

## A study of vector analysis in a rotating magnetic field using Scilab

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### Abstract

In electromagnetics, a rotating magnetic field in three-phase electrical machines is one of the most complicated topics to describe to the students in our class to understand the process of the stator of an electrical machine generates the rotating magnetic field. These problems focus on transforming these fields into phasors using mathematical ways. However, the mathematical proof is quite complex to conceive and difficult to image the space phasors rotation. In this research, a vector analysis is recommended to solve these problems and applied to study the rotating magnetic field in electrical engineering, which takes the mathematical approach to a Scilab simulation. To demonstrate the methodology, examples of application vector analysis for rotating magnetic field using Scilab under various values of the magnetic fluxes are presented. The results show that the students can comprehend the principles of rotating magnetic field.

**Keywords:** Electromagnetics, Magnetic field, Vector analysis, Scilab

### Introduction

Electrical Machinery as a device can use almost everywhere, whether at home or a hospital and in the general industry, such as lathes, electric cars, etc. The principle of electrical machinery is caused by the interaction between the magnetic field at the copper windings in the stator (Stator) and the magnetic field in the copper windings in the rotor (Rotor), causing the attraction and repulsion of both magnetic fields. And then use the energy obtained from that rotation. In electromagnetic theory, the interaction of a conductor with a rotating magnetic field is an essential topic. It is also frequently encountered in various applications in a variety of branches, such as electric generators, induction regulators, and induction motors

(Kraftmakher, 2001). To gain further insight into the procedure of such systems, the instructor begins by explaining the relationship between steps and processes as clearly as possible because students are thus far introduced to flux plots and the motion of magnetic fields.

Vector analysis plays an important role in explaining the direction of magnetic flux around the conductor. The vector field is particularly identified by the behavior of its components when the coordinate axes are rotated. Mathematics and Statistics have proven to be very efficient in presenting complicated phenomena and allowing for easy visualization and manipulation. Since its inception, the concept of vector has been extensively covered in numerous textbooks (Thidé, 2004; Guru and Hiziroglu, 2009), and its analysis has been widely used in Electromagnetics, Electromagnetic fields and Electromagnetic Waves subjects.

Numerous research studies have investigated the electromagnetic processes using vector analysis (Guru and Hiziroglu, 2009; Khokhlov et al., 2016; Kovivchak, 2017). However, there are always rotational questions of the interaction between the magnetic fields of movement magnet. Nowadays, vector analysis is one of the most popular topics in mathematics and commonly used to explain the movement of a rotating magnetic field. To illustrate this procedure, it can be difficult for students and is not enough to learn this lesson. This is because it is not clear image of rotation. Finding a suitable approach, these problems can be explained by vector analysis combine with the tool for calculation which is Scilab. In this article, vector analysis is applied to study the rotating magnetic field in electrical engineering which takes the mathematical approach to a Scilab simulation. In this article, vector analysis was used to investigate the rotating magnetic field in electrical engineering using a mathematical approach and a Scilab simulation. The paper is organized as follows: a brief physical description of the stator of three phase induction machines and the explanation of a complementary tool for analysis and simulation are given in section 2. The examples of application vector analysis using Scilab are calculated in section 3. Lastly, the conclusion is presented in section 4.

## Materials and Methods

### Rotating magnetic field

Throughout this section, we first introduce a basic physical description of three phase induction machines' stators. Three phase induction motors have symmetric three phase stator windings that are displaced in time from each other with an angle at  $120^\circ$  in each space (Frede, 2018). In this study, the sequence a'b'c' is denoted as the magnetic axis of three-coils. Therefore, the three-phase machine consists of 3 coils as follows:

a - a'

b - b'

c - c'

We can demonstrate three coils displayed in space around a primitive three-phase machine's stator as in figure 1.

