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Study of Rutherford Scattering Cross Section via Geant4 Methods

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ABSTRACT

The measurement of alpha scattering from gold was instrumental in developing the model of an atom. Rutherford was able to verify this model by predicting the number of particles scattering at each angle. The Rutherford scattering cross sections were studied using the Geant4 (Geometry and tracking) software. Most of the results from the simulation were found to correspond with the data calculated using the Rutherford scattering formula. The trajectories of alpha particles produced during the reaction could be visualized through the simulation. This research showed that Geant4 can simulate accurately the scattering of charged particles from materials. Moreover, Geant4 can be applied in the simulation of the scattering of other particles from different materials in an elastic collision. The simulation by Geant4 is also beneficial in the physics laboratory since it can assist students in their understanding of Rutherford's alpha scattering experiments and avoid any unnecessary exposure to radioactive substances.

Keywords: simulation, alpha particle, Rutherford scattering, elastic collision, Geant4

INTRODUCTION

Alpha particles are attenuated by three major interactions—namely, nuclear collision, elastic scattering and absorption. At high energy (in excess of 20 MeV) a nuclear collision with high atomic nuclei is the major concern. In this case, another new product is produced through the inelastic collision. The energy of alpha particles at 5.5 MeV (thermal energy) has been used in simulation to achieve the conditions of an elastic collision. (Rutherford, 1911; Melissinos, 1966)

Geant4 (an acronym from geometry and tracking) is a toolkit for the simulation of the passage of particles through matter. This is supported by the European Organization for Nuclear Research (CERN). Geant4 (or G4) is the fourth version of this toolkit and is based on the C^{++} language which supports object oriented programming that enables further development and easy design (The GEANT4 Collaboration, 2003). The aim of the current work was to study the concept of the Rutherford scattering formula using Geant4. The simulation of Rutherford's alpha scattering experiment used americium-241 (Am-241) as the alpha source of energy (5.5 MeV). The purpose of the experiment was to study the compliance of yield (number of the alpha particle deflections) at different angles of deflection using simulation by Geant4 and the yield at different angles based on the Rutherford scattering formula. The results from the simulation will show whether Geant4 can simulate the scattering of charged particles from materials. Moreover, Geant4 can be

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applied in the simulation of the scattering of other particles from different materials in an elastic collision. The simulation by Geant4 can be used to visualize the Rutherford scattering which can contribute to an understanding of the physics concepts by matching mental images to this concept. This simulation can also influence the development of an actual experiment so that it is more accurate.

MATERIALS AND METHODS

Rutherford scattering formula

The Rutherford scattering cross section ($\frac{d\sigma}{d\Omega}$) is given by Equation 1 (Rutherford, 1911; Melissinos, 1966; Arya, 1998; Marion and Thornton, 2004):

$$\frac{d\sigma}{d\Omega}(\theta)=\left(\frac{Z_1 Z_2 e^2}{4E}\right)^2 \frac{1}{\sin^4 \theta/2} \quad (1)$$

where Z_1 and Z_2 are the atomic numbers of the projectile and target; e is the charge of the electron; E is the energy of the incident projectile; and θ is the angle of scattering from the initial direction (angle of deflection).

Equation 1 can provide the correct answer for an infinitely heavy nucleus or for the center-of-mass reference frame that moves at a velocity less than that of the incident particle. For laboratory energies and angles, the cross section may be written as Equation 2 (Rutherford, 1911; Melissinos, 1966; Krane, 1987):

$$\frac{d\sigma}{d\Omega}(\theta)=1.296\left(\frac{Z_1 Z_2}{E}\right)^2\left[\frac{1}{\sin^4 \theta/2}-2\frac{M_1}{M_2}\right]$$

mb/steradian (2)

where the energy E is in MeV and M_1 and M_2 are the projectile and target masses, respectively; the units of mb are 1×10^{-27} cm^2, and one sphere = 4π steradian; E is 5.5 MeV; M_1 and M_2 are the atomic weights of alpha particles and gold, respectively (M_1 =4.002602 u, M_2=196.966543 u); Z_1 and Z_2 are the atomic numbers of alpha and gold, respectively (Z_1=2, Z_2=79). The logarithmic plot of the Rutherford scattering cross section at different angles of deflection is shown in Figure 1.

