should be performed. If the signal is false, then the process is still "in-control". Since searching for and determining a false signal takes time and incurs cost, compensatory maintenance is performed. There is also a signal in S2, similar to S1. When the signal is valid and the process shifts to an "out-of-control" state, this results in reactive maintenance. In S3 and S4, no signal occurs in the control chart before the scheduled time, so that at the $k+1$ th sampling interval, appropriate maintenance should be arranged. In S3, the process is always "in-control", and planned maintenance is performed. When the process shifts to an "out-of-control" state in S4, reactive maintenance takes place because the "out-of-control" condition occurred before the scheduled time and additional time and expense will be incurred to identify and solve the equipment problem.

Cycle time

The cycle time consists of the sum of the in-control time, out-of-control time and maintenance time. The conditional mean cycle time was derived for the four scenarios, respectively. The process stays "in-control" during the full cycle in S1.

Expected cycle time in S1 ($E[T|S_1]$) is provided by Equation 3:

$$E[T|S_1] = h\sum_{i=1}^{k} ip_i^I(1-F(ih)) + T_0 + T_C \quad (3)$$

Since S2 assumes that the process shifts to an "out-of-control" state prior to the planned maintenance and the process failure mechanism follows a Weibull distribution, the in-control time follows a truncated Weibull distribution (Equation 4):

$$f(t|(k+1)h) = \frac{f(t)}{F((k+1)h)} = \frac{\lambda^v v t^{v-1} e^{-(\lambda t)^v}}{1-e^{-(\lambda(k+1)h)^v}};$$

$$\theta \le t \le (k+1)h \quad (4)$$

So (Equations 5, 6 and 7):

$$E[T|S_2] = \int_0^{kh} tf(t|(k+1)h)dt + hARL_O - \tau + nE + T_A + T_R \quad (5)$$

$$E[T|S_3] = (k+1)h + T_P \quad (6)$$

$$E[T|S_4] = (k+1)h + T_R \quad (7)$$

Cycle cost

Expected Cycle Cost $E[C]$

The cycle cost also consists of three main components: the cost of quality loss incurred while operating the process, the cost of sampling and the cost of maintenance. The cost of quality loss includes both C_I and C_O. These two costs can be estimated using the Taguchi loss function (Equations 8, 9, 10 and 11):

$$E[C|S_1] = C_I[h\sum_{i=0}^{k} ip_i^I(1-F(ih))+\gamma_C T_C] + (C_F+nC_V)\sum_{i=0}^{k} ip_i^I(1-F(ih))+C_f+C_C] \quad (8)$$

$$E[C|S_2] = C_I\left[\int_0^{kh} tf(t|(k+1)h)dt\right] + C_O\left[hARL_O-\tau + nE+\gamma_A T_A+\gamma_R T_R\right]+\frac{1}{h}E[T|S_2](C_F+nC_V)+C_R \quad (9)$$

$$E[C|S_3] = C_I\left[(k+1)h + \gamma_P T_P\right] + k(C_F+nC_V)+ C_P \quad (10)$$

$$E[C|S_4] = C_I\left[\int_0^{kh} tf(t|(k+1)h)dt\right] + C_O\left[(k+1)h-\int_0^{kh} tf(t|(k+1)h)dt+\gamma_R T_R\right] + k(C_F+nC_V)+C_R \quad (11)$$

Expected hourly cost $E[H]$

The model can be considered as a renewal-reward process; hence the expected cost per hour $E[H]$ can be expressed as (Equations 12, 13 and 14)

$$E[H] = \frac{E[C]}{E[T]} \quad (12)$$

where $E[T] = E[T|S_1]P(S_1) + E[T|S_2]P(S_2) + E[T|S_3]P(S_3) + E[T|S_4]P(S_4)$ (13)

$$E[C] = E[C|S_1]P(S_1) + E[C|S_2]P(S_2) + E[C|S_3]P(S_3) + E[C|S_4]P(S_4) \quad (14)$$

and $P(S_1) = \sum_{i=1}^{k} P_i^I(1-F(ih))$

$$P(S_2) = \sum_{i=1}^{k}[F(ih)-F(i-1)h](1-\sum_{j=1}^{i-1}P_j^I)\sum_{l=1}^{k-i+1}P_l^O$$

$$P(S_3) = (1-F(kh)) - \sum_{i=1}^{k} P_i^I(1-F(ih))$$

$$P(S_4) = F(kh) - \sum_{i=1}^{k}[F(ih)-F(i-1)h]\,(1-\sum_{j=1}^{i-1}P_j^I)\sum_{l=1}^{k-i+1}P_l^O$$

In practice, there are four parameters (called policy variables) to be optimized: (n,h,L,k) A numerical experiment: grid-search approach was applied in this paper to find the optimal values, n^*,h^*,L^*,k^*, that minimize the hourly cost.

