

# Vibration Reduction in Slewing of a Very Flexible One-Link Manipulator Using Shaped Reference Inputs

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## ABSTRACT

Future trends suggest that manipulators will be required to be light and to carry large payloads and this encourages the study of flexible-link robots. However, after more than three decades of investigation, vibration control of the flexible-link robot is still an unsolved problem. Researchers have made much progress using active and passive controls and with structural modification and new sensors and actuators, but a simple, effective and reliable vibration controller has yet to be found. To reduce vibration, shaping the reference input so that no energy is injected around the flexible modes can be performed in the time domain, by convolution of the input with a sequence of impulses or in the frequency domain, by optimally constructing a new input from suitable basis functions. While many researchers have applied the time-domain method to the flexible-link robot, the frequency-domain method is rarely seen with this type of robot. This paper investigated the results of applying the frequency-domain command shaping method to a flexible one-link robot. The closed-loop system consisted of a feed-forward, computed-torque loop and a feedback PID loop. As vital information for this method, the range of natural frequencies with respect to the robot's parameters was obtained from an exact model, treating the flexible link as an Euler-Bernoulli's beam. Using minimum *a priori* knowledge of the robot, the experimental results showed that the method is effective in reducing the vibration of a slewing flexible-link robot.

**Keywords:** vibration reduction, flexible link, command shaping, shaped reference inputs, continuous system

## INTRODUCTION

A flexible-link robotic manipulator is one that has flexibility (or elasticity or compliance) in its link. An angling rod is a good example of a very flexible-link manipulator, with the flexibility for a good purpose. But who would want to make a robot link as flexible as an angling rod? The space station era in the 1980s (for which manipulators

were required to be light, long, automated, accurately positioned and able to carry large payloads) initiated the need to study flexible-link manipulators (Tosunoglu *et al.*, 1990).

The emerging industrial trend for light robot design with higher payload capacities and faster operation at a higher level of precision has attracted attention from researchers around the world, especially over the last fifteen years.

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Various challenges in research range from system design, structural optimization, sensing and construction to modeling and control (Wang and Gao, 2003).

Wang and Russell (1995) studied the optimum shape of the robot's link to increase the fundamental frequency under a finite range of tip loads. Using piezoelectric actuators and sensors with their capability of converting between mechanical deformation and electrical field, Ge *et al.* (1999) were able to design a distributed controller on an exact model which in theory can guarantee the globally exponential stability of the closed-loop system. Ge *et al.* (2006) used shape memory alloys (SMAs) as actuators and sensors in controlling flexible robots. Compared to the piezoelectric actuators, the SMA actuators can produce larger forces, withstand larger stresses and recover larger strains but with slower responses. Pratiher and Dwivedy (2008) studied the nonlinear behavior of a single-link flexible visco-elastic manipulator. The visco-elastic material had good vibration absorbing behavior.

In the past, vibration reduction was performed passively by adjusting the spring and damping constants. More recently, a group of researchers at the Massachusetts Institute of Technology devised a passive control technique called command input shaping in which a system's command input is altered so that no energy is injected around the flexible modes either by construction of input from versine or ramped sinusoidal functions (Meckl and Seering, 1988) or by using a notch filter (Singer and Seering, 1988) or by convolution of the input with a sequence of impulses (Singer and Seering, 1990).

The convolution of the input with a sequence of impulses has gained much attention due to their straight-forward time-domain operation. Tzes and Yurkovich (1993) and Khorrami *et al.* (1995) identified the modal frequencies of a flexible-link robot online using their time-varying transfer function estimation

scheme and used them to adjust their controller based on the input shaping method proposed by Singer and Seering (1990). Nevertheless, the online adaptation scheme is only suitable for a few links having a few modal frequencies. Lim *et al.* (1999) proposed an input-shaping filter for multi-input flexible systems using convex optimization by which additional performance requirements and robustness against model uncertainties can be included in the design formulation. Mohamed and Tokhi (2004) presented feed-forward control schemes for vibration control of a flexible robot using input shaping and filtering techniques and provided a comparative assessment of these techniques. Shan *et al.* (2005) proposed a modified input shaping technique for robustness and multimode vibration suppression of a single-link flexible robot. The modified technique was compared to the traditional input shaping (Singer and Seering, 1990) and was found to be more convenient. Alam and Tokhi (2008) incorporated the genetic algorithms into a multi-objective optimization to determine a set of solutions for the amplitudes and corresponding time locations of impulses of a multi-modal command shaper. Mimmi and Pennacchi (2001) used modified trapezoidal motion input to reduce residual vibration.

Nevertheless, there are some limitations of the input convolution technique when implemented in a closed-loop system. As pointed out by Zuo and Wang (1992), a closed-loop shaped input controller requires the assumption that the fundamental frequency of the elastic vibrations is sufficiently higher than the bandwidth of the closed loop system employing a rigid body controller. Moreover, since this is a time-domain filtering technique, there will be certain frequencies from the input which cannot be reproduced at the output.

The frequency-domain command shaping technique in Meckl and Seering (1988) does not have these limitations. Chatlatanagulchai

*et al.* (2006) applied this technique to a two-link flexible-joint robot. Because of the configuration-dependent natural frequencies, the shaped command based on a versine basis function was divided into segments to better attenuate the vibration at a specific configuration. However, this technique has rarely been applied to a flexible-link robot.

