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A Study of Optimal Burn-In Time to Minimize Cost for a Series System Sold Under Warranty

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ABSTRACT

This research aimed to study a series system undergoing a two-level burn-in procedure. A cost model was developed to calculate the optimal burn-in time and minimal total cost of products. Then the system cost model was used to analyze the effect of components and the systems that undergo a burn-in process. The numerical examples illustrated the failure times of products followed a mixed exponential distribution. The results indicated that the optimal burn-in time could minimize the total mean cost and maximize the system reliability based on nonlinear programming.

Key words: two-level burn-in, infant mortality failure, system reliability, renewable full-service warranty

INTRODUCTION

The burn-in process is a widely used technique to detect the quality of products after production; it is used to screen out defective components before they are delivered to customers or put into field operations. Before shipping to customers, components are tested, for example, under electrical or thermal conditions that resemble working conditions in field operations. Those components that fail during the burn-in procedure will be scrapped or repaired and only those that survive the burn-in process will be considered as good quality. Therefore, utilization of the burn-in process can reduce the warranty cost. However, in situations where the burn-in process is used to increase product reliability for products with a monotonically decreasing failure rate, cost is not considered. For this reason, several researchers have attempted to calculate the optimal burn-in time to maximize the expected profits or minimize the expected costs in the burn-in process. Nguyen and Murthy (1982) determined the optimal burn-in time from considering repairable and non-repairable products sold under various warranty policies to minimize the expected total cost. Later, Chien and Sheu (2005) considered a general repairable product sold under warranty and proposed an optimal burn-in time that minimized the expected total cost. In each system, the manufacturers often considered a combination between components and a system burn-in process, the so-called “two level burn-in process”. In general, competition between cases of component failure and connection failure occurs in every system. Kim and Kuo (2004) studied the systems that were repaired during the burn-in procedure and put back into the test chamber to continue undergoing the burn-in process. They developed a probabilistic model, which was useful for a two-level burn-in procedure to optimize reliability and the economy of production when compatibility existed in components, as well as in connection. The warranty cost usually involves the product

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failure cost during the initial high failure rate period (infant mortality). Since the cost of failure during production is usually lower than that during the warranty period, the burn-in process is often used as a means of reducing the warranty cost and ensuring product quality.

This research aimed to study a series system undergoing a two-level burn-in procedure, in which the compatibility, environment and stress of each system was fixed under normal operating conditions. After surviving a burn-in process, products will be sold under the full renewable service warranty. By assuming that a component is replaced at the time of its failure with a statistically identical component and a connection is repaired upon failure, the optimal burn-in time required before the product is put on sale was determined. Finally, using the developed models, the total cost of products with burn-in time and warranty period was minimized.

MATERIALS AND METHODS

Firstly, the burn-in procedure was considered by deriving system performance in terms of the component and system burn-in time. This model was used to explain the series system under two failure cases: position failure and connection failure. It was assumed that a component was replaced at the time of its failure with a statistically identical component and a connection was repaired upon failure. Many researchers found that a mixed distribution provided a good model to describe the lifetime of a component from an overall population of indistinguishable components. Therefore, the overall failure distributions are a linear combination of the individual failure distribution. Considering a mixed distribution in a two-level burn-in procedure, components of type i are independently and identically distributed (i.i.d.) with a distribution of F_i as described by Equation 1:

$$F_i = \sum_{j=1}^{m} p_{ij} F_{ij} \qquad \text{with } \sum_{j=1}^{m} p_{ij} = 1, i = 1,2,...n, \quad (1)$$

where, m is a number of subcomponent of components type i and n is a number of components type i and probability of a subcomponent j of components type i denoted by p_{ij}.