The change of cross section $\{\frac{d\sigma}{d\Omega}(\theta)\}$ is usefully obtained from Rutherford's experiment because the cross section is difficult to measure in the laboratory. The relationship between yield (number of the alpha particle deflections) and the Rutherford scattering cross section is given by Equation 3:

$$Y(\theta)=n_0 Nx\frac{d\sigma}{d\Omega}\Delta\Omega \quad ;$$

$$N=\frac{6.024\times 10^{23}\times \textit{density of gold }(gm/cm^3)}{\textit{mass of gold }(gm/mole)} \quad (3)$$

where n_0 is the number of incident particles; N is the density of target atoms; x is the target thickness; and $\Delta\Omega$ is the solid angle subtended by the detector which can be given by Equation 4:

$$\Delta\Omega=\frac{A}{4\pi r^2}\ , \quad (4)$$

where A is the area of the detector that is used; and r is the distance from the target to the slit that defines the area of the detector (Rutherford, 1911; Melissinos, 1966; Lapp and Andrews, 1972; Krane, 1987).

The relationship between the yield and the angle of deflection (θ) is given by Equation 5:

$$Y(\theta) = n_0 Nx\frac{d\sigma}{d\Omega}\Delta\Omega$$

$$= n_0 Nx\left(\frac{Z_1 Z_2 e^2}{4E}\right)^2 \frac{1}{\sin^4 \theta/2}\frac{A}{4\pi r^2}$$

$$= K\frac{1}{\sin^4 \theta/2} \quad ; \quad K=n_0 Nx\left(\frac{Z_1 Z_2 e^2}{4E}\right)^2\frac{A}{4\pi r^2}$$

(5)

Equation 5 is the yield at different angles of deflection from the Rutherford scattering formula.

Details of simulation

In order to carry out a simulation, the main program must be executed, which will vary according to the needs of a given simulation application that must be supplied by the user. The Geant4 toolkit does not provide a main program, so that the user has to build it by including subprograms needed to run the application. The main program is implemented by two toolkit classes: the G4RunManager and the G4UImanager. The G4RunManager consists of five subprograms; DetectorConstruction, PhysicsList, PrimaryGeneratorAction, SteppingAction and TrackingAction. The G4UImanager creates a pointer to the interface manager. This manager class is created when the run manager is created. The pointer UI is created for the user to issue commands to the program (Colonna and Altieri, 2002; Incerti *et al.*, 2003; Anonymous, 2004; Cirrone *et al.*, 2006). The simulation of alpha particle scattering using Geant4 is illustrated in Figure 2.

Working procedure

The working procedure was divided into six steps (Anonymous, 2005; Cirrone *et al.*, 2006) shown in Figure 3.

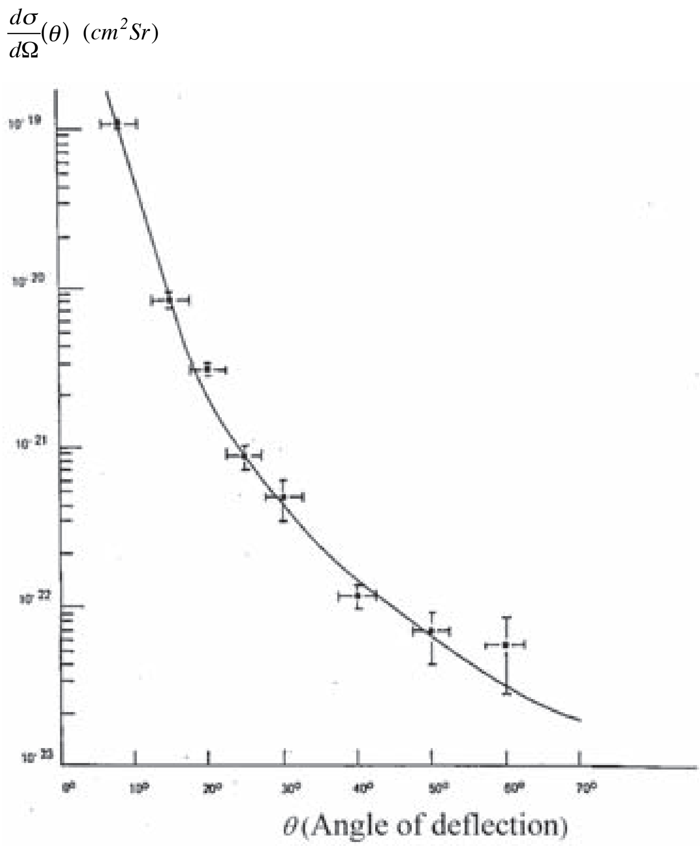

Figure 1 Rutherford scattering cross section at different angles of deflection (as calculated from Equation 2).