In this paper, experience in applying the frequency-domain command shaping technique to a flexible-link robot is presented. Natural frequencies of the one-link flexible-link robot were obtained from experiments. Iemsamai and Chatlatanagulchai (2008) compared an exact model and an approximated model, obtained from lumping of the distributed mass, to the experiment. Better accuracy in computing modal frequencies and locating the payload position were obtained from the exact model. The exact model based on the Euler-Bernoulli beam was used to obtain the variation of natural frequencies due to changes in system parameters. This variation was used in the algorithm to reduce the spectral magnitude in a sufficiently wide band of frequencies surrounding the nominal natural frequencies.

The proposed closed-loop system consists of a computed-torque feed-forward loop and a PD feedback loop. The feed-forward loop tries to cancel the plant as much as it can without changing the frequency content of the shaped reference command. The feedback loop provides stability from imperfect cancellation. The experimental results have shown that this algorithm effectively reduced residual vibration of a slewing flexible-link robot resulting in faster move time.

## MATERIALS AND METHODS

### Exact model of the flexible-link robot

Consider a diagram of a one-link flexible-link robot (Figure 1). The coordinates  $x_0 - y_0$  are fixed while the coordinates  $x - y$  rotate with the

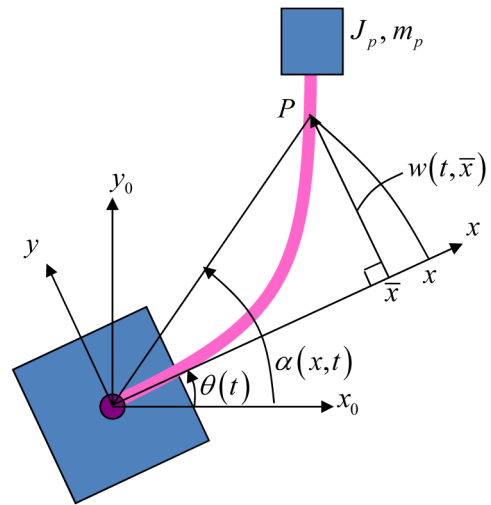


**Figure 1** Diagram of a one-link flexible-link robot.  $x$  and  $y$  are the coordinates rotating with the base;  $x_0$  and  $y_0$  are fixed coordinates;  $\theta$  is the base angular position;  $\alpha$  is the angular position of a point  $P$  on the link;  $w(t, \bar{x})$  is transversal deflection due to link flexibility;  $\bar{x}$  and  $x$  are corresponding positions on the  $x$  axis;  $m_p$  and  $J_p$  are payload mass and mass moment of inertia around center of payload;  $t$  is time.

base  $\theta$ . is the base angular position, and  $\alpha$  the angular position of a point  $P$  on the link. The aim is to control  $\alpha$  when the point  $P$  is at the payload  $m_p$ .  $w(t, \bar{x})$  is transversal deflection due to link flexibility.

Assuming that the link moves in the horizontal plane and only transversal deflection is considered and applying Newton's second law to a small portion of the link in Figure 2, where  $M$  is bending moment and  $Q$  is shear force, the first equation of motion is obtained (Equation 1):

$$EI \frac{\partial^4 w(x, t)}{\partial x^4} + \rho \left[ \frac{\partial^2 w(x, t)}{\partial t^2} + x \frac{d^2 \theta(t)}{dt^2} \right] = 0, 0 < x < L \quad (1)$$

where  $E$  is modulus of elasticity,  $I$  is uniform cross-sectional area moment of inertia,  $\rho$  is uniform linear mass density and  $L$  is total length of the link.

Consider a diagram of the whole robot assembly (Figure 3). Letting  $T$  be the driving torque,  $J_0$  be the base (motor and hub) mass

moment of inertia about center of base,  $J_b$  be the link mass moment of inertia about center of base,  $m_p$  and  $J_p$  be payload mass and mass moment of inertia around center of payload, and applying again Newton's second law, the second equation of motion is (Equation 2):

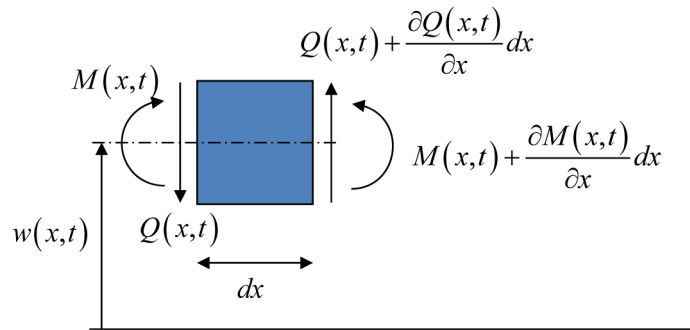


**Figure 2** Diagram of a small portion of the link.  $w(t, \bar{x})$  is transversal deflection due to link flexibility;  $Q$  is shear force;  $M$  is bending moment;  $x$  is the coordinate;  $t$  is time.