Suppose that components type i undergo component burn-in for a time $b_i > 0$ are i.i.d. with a distribution described by Equation 2:

$$F_i^*\left(t \middle| b_i\right) = \sum_{j=1}^{m} p_{ij}^{'} \frac{F_{ij}\left(b_i + t\right) - F_{ij}(b_i)}{\bar{F}_{ij}(b_i)} \quad (2)$$

where, $p_{ij}^{'} \equiv \dfrac{p_{ij}[1 - F_{ij}\left(b_i\right)]}{\sum_{j=1}^{m} p_{ij}[1 - F_{ij}\left(b_i\right)]}$ for $t \geq 0 \;\; i = 1,2,...n, j = 1,2,...,m$

Survival function of system without burn-in process

When considering an assembly population for an identical system, only the survival components positioned at i, with the survival function $S_i^*(t)$ are assembled into a system. Suppose that $S_i^*(t)$ is a mixed distribution for components causing infant mortality failure (decrease failure) and normal failure. Assume that each defect in a connection results in an independent connection failure with a common survival $\bar{G}(t)$. If components and connections are independent, a component failure and a component-connect failure will compete with each other within component type i. Given that $[K_i = k_i]$, the survival function of component position i is shown in Equation 3:

$$S_i^*\left(t\right) \bullet \left[\bar{G}(t)\right]^{K_i} \quad (3)$$

where, K_i is number of assembly defects in component-connection position i.

Thus, from Equation 3, the survival function of the systems assembled from components surviving will be expressed as Equation 4:

$$R_s^*(t) = \prod_{i=1}^{n} S_i^*\left(t\right) \bullet \left[\bar{G}(t)\right]^{K_i} \quad (4)$$

Survival function of system undergoing burn-in process

Suppose that each system undergoes a burn-in process for time $b > 0$. A system that fails burn-in will be taken out of the test, replaced, and after replacement, the system still remains in the burn-in process. The first component in position i has a distribution $F_i^*(t|b_i)$ and all subsequent components have a distribution $F_i(t|b_i)$, which is a delayed renewal process. The mean cumulative number of failures in position i during system burn-in will be defined by Equation 5:

$$E\left[N_i\left(b\right)\middle|b_i\right] = \sum_{n=1}^{\infty} F_i^*\left(b\middle|b_i\right) \bullet F_i^{n-1}\left(b\middle|b_i\right) \quad (5)$$

where, $F_i^*(b|b_i) \bullet F_i^{n-1}(b|b_i)$ is a convolution of two distributions $F_i^*(b|b_i)$ and $F_i^{n-1}(b|b_i)$

Assume that a connection defect, a cause of system failure, is removed perfectly. According to the model of Kim and Kuo (2004), where N is a number of connection defects initially present in a system, it is clear that $N(b)$ is a nonhomogeneous Poisson process and the condition distribution of $N(b)$, if given N=n, is a binomial distribution, with parameters n and $G(b)$. Thus, the expected number of connection failures during system burn-in is (Equation 6):

$$E[E[N(b)|N]] = G(b)E(N) \quad (6)$$

where, $G(b)$ is time to failure distribution of connection defects.

Next, consider a test during detection of a component and/or a connection failure before the system fail occurs. If the failures are found, the causes of the two failures, which compete at each component position, will be replaced. That is, a failed component is replaced with a statistically identical component, while a failed connection is replaced by removing the assembly defect. Suppose that each system undergoes a system burn-in process for time $b \geq 0$ and each component undergoes a component burn-in for a time $b_i \geq 0$. Assume that components and connections are independent, and that successive component failures can be described independently from successive connection failures in component position i. If reassembly of the component is perfect, then the component at position i will have a survival function $S_i^*(t|b_i)$ and all subsequent components will have a survival function $S_i(t|b_i)$. Thus, the survival function of component position i after the system burn-in process is the survival function of the excess life of the delayed renewal process (Ross, 1983) as shown in Equation 7:

$$S_i^*\left(t\middle|b_i,b\right) = S_i^*\left(t+b\middle|b_i\right) + \int_0^b S_i\left(t+b-x\right) dE\left[N_i\left(b\right)\middle|b_i\right] \quad (7)$$

From Equation 7, if reassembly still results in decreasing failures, the survival function of components type i after the system burn-in process will be the survival function of the excess life of the renewal process, which is given by Equation 8:

$$S_i^*\left(t\middle|b_i,b\right) = S_i^*\left(t+b\middle|b_i\right) + \int_0^b S_i^*\left(t+b-x\right) dE\left[N_i\left(b\right)\middle|b_i\right] \quad (8)$$

where, $E\left[N_i\left(b\right)\middle|b_i\right] = \sum_{n=1}^{\infty} F_i^{*n}\left(b\middle|b_i\right)$