Stand-alone models

Maintenance

In this model, only planned maintenance was assumed (Equations 15, 16 and 17):

$$E_{\mathrm{PM}}[T] = (k+1)h+T_R F((k+1)h)+T_P(1-F((k+1)h)) \quad (15)$$

$$E_{\mathrm{PM}}[C] = \{C_I\int_0^{kh} tf(t|(k+1)h)dt+C_O[(k+1)h-\int_0^{kh} tf(t|(k+1)h)dt+\gamma_R T_R]+C_R\} \quad (16)$$
$$\times F((k+1)h)+\{C_I((k+1)h)+\gamma_P T_P+C_P\}[1-F((k+1)h)]$$

$$E_{\mathrm{PM}}[H] = \frac{E[C]}{E[T]} \quad (17)$$

Statistical process control model

This model has been extensively investigated in the literature; it follows the second scenario in the integrated model. When k approaches infinity in the integrated model, it degenerates to the statistical process control model. Then, the expressions of the expected cycle time and cycle cost are (Equations 18, 19 and 20):

$$E_{\mathrm{SPC}}[T] = \int_0^{\infty} tf(t|(k+1)h)dt+hARL_O-\tau'+nE+T_A+T_R \quad (18)$$

$$E_{\mathrm{SPC}}[C] = C_I\int_0^{\infty} tf(t|(k+1)h)dt+C_O[hARL_O-\tau'+nE+\gamma_A T_A+\gamma_R T_R]+\frac{1}{h}E_{SPC}[T](C_F+nC_V)+C_R \quad (19)$$

$$E_{\mathrm{SPC}}[H] = \frac{E[C]}{E[T]} \quad (20)$$

where, $\tau' = \sum_{i=0}^{(i+1)h} (t-ih) f(t/(k+1)h)$

X-bar control chart

The lower and upper control limits associated with the Shewhart X-bar control chart are such that

$$\text{LCL} = \mu_0 - L(\frac{\sigma_0}{\sqrt{n}})$$

$$\text{UCL} = \mu_0 + L(\frac{\sigma_0}{\sqrt{n}})$$

where, μ_o is mean and standard deviation $\frac{\sigma_0}{\sqrt{n}}$ and L is the control is limited parameter and n is the sample size.

At any sampling instant, t, the sample average $\overline{X}_t$ is compared against these limits, and if it is outside the limit, a search for an assignable cause is started.

EWMA control chart

The chart statistic for the EWMA mean chart at sampling instant, t, is computed iteratively from

$$Z_t = r\overline{X}_t + (1-r)Z_{t-1}$$

For the EWMA mean chart, the lower and upper control limits (LCL_{ewma} and UCL_{ewma}) are computed based on the asymptotic in-control standard deviation of the EWMA chart statistic Z such that

$$\text{UCL} = \overline{\overline{X}} + k\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{r}{2-r}}$$

$$\text{LCL} = \overline{\overline{X}} - k\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{r}{2-r}}$$

Considering Equation 21:

$$\alpha = P(\overline{\overline{X}} < LCL \mid \mu = \mu_0) + P(\overline{\overline{X}} > UCL \mid \mu = \mu_0) \quad (21)$$

$$= 1 - \Phi(k) + \Phi(-k)$$

where, $\Phi(x)$ is a cumulative distribution function of standard normal distribution

α is Type I error probability,

then (Equation 22):

$$\beta = P(\overline{\overline{X}} < UCL \mid \mu = \mu_0 + \eta) - P(\overline{\overline{X}} < LCL \mid \mu = \mu_0 + \eta)$$

$$= \Phi\left(k - \frac{\eta}{\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{r}{2-r}}}\right) - \Phi\left(-k - \frac{\eta}{\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{r}{2-r}}}\right) \quad (22)$$

where, β is Type II error probability

RESULTS AND DISCUSSION

Equation (12) indicates that optimizing the four policy variables (n^*,h^*,L^*,k^*) is not a straightforward process. To illustrate the nature of the solutions obtained from the economic design of the integrated model, an industrial case is presented. This paper used the EWMA control chart to monitor the manufacturing process. The EWMA control chart, with center line μ_0 and upper and lower control limits was used to monitor the process. By type I error probability (α) and type II error probability (β) substitute Equation 23:

$$p_i^I = \alpha * (1-\alpha)^{i-1} \text{ and } p_i^O = (1-\beta) * \beta^{i-1} \quad (23)$$

The initial values of the necessary parameters are given in Table 1.

Optimum values for the four design policy variables (n^*,h^*,L^*,k^*) that minimize $E[H]$ can be determined using grid searching in the optimization Toolbox of the MATLAB 7.6.0 (R2008a) software.

The numerical results are summarized in Table 2. The optimal values of the policy variables that minimize $E[H]$ are $n^*=4$, $h^*=1.15$, $k^*=22$, $L^*=1.1$, and the corresponding hourly cost is $E[H]$ = 153.02.

For the two stand-alone models, same values of the corresponding parameters and policy variables are assigned and corresponding hourly costs are obtained, which are $E_{PM}[H]$ = 195.23 and $E_{SPC}[H]$ = 162.71 by EWMA control chart. This proves that the integrated model has better economic behavior than the model in isolation.

Table 1 Parameter value of initial value in the model.

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
E	0.1	C_C	1000	T_A	0.3	C_V	0.1
C_O	200	C_R	2000	T_R	1	C_I	10
C_F	10	C_P	3000	λ	0.05	T_O	0.2
C_f	100	T_P	0.8	ν	2	T_C	0.6

Table 2 Numerical results in the integrated model.

Variable	Integrated model (X-bar)	Integrated model (EWMA)
n^*	4	4
h^*	1.23	1.15
L^*	2.91	1.1
k^*	22	22
$E[H]$	158.32	153.02
$E_{PM}[H]$	197.25	195.23
$E_{SPC}[H]$	165.75	162.71

CONCLUSION

This research proposed statistical process control (SPC) and maintenance management (MM) by an EWMA control chart. Four different scenarios were covered by the model and the expressions for the corresponding cycle times and cycle costs were derived. This method was applied to find the optimal values n^*,h^*,L^*,k^* that minimized the hourly cost, $E[H]$. However, Zhou and Zhu (2008) found that the integrated model was more effective than two stand-alone models. The result of this case study showed that the integrated model managed with the inclusion of an EWMA control chart was better than the integrated model with X-bar control chart models.

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