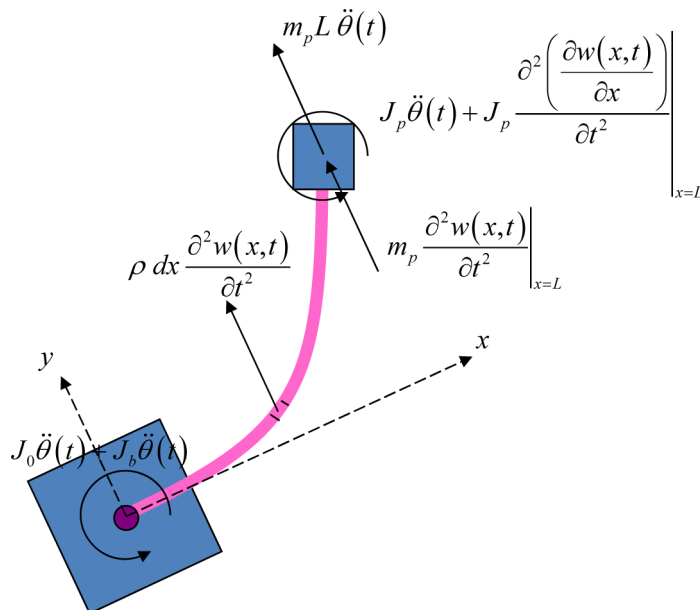


**Figure 3** Diagram of the whole robot assembly.  $J_0$  is the base mass moment of inertia about center of base;  $J_b$  is the link mass moment of inertia about center of base;  $m_p$  and  $J_p$  are payload mass and mass moment of inertia around center of payload;  $\rho$  is the density of the link;  $w(t, \bar{x})$  is transversal deflection due to link flexibility;  $L$  is the length of link;  $\theta$  is the base angular position;  $x$  and  $y$  are the coordinates;  $t$  is time.

$$T(t) = J_0 \ddot{\theta}(t) + J_b \ddot{\theta}(t) + J_p \ddot{\theta}(t) + m_p L^2 \ddot{\theta}(t) + \int_0^L \rho x \frac{\partial^2 w(x,t)}{\partial t^2} dx + m_p L \frac{\partial^2 w(x,t)}{\partial t^2} \bigg|_{x=L} + J_p \frac{\partial^2 \left( \frac{\partial w(x,t)}{\partial x} \right)}{\partial t^2} \bigg|_{x=L} \quad (2)$$

Two obvious boundary conditions are provided by Equations 3 and 4:

$$w(0,t) = 0 \quad (3)$$

and

$$\frac{\partial w(x,t)}{\partial x} \bigg|_{x=0} = 0 \quad (4)$$

The other two boundary conditions can be found from applying Newton's second law to the payload. A diagram of the payload is given in Figure 4. The resulting boundary conditions are (Equations 5 and 6):

$$EI \frac{\partial^3 w(x,t)}{\partial x^3} \bigg|_{x=L} = m_p L \ddot{\theta}(t) + m_p \frac{\partial^2 w(x,t)}{\partial t^2} \bigg|_{x=L} \quad (5)$$

and

$$EI \frac{\partial^2 w(x,t)}{\partial x^2} \bigg|_{x=L} = -J_p \ddot{\theta}(t) - J_p \frac{\partial^2 \left( \frac{\partial w(x,t)}{\partial x} \right)}{\partial t^2} \bigg|_{x=L} \quad (6)$$

Next, the differential eigenvalue problem is considered. By solving for  $\ddot{\theta}$ , letting  $T = 0$ , substituting  $\ddot{\theta}$  into , and separating the variables  $w(x,t) = \psi(x)\delta(t)$ , two ordinary differential equations (ODE) are obtained in terms of  $\psi$  and  $\delta$ . Since there is only interest in computing the natural frequencies, the ODE of  $\psi$  is considered as Equation 7:

$$EI \frac{\partial^4 \psi(x)}{\partial x^4} - \omega^2 \rho \psi(x) + \frac{x \omega^2 \rho}{[J_0 + J_b + J_p + m_p L^2]} \left[ \int_0^L \rho x \psi(x) dx + m_p L \psi(x) \bigg|_{x=L} + J_p \frac{\partial \psi(x)}{\partial x} \bigg|_{x=L} \right] = 0 \quad (7)$$

where  $\omega$  is the natural frequency. The solution of Equation 7 can be found using the method of undetermined coefficients (Equation 8):

$$\psi(x) = C_1 \sinh(\beta x) + C_2 \cosh(\beta x) + C_3 \sin(\beta x) + C_4 \cos(\beta x) + C_5 x \quad (8)$$

where  $\beta = (\omega^2 \rho / EI)^{1/4}$ .

By substituting in , we can find  $C_5$  as a function of  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$ . Using the four boundary conditions (Equations 3–6), after a considerable amount of algebra, four linear equations are obtained (Equations 9–12):

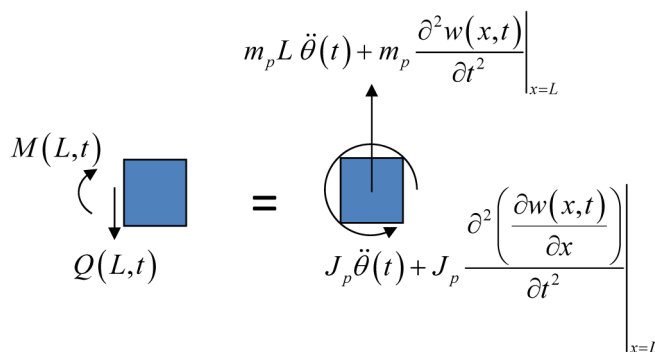


**Figure 4** Diagram of payload.  $M$  is bending moment;  $Q$  is shear force;  $L$  is the length of link;  $m_p$  and  $J_p$  are payload mass and mass moment of inertia around center of payload;  $\theta$  is the base angular position;  $w(t, \bar{x})$  is transversal deflection due to link flexibility;  $x$  is the coordinate;  $t$  is time.