Consider an assembly of a population of identical systems, in which components type i undergo a component burn-in process for a time $b_i > 0$ before being assembled into the system. Only the survival components of type i with the survival function $S_i(t|b_i)$ are assembled into the system. After installing the component type i into the system, the component survival type i is given by $S_i^*(t|b_i)$. Suppose that $S_i(t|b_i)$ is a mixed distribution for deviant and normal components. Then, $S_i^*(t|b_i)$ is a mixed distribution for components causing infant mortality failure and normal failure. Assume that each defect in a connection results in an independent connection failure with a common survival $\bar{G}_b(t)$ undergoing a burn-in process at a time $b > 0$.

$$\bar{G}_b(t) = P\{T > t + b | T > b\} = \frac{\bar{G}(t+b)}{\bar{G}(b)} \quad (9)$$

If components and connections are independent, a component failure and a component-connect failure at components type i will compete with each other. The survival function of components type i undergoing a burn-in is given by

$$S_i^*(t|b_i)\left[\bar{G}_b(t)\right]^{K_i}$$

Therefore, the reliability of the system at the end of burn-in process is provided by Equation 10:

$$R_s^*(t|b) = \prod_{i=1}^{n} S_i^*(t|b_i, b)\left[\frac{\bar{G}(t+b)}{\bar{G}(b)}\right]^{K_i} \quad (10)$$

Component burn-in cost

The random variable h_i-1 denotes the number of replacements until the first subcomponent of component type i surviving the burn-in process is obtained. Let $X_{i1}, X_{i2},\ldots, X_{in}$ be independently and identically distributed lifetimes of all components in type i. Then all components will have a distribution $F_i(b_i)$. Denoting $X_{ij}^{b_i}$ as $X_{ij} | X_{ij} \le b_i$, the manufacturing cost incurred until the first subcomponent survival of component type i from the burn-in time b_i is as presented by Mi, 1997 (Equation 11):

$$C_{CT_i}(b_i) = C_{oi} + C_{fi} + C_{bi}T_{total,i} + C_{ri}(\eta_i - 1) \quad (11)$$

where, $T_{total,i}$ is the total burn-in time until the first subcomponent of component type i survives burn-in time b_i.

Let. $T_{total,i} = \sum_{j=1}^{\eta_i - 1} X_{ij}^{b_i} + b_i$

and $$E[\eta_i - 1] = \frac{1 - S_i(b_i)}{S_i(b_i)} = \frac{F_i(b_i)}{S_i(b_i)} \quad (12)$$

Let η_i-1 be a stopping time with respect to an i.i.d. random variable. So from Wald's equation (Ross, 1982) we obtain $E\left[\sum_{j=1}^{\eta_i-1} X_{ij}^{b_i}\right] = E[\eta_i - 1]E[X_{ij}^{b_i}] =$

$$\frac{1}{S_i(b_i)}\ b_iF_i(b_i) - \int_0^{b_i} F_i(x)dx$$

Thus, Equation 13 is derived:

$$E\left[T_{total,i}\right] = E\left[\sum_{j=1}^{\eta_i-1} X_{ij}^{b_i} + b_i\right] = E[\eta_i - 1]E[X_{ij}^{b_i}] + b_i$$

$$= \frac{\int_0^{b_i} S_i(x)dx}{S_i(b_i)} \quad (13)$$

The expected manufacturing cost per burn-in component type i is given by Equation 14:

$$E\left[C_{CT_i}(b_i)\right] = C_{oi} + C_{fi} + \frac{C_{bi}\int_0^{b_i} S_i(x)dx + C_{ri}F_i(b_i)}{S_i(b_i)} \quad (14)$$

Then, the expected system manufacturing cost after assembly is given by Equation 15:

$$C_{SC}(b) = \sum_{i=1}^{n} E\left[C_{CT_i}(b_i)\right] \quad (15)$$

where, C_{oi} is the manufacturing cost per component of type i for i=1,2,…, n,

C_{fi} is the fixed setup cost of burn-in per component of type i for i=1,2,…, n,

C_{bi} is the cost per unit time of burn-in per component of type i for i=1,2,…, n, and

C_{ri} is the replacement cost during burn-in per component of type i for i=1,2,…, n,.