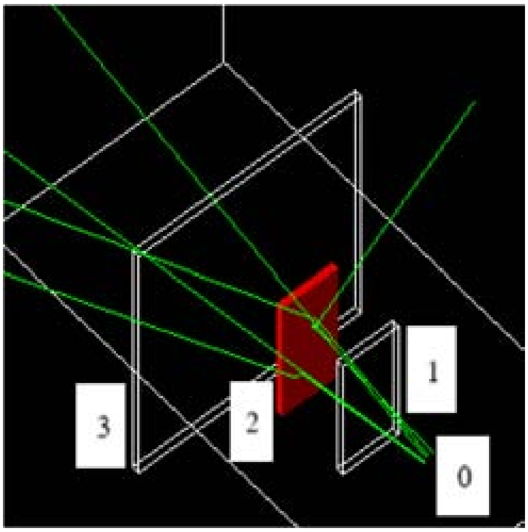

Figure 2 Illustration of the simulation of alpha particle scattering using Geant4, where point 0 represents a generator, point 2 represents a target which is gold foil, point 1 represents a vacuum measuring the number of alpha particles from a generation and point 3 represents a vacuum measuring the number of alpha particle scattering from a target.

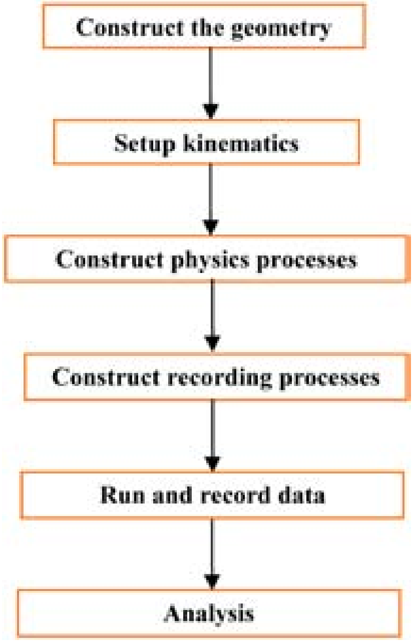

Figure 3 Flow chart of working procedure.

RESULTS AND DISCUSSION

The simulation of Rutherford's alpha scattering experiment used americium-241 (Am-241) as the alpha source of energy (5.5 MeV). The Am-241 was placed 5 cm from the target. Americium-241 emits alpha particles, is made in a nuclear reactor and is used as an alpha source in nuclear physics experiments. In the Rutherford scattering experiment, the emission of alpha particles (5.5 MeV for each particle) is controlled by a collimator which is 5 cm long and under a vacuum. Thus, the alpha particles are emitted from the Am-241 source as a beam and have energy of 5.5 MeV for each particle. The target is gold foil 2 μm in thickness and 36 alpha detectors are placed around the target at a distance of 5.75 cm from the target. Each detector has a radius of 0.5 cm, 0.1 cm thickness and is subtended with 10° of deflection angle (Figure 4).

A simulation of Rutherford's alpha scattering experiment by Geant4 can be plotted as the logarithm of yield at different angles of deflection as shown in Figure 5 by the solid line.

The data of the simulation setup (n_0 = 1,000,000 particles, ρ (density of gold) = 19.3 gm.cc^{-1}, x = 2 μm, $Z_1 = 2$, $Z_2 = 79$, $e = 1.6 \times 10^{-19}$ coulomb, E = 5.5 MeV and $A/4\pi r^2 = 10° = 0.17$ steradian) can be used to calculate the yield at different angles of deflection from the Rutherford scattering formula (Equation 5). The results of the Rutherford scattering formula are shown in Figure 5 as the dotted line.

Most of the results from the simulation showed good agreement with the data calculated from the Rutherford scattering formula. Deviation can be explained by quantum mechanics; the majority of results from the simulations at an angle of deflection of more than 90° correlate with the theory very well because the incident alpha rays that deflected more than 90° can penetrate the