$$\begin{aligned} & \left( \beta + \frac{1}{J_0} \left( \rho \left( \frac{L \cosh \beta L}{\beta} - \frac{\sinh \beta L}{\beta^2} \right) + m_p L \sinh(\beta L) + J_p \beta \cosh(\beta L) \right) \right) C_1 \\ & + \left( \frac{1}{J_0} \left( \rho \left( \frac{L \sinh \beta L}{\beta} - \frac{\cosh \beta L}{\beta^2} \right) + \frac{\rho}{\beta^2} + m_p L \cosh(\beta L) + J_p \beta \sinh(\beta L) \right) \right) C_2 \\ & + \left( \beta + \frac{1}{J_0} \left( \rho \left( \frac{\sin \beta L}{\beta^2} - \frac{L \cos \beta L}{\beta} \right) + m_p L \sin(\beta L) + J_p \beta \cos(\beta L) \right) \right) C_3 \\ & + \left( \frac{1}{J_0} \left( \rho \left( \frac{\cos \beta L}{\beta^2} + \frac{L \sin \beta L}{\beta} \right) - \frac{\rho}{\beta^2} + m_p L \cos(\beta L) - J_p \beta \sin(\beta L) \right) \right) C_4 = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} & \left( \frac{\cosh(\beta L)}{\beta} + \frac{m_p \sinh(\beta L)}{\rho} \right) C_1 + \left( \frac{\sinh(\beta L)}{\beta} + \frac{m_p \cosh(\beta L)}{\rho} \right) C_2 \\ & + \left( \frac{-\cos(\beta L)}{\beta} + \frac{m_p \sin(\beta L)}{\rho} \right) C_3 + \left( \frac{\sin(\beta L)}{\beta} + \frac{m_p \cos(\beta L)}{\rho} \right) C_4 = 0 \end{aligned} \quad (10)$$

$$\begin{aligned} & (\beta^2 \sinh(\beta L) - \beta^5 \cosh(\beta L)) C_1 + (\beta^2 \cosh(\beta L) - \beta^5 \sinh(\beta L)) C_2 \\ & + (-\beta^2 \sin(\beta L) - \beta^5 \cos(\beta L)) C_3 + (-\beta^2 \cos(\beta L) + \beta^5 \sin(\beta L)) C_4 = 0 \end{aligned} \quad (11)$$

$$C_2 + C_4 = 0 \quad (12)$$

whose determinant is set to zero to determine the natural frequency  $\omega$ .

### Experimental set-up

A photograph of the flexible-link robot is shown in Figure 5. An accelerometer was mounted with the payload whose location was at the tip. Three strain gauges were used to measure transversal deflection, and an optical encoder was used to measure motor and hub angle. A steel ruler was used as the flexible link with an effective

length of 0.54 of  $m$ .

Figure 6 is a block diagram of the experimental arrangement. A host computer, with the necessary software (Labview version 8.51, National Instruments, Austin, Texas; Matlab version 2009a, Mathworks, Natick, MA, USA) was used to communicate with the user and a target computer. The target computer contained a data acquisition card whose functions were to acquire sensor signals and to send out actuator commands from the control algorithm. The host and target computers were connected to each other via a local area network line. A control signal was sent as voltage to a motor amplifier board to amplify to a level that could drive the DC motor. An IC chip accelerometer was mounted at the tip to measure linear acceleration. An in-house-design op-amp circuit amplified and filtered the signals for the strain gauges. A DC power supply provided the required current to the motor amplifier board.

Iemsamai and Chatlatanagulchai (2008) validated the exact model given in Equation 1 by comparing the tip angle output ( $\alpha$ ) and the natural frequencies ( $\omega_i$ ) of the model to those of the actual robot, using nominal parameter values as  $E = 2.1 \times 10^{11}$ ,  $I = 1.67 \times 10^{-11}$ ,  $L = 0.54$ ,  $\rho = 0.25$ ,  $J_0 = 17.55 \times 10^{-5}$ ,  $J_p = 0.012$ , and  $m_p = 0.09917$ . The nominal parameter values were obtained from real physical measurements.

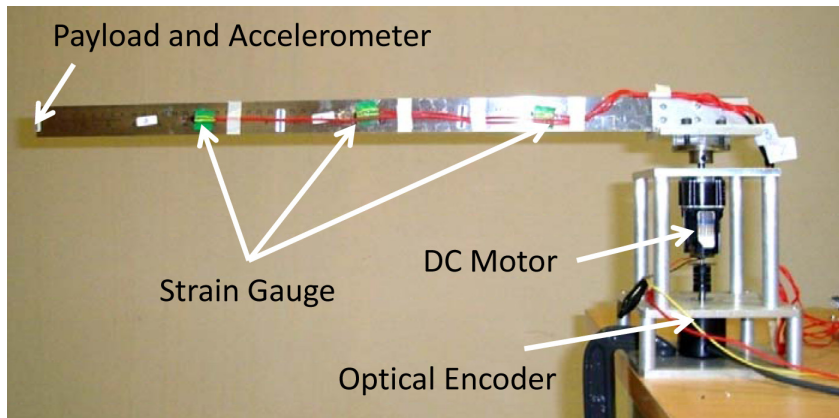


**Figure 5** Photograph of the flexible-link robot in our laboratory.



$$\Phi_l^*(t) = \alpha_l \left( \frac{1}{2} - \frac{t}{T_f} \right) + \sin \alpha_l \frac{t}{T_f} - \frac{\alpha_l}{2} \cos \alpha_l \frac{t}{T_f}$$

where,  $\alpha_l$  is the characteristic number associated with each harmonic as defined in Meckl (1984) and can be used as a basis function in the reconstructed series. The ramped sinusoidal function is selected for its smooth transition in slope that results in the narrow frequency spectra. Any input function can be represented by the series (Equation 15):

$$f(t) = \sum_{l=1}^L \frac{B_l}{\alpha_l^2} \Phi_l^*(t) \quad (15)$$

where  $B_l$  is the coefficient for each harmonic.