System burn-in cost

The system burn-in cost incurred during system burn-in process includes:

1. fixing; variable cost incurs during the

burn-in.

2. system repairing; cost due to burn-in process is the reassembly cost in component type i and the connection repair cost in system.

$$C_{SB}(b) = C_{sf} + C_{sb}b + \sum_{i=1}^{n}\left\{E\left[C_{CT_i}(b_i)\right] + C_{ai}\right\} E\left[N_i(b)\middle|b_i\right] + C_{ca}E\left[E\left[N(b)\middle|N\right]\right] \quad (16)$$

where, C_{sf} is the fixed setup cost of burn-in per system,

C_{sb} is the cost per unit time of burn-in per system,

C_{ai} is the reassembly cost in component type i for i=1,2,…, n, and

C_{ca} is the connection repair cost in system.

Warranty cost

Consider the warranty cost incurred from a product with warranty length w and from burn-in time b under renewable full-service warranty (RFSW) policies for a series system product. Assume that the seller is required to provide a new product without cost to the customer, up until a product having a life time of at least w. T is a time interval starting from the date of sale until the warranty period w. Let $t_1+t_2+\ldots+t_{N_S}$ be the inter-arrival failure time within T, and N_s be the actual failure inter-arrival time. Then T can be expressed as $T = t_1+t_2+\ldots+t_{N_S}+w$. If $C_{ci}(b,w)$ is the manufacturing cost per burn-in component type i within T, then the expected manufacturing cost per burn-in component type i under the RSFW policy is given by Equation 17:

$$E\left[C_{ci}(b,w)\right] = \sum_{i=1}^{n}\left(C_{rp_i} + C_m\right)E\left[N_i(b+w) - N_i(b)\middle|b_i\right] \quad (17)$$

and the expected warranty cost per system for a series system under the RSFW policy is given by Equation 18:

$$C_{SW}(b,w) = \sum_{i=1}^{n}\left\{E\left[C_{ci}(b,w)\right] + C_{wi}\right\} + C_{cw}E[E(N(b+w) - N(b)|N)] \quad (18)$$

where, C_{rp_i} is replacement cost of component type i under warranty period,

C_m is system maintenance cost under warranty period,

C_w is connection replacement cost under warranty period, and

C_{wi} is the reassembly cost in component type i under warranty period.

Life-cycle cost

The life-cycle cost of a system consists of the burn-in cost and warranty cost. Let $TC(b,w)$ be denoted as the expected system life-cycle cost of a series system with the burn-in time b and warranty length w under RSFW policy, given by Equation 19:

$$E[TC(b,w)] = C_{SC}(b) + C_{SW}(b,w) \quad (19)$$

RESULTS AND DISCUSSION

Suppose that the original components of type i before the burn-in process are i.i.d. following a mixed exponential distribution with parameter λ_{i1}, λ_{i2}, given by

$$F_i(t|b_i) = p_{i1}'(1 - e^{-\lambda_{i1}t}) + (1 - p_{i1}')(1 - e^{-\lambda_{i2}t})$$

where, $1 \le i \le 5$, λ_{i1}, $\lambda_{i2} > 0$.

Let the lifetime distribution of a component type i for fixing $b_i \ge 0$ be

$$F_i^*(t|b_i) = p_{i1}'(1 - e^{-\lambda_{i1}t}) + (1 - p_{i1}')(1 - e^{-\lambda_{i2}t})$$

for $1 \le i \le 5$

$$= \frac{p_{i1}e^{-\lambda_{i1}b_i}}{p_{i1}e^{-\lambda_{i1}b_i} - (1 - p_{i1})e^{-\lambda_{i2}b_i}}(1 - e^{-\lambda_{i1}b_i}) +$$

$$(1 - \frac{p_{i1}e^{-\lambda_{i1}b_i}}{p_{i1}e^{-\lambda_{i1}b_i} - (1 - p_{i1})e^{-\lambda_{i2}b_i}})(1 - e^{-\lambda_{i2}b_i})$$

where, $0 \le p_{i1} \le 1$

Table 1 lists the input parameters and cost values, which were chosen arbitrarily to illustrate the following examples.