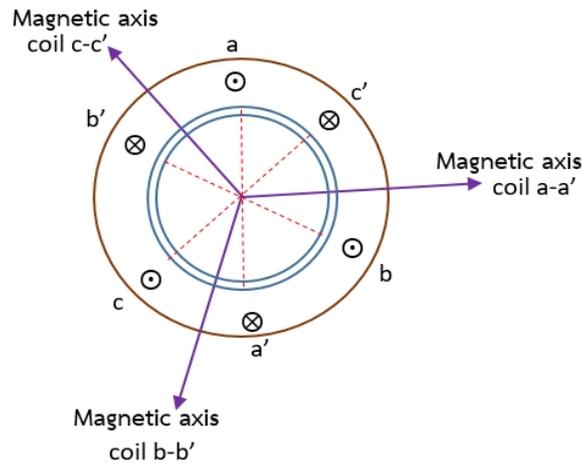


Figure 1 Illustration of Magnetic axis of 3-coils.

We can see that each coil's magnetic axis is perpendicular to its corresponding plane. The three coils placed in the space around the stator are shown in figure 1. A magnetic field is generated when a three-phase winding is energized by a three-phase supply. Besides that, the resultant component will rotate in space around the electric machine's air gap. As a result, it is referred to as a rotating field. Additionally, each winding's conductor is distributed in the pattern of a sinusoidal wave as shown in figure 2. Hence, when current flows into the three-phase windings, it produces the sinusoidal waveform's magneto motive force (MMF).

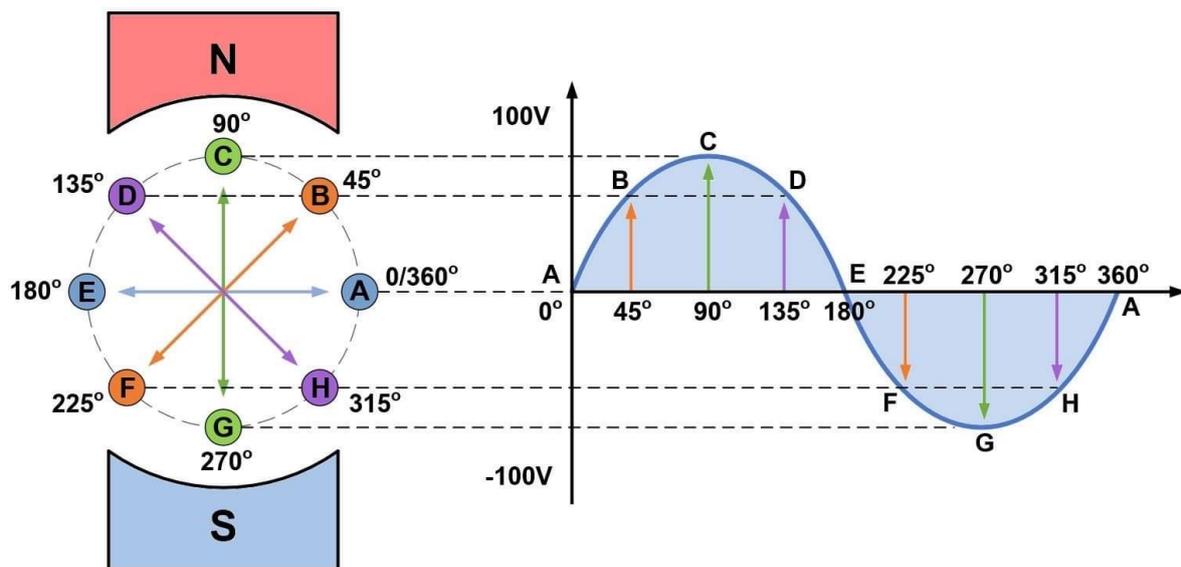


Figure 2 Illustration of a sinusoidal waveform.