Since a single cycle of the square-wave acceleration profile is known to give optimal move time for a rigid body,  $B_l$  can be chosen to minimize the move time by having  $f(t)$  to be as close as possible to the square wave. In addition,  $B_l$  must be chosen so that  $f(t)$  may have minimum spectral magnitudes over a range of frequencies surrounding the system natural frequency. To achieve this,  $B_l$  can be chosen to minimize the following cost function (Equation 16):

$$J = \frac{1}{T_f} \left\{ \int_0^{T_f/2} [1 - f(t)]^2 dt + \int_{T_f/2}^{T_f} [-1 - f(t)]^2 dt \right\} + \rho \sum_{i=1}^{11} (\omega_i T_s)^2 |F^*(\omega_i T_f)|^2 \quad (16)$$

where  $\rho$  is a relative weighting between two objectives and  $T_s$  is the move time to cover a distance  $y_f$  when the input is a single cycle of a square wave of amplitude  $F_{\max}$ . The cost function contains 11 frequencies surrounding resonance; this number can be changed arbitrarily. In the case of  $\pm 17\%$  deviation for the first three modes, the bounds on the frequency  $\omega_i$  are  $0.83\omega_n < \omega_i < 1.17\omega_n$ .

To minimize  $J$ ,  $\partial J / \partial B_l$  is set to zero. The resulting expression is  $B_l$  as a function of  $T_f$ . Therefore, an iterative routine must be performed by first guessing a  $T_f$ , then after computing the corresponding  $B_l$ , recompute a true  $T_f$  and iterate until the guessed  $T_f$  falls within an acceptable margin from the true  $T_f$ .

To implement the command shaping technique in a closed-loop system, consider the feed-forward/feedback block diagram in Figure 7.  $P$  is the actual robot,  $v$  is the command voltage to the motor amplifier board,  $G$  is a controller,  $\alpha$  is the angular position of the payload,  $\hat{P}$  is a simplified robot model,  $\hat{P}^{-1}$  is its inverse mapping,  $v_1$  and  $v_2$  are command inputs from the inverse mapping and the controller and  $\alpha_d$  and its derivatives are desired trajectories.

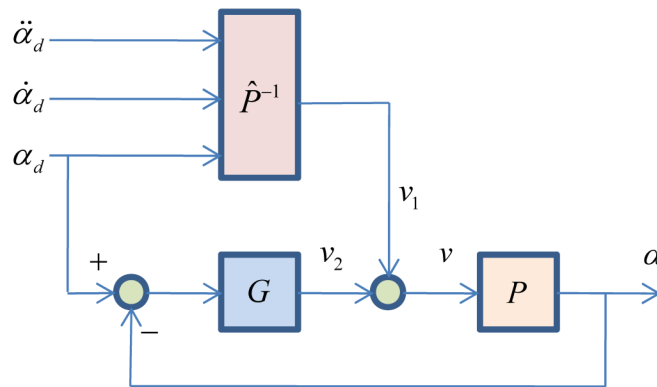


**Figure 7** Diagram of a feed-forward/feedback system.  $P$  is the actual robot;  $v$  is the command voltage to the motor amplifier board;  $G$  is a controller;  $\alpha$  is the angular position of the payload,  $\hat{P}$  is a simplified robot model;  $\hat{P}^{-1}$  is its inverse mapping;  $v_1$  and  $v_2$  are command inputs from the inverse mapping and the controller and  $\alpha_d$  and its derivatives are desired trajectories.

$\hat{P}$  is a simplified second-order transfer function that has resulted from a system identification process with the actual robot, where the emphasis was on obtaining a minimum-phase, stable model suitable for inversion (Equation 17):

$$\hat{P} = \frac{1.2}{0.1115s^2 + 0.5708s + 1} \quad (17)$$

To obtain the simplified transfer function, a frequency-varying sine wave was given as an input voltage  $v_1$  to the motor, which drives the robot. The resulting angular position output  $\alpha$  was recorded. The transfer function from the input  $v_1$  and the output  $\alpha$  was obtained using the Matlab command `<ident>`. Afterward, model validation was performed by inputting a different input  $v_1$  to the motor and comparing the model output with the actual output. Figure 8(a) shows the model validation, where the solid line is the model output and the dotted line is the actual output. Because  $\hat{P}^{-1}$  does not perfectly cancel  $P$ , a feedback controller  $G$  is needed (in this case, a PD controller) to handle the uncertainty.

If  $\ddot{\alpha}_d$  is a single cycle of a square wave as in Figure 8(b), whose spectral magnitudes are large over the whole frequency range, its integrals  $\dot{\alpha}_d$  and  $\alpha_d$  will have large spectral magnitudes over the whole frequency range as well. As a result,  $v_1$ , which merely comes out of a linear inverse operation, will have large spectral magnitudes and will excite the natural frequencies of the robot  $P$ , resulting in a high level of residual vibration.