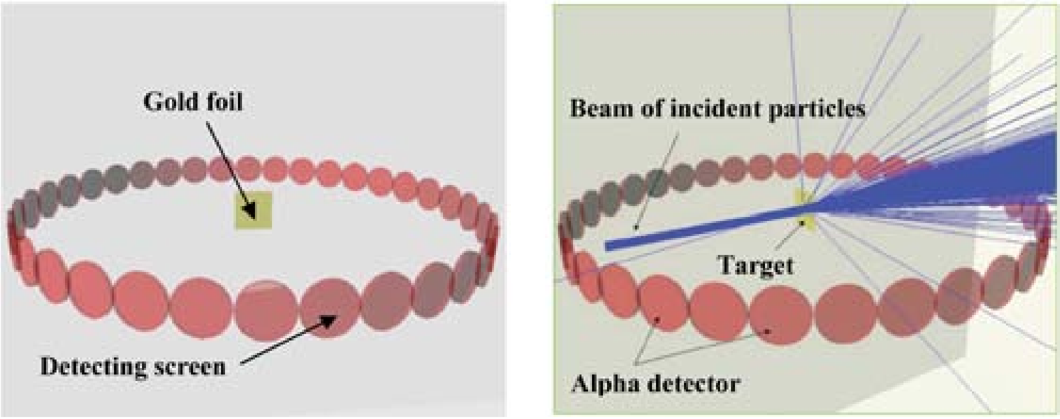

Figure 4 Simulation of Rutherford's experiment by Geant4: (a) Simulation setup. The gold target is 1 cm × 1 cm × 2 μm and is positioned at the center of the experimental hall. The 36 detectors are 5.75 cm away from the gold target. All detectors have a radius of 0.5 cm and a thickness of 0.1 cm; (b) A total of 1,000,000 events was simulated. The alpha source energy was 5.5 MeV and was 5 cm away from the gold target. The alpha detector represents a vacuum measuring the number of alpha particles scattering from the target.

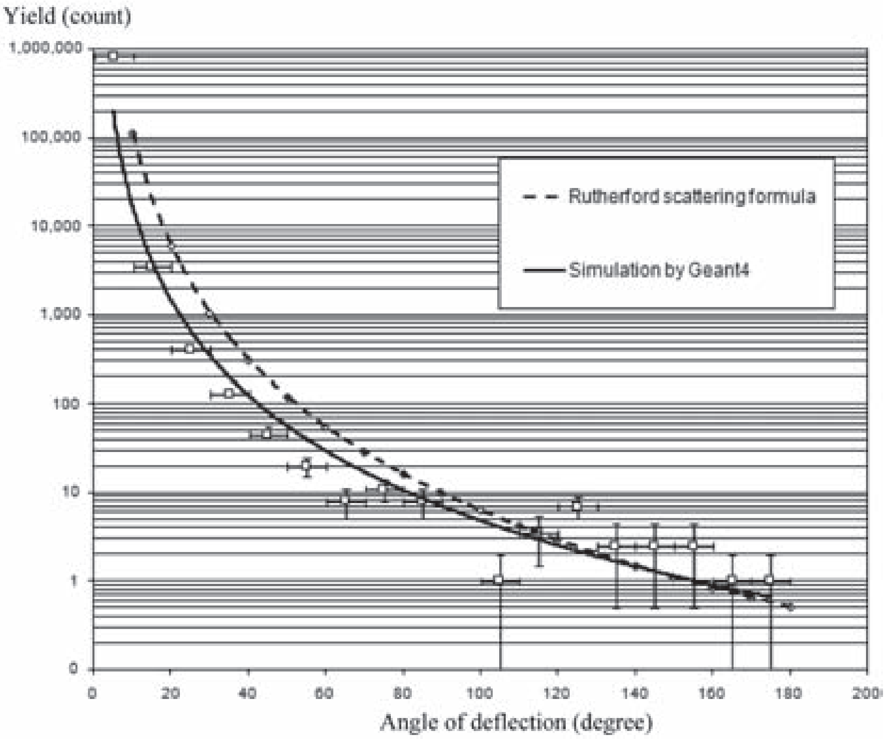

Figure 5 Plot of yield at different angles of deflection from simulation by Geant4 (solid line) and by Rutherford scattering formula (dotted line).

Coulomb potential barrier. The results indicate a finite probability for penetration of the Coulomb potential barrier (Rutherford, 1911; Melissinos, 1966; Krane, 1987). A similar explanation is necessary to explain the alpha particle emission during radioactive decay (Marion, 1960). An incident alpha particle which can penetrate the Coulomb potential barrier will have a hyperbolic trajectory. The Rutherford scattering formula can provide a good result as long as the perihelion distance is greater than 1×10^{-14} m. Thus, it can be concluded that the positive charge of the nucleus must be concentrated in a sphere with a radius of less than 1×10^{-14} m (Arya, 1998; Marion and Thornton, 2004; Fowles and Cassiday, 2005).

The results from the simulation show that Geant4 can simulate the scattering of charged particles from materials. Moreover, Geant4 can be applied in the simulation of scattering of the other particles from different materials in an elastic collision. The simulation by Geant4 is also beneficial in a physics laboratory since it can help students to understand Rutherford's alpha scattering experiment and visualize the path of scattered alpha particles while avoiding unnecessary exposure to radioactive substances during the experiment in the laboratory. This simulation also influences the actual experiment results to be the most accurate.

CONCLUSION

Geant4 can simulate accurately the scattering of charged particles from materials. Furthermore, Geant4 can be applied in the simulation of scattering of other particles from different materials in the case of an elastic collision such as from Rutherford scattering and Compton scattering, among others.

The simulation by Geant4 is also beneficial in a physics laboratory since it can help students to understand Rutherford's alpha scattering experiment and visualize the path of scattered alpha particles while avoiding unnecessary exposure to radioactive substances during the experiment in the laboratory. This simulation also influences the actual experiment results to be the most accurate.

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