Mathematical analysis of rotating magnetic field

In a balanced system, the mathematical equations for the instantaneous values of the three fluxes  $M_x$ ,  $M_y$  and  $M_z$  can be written as functions of time as follows:

$$M_x = M_m \sin \omega t \tag{1}$$

$$M_y = M_m \sin (\omega t - 120) \tag{2}$$

$$M_z = M_m \sin (\omega t + 120) \tag{3}$$

where  $M_m$  is the magnitude of each flux (or amplitude of the flux),  $t$  is time, and  $\omega$  is an angular velocity. In addition, the instantaneous fluxes  $M_x$ ,  $M_y$  and  $M_z$  can be represented as the fluxes wave in the space corresponding green, red and purple phase winding, respectively in the following:

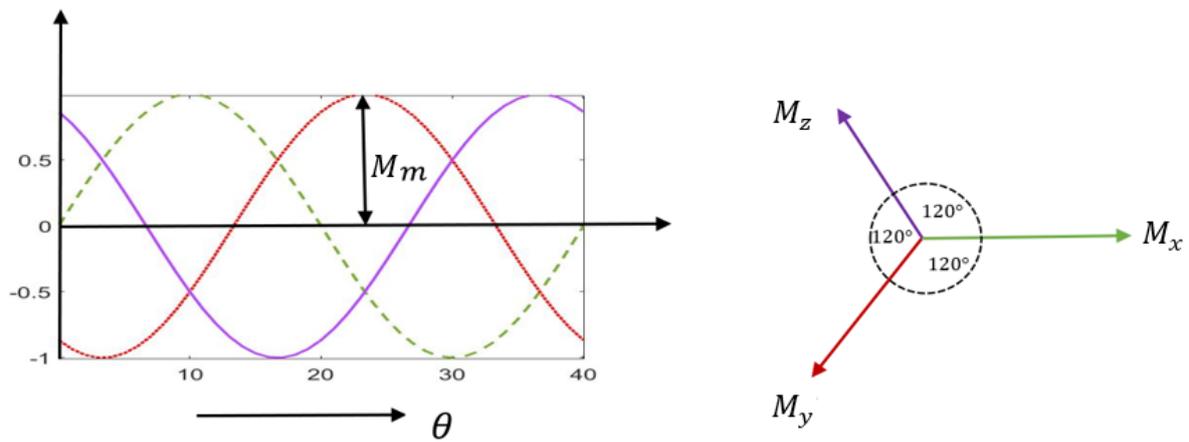


Figure 3 Illustration of three phases voltage sinusoidal waveform.

Next, we consider the magnitude of the resultant flux by using the phasor diagram of the three fluxes as shown in figure 4.

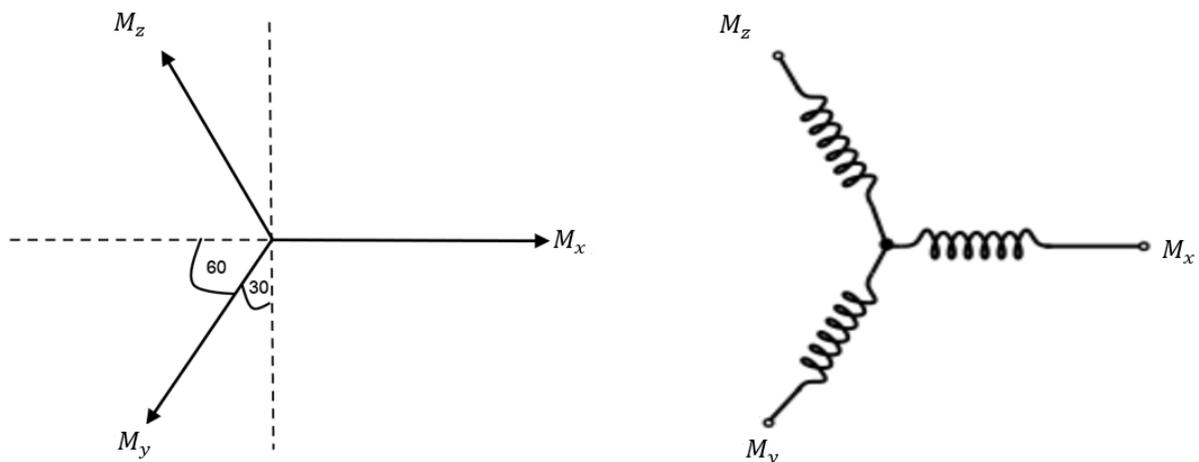


Figure 4 Illustration of the phasor diagram of the three fluxes.