Using the command shaping technique, in the case under consideration, first the natural frequencies of the robot were found by fast Fourier transform of the robot's step acceleration response. Since the robot is a continuous system, there are infinite numbers of modes, but for a finite bandwidth control usually the first three modes suffice. In this case, the result was three modes—namely,  $\omega_1 = 8 \text{ rad/s}$ ,  $\omega_2 = 20 \text{ rad/s}$ , and  $\omega_3 = 80 \text{ rad/s}$ —as shown in Figure 9.

Then,  $\ddot{\alpha}_d$  was constructed using the series (Equation 15). The cost function (Equation 16) was minimized using the design parameters:  $L = 10$ ,  $\rho = 3$ ,  $\omega_{n1} = 8 \text{ rad/s}$ ,  $\omega_{n2} = 20 \text{ rad/s}$ ,  $\omega_{n3} = 80 \text{ rad/s}$ , and  $F_{\max} = 10$ . Note that  $L$  is the

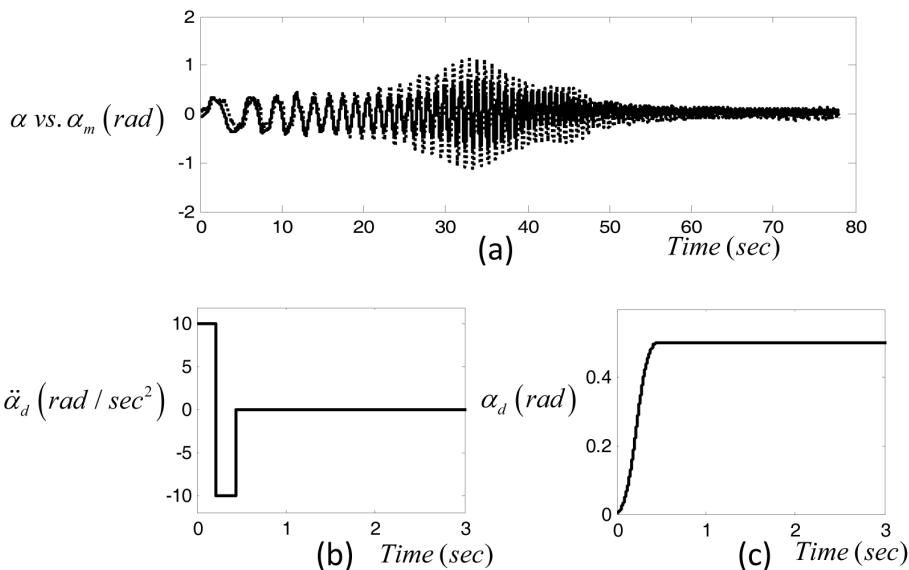


**Figure 8** (a)  $\hat{P}$  model validation. Solid line is model output  $\alpha_m$  and dotted line is actual output  $\alpha$ . (b) Square-wave acceleration profile. (c) Resulting position profile.

number of terms in the series, whereas  $\rho$  is the relative weighting of the cost function ; they are not the same as  $L$  and  $\rho$  in the exact model. To accommodate three natural frequencies, instead of one, there are three summations in Equation 16. Therefore,  $\omega_i$  becomes  $\omega_{ij}$ , where  $j$  represents the index of the natural frequency. In the case of  $\pm 17\%$  deviation for the first three modes, the bounds on the first three natural frequencies are  $0.83\omega_{nj} < \omega_{ij} < 1.17\omega_{nj}$ ,  $j = 1, 2, 3$ . The desired

move distance is  $y_f = 0.5 \text{ rad}$ . Figure 10(a) shows the frequency spectrum of the shaped  $\ddot{\alpha}_d$ . The spectrum magnitudes around the three natural frequencies were greatly reduced. The time plot of  $\ddot{\alpha}_d$  and corresponding  $\alpha_d$  are given in Figure 10(b) and (c). Note that the desired move time increased from  $0.45 \text{ s}$  in the square-wave case (Figure 8(c)) to  $1.4 \text{ s}$  in the ramped sinusoidal case (Figure 10(c)) because the energy that was pulled out from the spectrum around the three