For any fixed $t, b_i, w \geq 0$, suppose that maximizing system reliability subject to a total cost of \$4,000 is formulated under a constraint:

Maximizing $R_s^*(t|b)$

and subject to $E[TC(b,w)] \leq 4{,}000$ for $b \geq 0$

Table 2 shows that the optimal burn-in times depended on the warranty times, that is, the optimal burn-in time b^* decreased when the w warranty time increased. For a fixed warranty time, optimal burn-in times and mission times were slightly different. The reliability of the system that underwent a burn-in process was better than the system without a burn-in process, while the total mean cost of the system without the burn-in process was less than the system that underwent the burn-in process. This implied that a longer burn-in time led to a higher total mean cost, but the system reliability was higher than in the system without the burn-in process.

CONCLUSION

In this paper, a reliability function and cost function were developed to determine the optimal burn-in time to minimize the total mean cost of a series system with a burn-in time b and warranty length w under RSFW policy. The function developed showed that the system that underwent a burn-in process had system reliability higher than for the system without any burn-in process, but it had a high cost. In practice, if a product requires a complex production system or is expensive to produce, manufacturers must use a burn-in process to eliminate the early failure rate before selling the products under a warranty policy. Since the cost of failure occurring during production is usually lower than during the warranty period, often burn-in is used to reduce the warranty cost. For future work, there still are

Table 1 Associated exponential parameters and cost factors.

Component parameter					Component cost							
i	λ_{i1}	λ_{i2}	K_i	p_{i1}	C_{oi}	C_{fi}	C_{bi}	C_{ri}	C_{ai}	C_m	C_{rpi}	C_{wi}
1	0.001	0.000000001	1	0.15	2	0.1	0.2	5	15	20	10	100
2	0.001	0.000000001	2	0.1	2	0.1	0.2	5	15	20	10	180
3	0.001	0.000001	1	0.05	1	0.1	0.2	5	15	20	10	100
4	0.001	0.0000001	3	0.1	3	0.1	0.2	5	15	20	10	150
5	0.001	0.000000001	5	0.15	4	0.1	0.2	5	15	20	10	200
Connection parameters					Connection cost							
μ		λ			C_{sf}		C_{sb}		C_{ca}		C_{cw}	
1		0.00001			1		4		10		500	

Table 2 Optimal burn-in time (h) to maximize system reliability, $R_s^*(t|b)$.

								Undergo burn-in process		without burn-in process	
t(hrs)	w (month)	b_1	b_2	b_3	b_4	b_5	b^*	$E[TC(b^*,w)]$	$R_s^*(t\|b)$	$E[TC(w)]$	$R_s^*(t)$
15×10^3	1	1,890	1,500	814	1,452	1,876	42	3,839.91	0.82	743.52457	0.49
	6	1,792	1,409	698	1,359	1,791	39	3,632.50	0.81	744.11436	0.49
	12	1,767	1,376	668	1,325	1,748	38	3,561.93	0.80	744.72567	0.49
	18	1,743	1,346	638	1,296	1,712	38	3,502.13	0.80	745.33398	0.49
	24	1,719	1,316	608	1,268	1,677	37	3,439.03	0.79	745.93929	0.49
30×10^3	1	1,890	1,500	814	1,452	1,876	42	3,839.91	0.81	743.60263	0.48
	6	1,792	1,409	698	1,358	1,791	39	3,632.10	0.80	744.11436	0.48
	12	1,767	1,376	668	1,325	1,748	38	3,561.93	0.79	744.72567	0.48
	18	1,743	1,346	638	1,296	1,711	38	3,501.72	0.79	745.33398	0.48
	24	1,720	1,316	608	1,266	1,676	37	3,384.36	0.78	745.93929	0.48

several potential extensions to the study of the burn-in process. One, which is being carried out by the authors, is the use of a burn-in process for complex products (parallel system, series-parallel system, and parallel-series system) under RFSW policies.

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