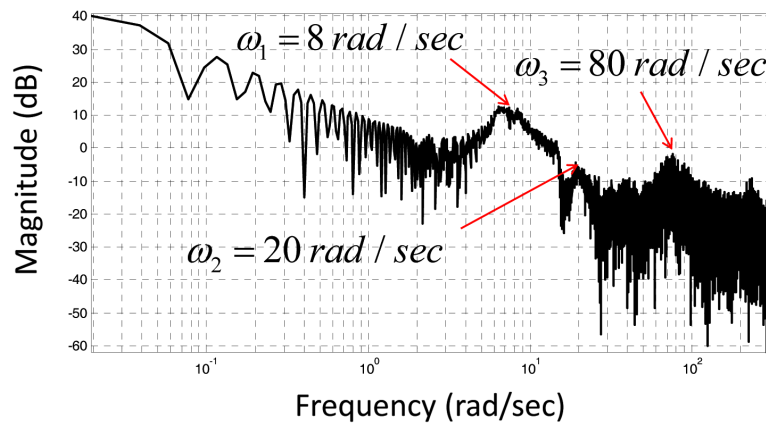


**Figure 9** Frequency spectrum of robot's step response.

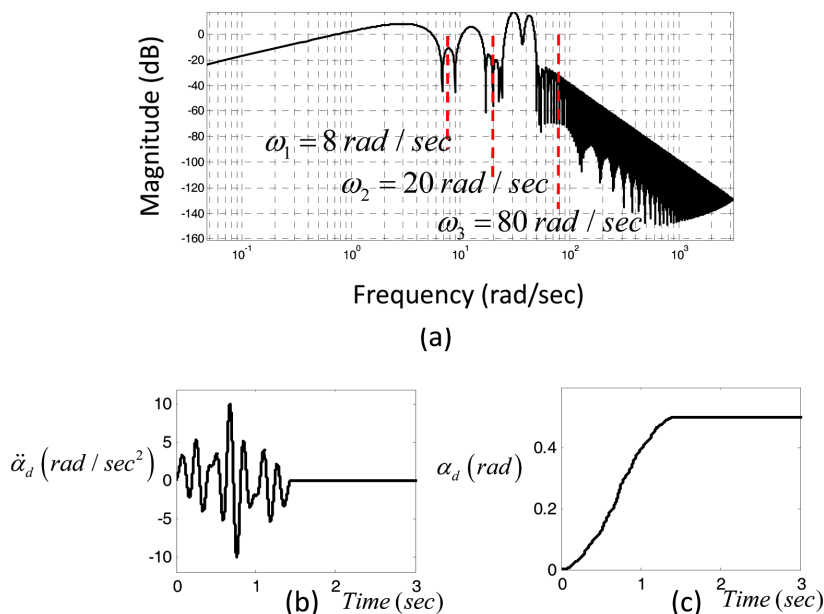


**Figure 10** (a) Frequency spectrum of the shaped  $\ddot{\alpha}_d$ . (b) Time plot of  $\ddot{\alpha}_d$ . (c) Corresponding angular position  $\alpha_d$ .

natural frequencies results in a slower desired position profile. However, as will be seen in the experimental result, the ramped sinusoidal profile greatly reduced the robot's move time because it induced much less residual vibration than the square-wave profile.

## RESULTS

Figure 11(a) shows the experimental results of the robot's angular position  $\alpha$  when the square wave is used as  $\ddot{\alpha}_d$  (dotted line, without command shaping) and when the shaped input is used as  $\ddot{\alpha}_d$  (solid line, with command shaping). It is clear that the robot settled at its desired trajectory much faster in the shaped input case with much less overshoot. The low-frequency vibration disappeared after 7 sec in the shaped input case while, in the square-wave case, the low-frequency vibration was still apparent as a result of the robot being very flexible. To quantify the result, the robot's settling time reduced from 3.3 sec to 1.2 sec using the shaped input, and the overshoot reduced from 120% to 30% using the shaped input.

The driving voltage inputs are given in Figure 11(b) where the dotted line belongs to the square-wave case, and the solid line belongs to the shaped input case. During the move period, in the shaped input case, the input was more active but with lower amplitude. As a result, the trajectory was actively controlled with less overshoot. The robot could follow the human command closely with significantly less vibration. Hence the robot could achieve faster point-to-point motion despite its very flexible link. The design can be extensively applied to actual cranes or industrial robot manipulators for faster moves and reduced link damage due to vibration.

Table 1 contains transient responses and control effort comparisons between the proposed method and other active control systems. All active control systems are designed to move the robot from its initial position at 0 *rad* to its end position

at 0.5 *rad*. They were tuned for the best results possible in terms of settling time and overshoot while the peak control effort was also recorded.

When the command shaping was turned on, the proposed method had the shortest settling time and smallest overshoot with the least peak control effort used. This paper has shown that using command shaping with a simple active controller, like the PD, can make the following contributions:

1) With less settling time, the very flexible-link robot can be moved faster from point to point; therefore, productivity can be increased.

2) Less overshoot results in safer operations because the very flexible-link robot does not move beyond the safety zone, where obstacles may be hit. Less overshoot can also result

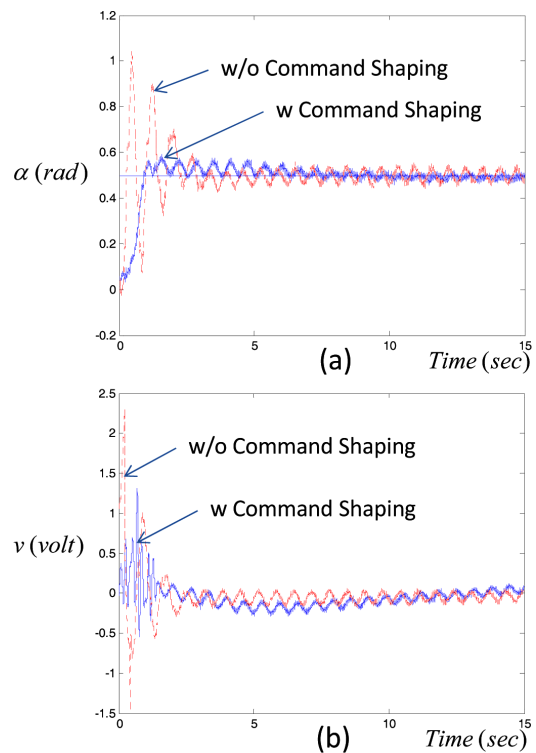


**Figure 11** (a) Robot's angular position with (w) and without (w/o) command shaping. (b) Input voltages for the two cases.

**Table 1** Transient responses and control effort comparison between proposed method and other active control systems.

| Method                                                  | Settling Time (sec) | Overshoot (%) | Peak Control Effort (volt) |
|---------------------------------------------------------|---------------------|---------------|----------------------------|
| Proposed method<br>(command shaping<br>with PD control) | 1.2                 | 30            | 1.3                        |
| PID control                                             | 3.3                 | 120           | 2.3                        |
| Fuzzy logic                                             | 3.0                 | 80            | 1.5                        |
| Quantitative<br>feedback control                        | 5.7                 | 200           | 3.5                        |

PD is the proportional-derivative controller.

PID is the proportional-integral-derivative controller.

in a longer life and lower maintenance cost for the robot.

3) With lower peak control effort, a smaller motor can be selected, which reduces the cost of the robot, and the running cost for items such as electricity can be lower.

## CONCLUSION

The desired acceleration profile was shaped using a series of ramped sinusoidal functions. The spectrum magnitude of the new profile was shaped to avoid energizing the system at its natural frequencies. As a result, the flexible-link robot achieved a faster settling time with less overshoot.

The work in this paper gives an alternative and practical way of controlling the slewing of the flexible-link robot. Since minimum knowledge of the system was required in the algorithm, it could easily be extended to the multi-link case or to other robots without much redesign effort.

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## LITERATURE CITED

- Alam, M.S. and M.O. Tokhi. 2008. Designing feedforward command shapers with multi-objective genetic optimization for vibration control of a single-link flexible manipulator. **Eng. Appl. Artif. Intell.** 21: 229–246.
- Chatlatanagulchai, W., V.M. Beazel and P.H. Meckl. 2006. Command shaping applied to a flexible robot with configuration-dependent resonance, pp. 1766–1771. *In Proceedings of the American Control Conference*. Minneapolis. Minnesota.
- Ge, S.S., T.H. Lee and J.Q. Gong. 1999. A robust distributed controller of a single-link SCARA/Cartesian smart materials robot. **Mechatronics** 9: 65–93.
- Ge, S.S., K.P. Tee, I.E. Vahhi and F.E.H. Tay. 2006. Tracking and vibration control of flexible robots using shape memory alloys. **IEEE/ASME Trans. Mechatron.** 11: 690–698.
- Iemsamai, K. and W. Chatlatanagulchai. 2008. Exact mathematical model of the flexible-link robot, pp. 399–404. *In Proc. of the 22nd Conference of Mechanical Engineering Network of Thailand*. Patum Thani.
- Khorrami, F., S. Jain and A. Tzes. 1995. Experimental results on adaptive nonlinear control and input preshaping for multi-link flexible manipulators. **Automatica** 31: 83–97.

- Lim, S., H.D. Stevens and J.P. How. 1999. Input shaping design for multi-input flexible systems. **J. Dyn. Syst., Meas., Control** 121: 443–447.
- Meckl, P.H. 1984. **Minimizing Residual Vibration of a Linear System using Appropriately Shaped Forcing Functions**. SM thesis. Department of Mechanical Engineering. Massachusetts Institute of Technology. Cambridge. Massachusetts.
- Meckl, P.H. 1988. **Control of Vibration in Mechanical Systems using Shaped Reference Inputs**. PhD dissertation. Department of Mechanical Engineering. Massachusetts Institute of Technology. Cambridge. Massachusetts.
- Meckl, P.H. and W.P. Seering. 1988. Reducing residual vibration in systems with uncertain resonances. **IEEE Control Syst. Mag.** 8: 73–76.
- Mimmi, G. and P. Pennacchi. 2001. Pre-shaping motion input for a rotating flexible link. **Int. J. Solids Struct.** 38: 2009–2023.
- Mohamed, Z. and M.O. Tokhi. 2004. Command shaping techniques for vibration control of a flexible robot manipulator. **Mechatronics** 14: 69–90.
- Pratiher, B. and S.K. Dwivedy. 2008. Non-linear vibration of a single link viscoelastic Cartesian manipulator. **Int. J. Non Linear Mech.** 43: 683–696.
- Shan, J., H.T. Liu and D. Sun. 2005. Modified input shaping for a rotating single-link flexible manipulator. **J. Sound Vib.** 285: 187–207.
- Singer, N.C. and W.P. Seering. 1988. Using acausal shaping techniques to reduce robot vibrations, pp. 1434–1439. *In Proceedings of the IEEE International Conference on Robotics and Automation*. Philadelphia. PA.
- Singer, N.C. and W.P. Seering. 1990. Preshaping command inputs to reduce system vibration. **ASME Trans. J. Dyn. Syst., Meas., Control** 112: 76–82.
- Tosunoglu, S., S. H. Lin and D. Tesar. 1990. Complete accessibility of oscillations in robotic systems by orthogonal projections. **J. Dyn. Syst., Meas., Control** 112: 194–202.
- Tzes, A. and S. Yurkovich. 1993. An adaptive input shaping control scheme for vibration suppression in slewing flexible structures. **IEEE Trans. Control Syst. Technol.** 1: 114–121.
- Wang, F.Y. and J.L. Russell. 1995. Optimum shape construction of flexible manipulators with total weight constraint. **IEEE Trans. Syst. Man Cybern.** 25: 605–614.
- Wang, F.Y. and Y. Gao. 2003. **Advanced Studies of Flexible Robotic Manipulators: Modeling, Design, Control, and Applications**. World Scientific. Singapore. 456 pp.
- Zuo, K. and D. Wang. 1992. Closed loop shaped-input control of a class of manipulators with a single flexible link, pp. 782–787. *In Proceedings of the IEEE International Conference on Robotics and Automation*. Nice. France.