In figure 4, we will obtain the magnitude of the resultant flux by considering phasor sum of horizontal and vertical components flux. Firstly, we can calculate the resulting horizontal flux component in the following equations:

$$\begin{aligned} M_H &= M_x - M_y \cos 60^\circ - M_z \cos 60^\circ \\ &= M_x - (M_y + M_z) \cos 60^\circ \\ &= M_x - (M_y + M_z) \frac{1}{2} \end{aligned}$$

To substitute the equations (1)-(3) into the above equation,  $M_H$  can be computed as:

$$\begin{aligned} M_H &= M_m \sin(\omega t) - \frac{1}{2} [M_m \sin(\omega t - 120) + M_m \sin(\omega t + 120)] \\ &= M_m \sin(\omega t) - \frac{1}{2} [M_m (\sin(\omega t - 120) + \sin(\omega t + 120))] \\ &= M_m \sin(\omega t) - \frac{M_m}{2} [\sin(\omega t) \cos(120) - \cos(\omega t) \sin(120) + \sin(\omega t) \cos(120) + \cos(\omega t) \sin(120)] \\ &= M_m \sin(\omega t) - \frac{M_m}{2} [2 \sin(\omega t) \cos(120)] \\ &= M_m \sin(\omega t) - \frac{M_m}{2} [2 \sin(\omega t) \left(-\frac{1}{2}\right)] \\ &= \frac{3}{2} M_m \sin(\omega t). \end{aligned} \tag{4}$$

As same as the procedure above, we can prove that

$$M_V = \frac{3}{2} M_m \cos(\omega t). \tag{5}$$

Consequently, from the equations (4)-(5), the resultant flux is denoted by considering the magnitude of vector in the following equation:

$$\begin{aligned} M_R &= \sqrt{(M_H)^2 + (M_V)^2} \\ &= \sqrt{\left(\frac{3}{2} M_m \sin(\omega t)\right)^2 + \left(\frac{3}{2} M_m \cos(\omega t)\right)^2} \\ &= \frac{3}{2} M_m \sqrt{(\sin(\omega t))^2 + (\cos(\omega t))^2} \\ &= \frac{3}{2} M_m \end{aligned} \tag{6}$$

We can see that the resultant of these fluxes at instant  $M_R$  is equal to  $\frac{3}{2}$  times of the magnitude of each flux ( $M_m$ ) per phase.

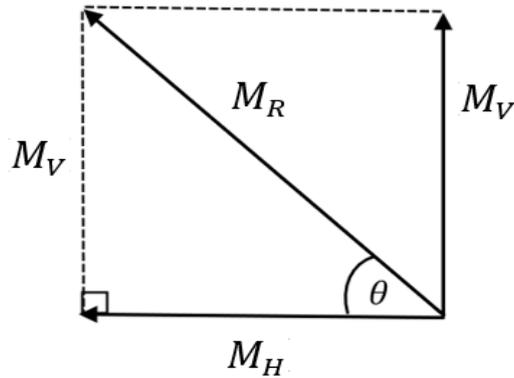


Figure 5 The horizontal ( $M_H$ ) and vertical ( $M_V$ ) components of the flux.

The horizontal and vertical components of the flux are obtained and presented in figure 5. It was observed that the relation of the two components can be expressed as follows:

$$\begin{aligned}\tan(\theta) &= \frac{M_V}{M_H} = \frac{3}{2} M_m \cos(\omega t) / \frac{3}{2} M_m \sin(\omega t) \\ &= \frac{\cos(\omega t)}{\sin(\omega t)} \\ &= \cot(\omega t) \\ &= \tan(90^\circ - \omega t)\end{aligned}$$

Therefore, we obtain  $\theta$  which is equal to  $90^\circ - \omega t$ . Obviously, it is the function of time. On the other hand, equations (1) – (3) can be rewritten as a polar form:

$$M_x = M_m \sin \omega t [\cos(0) + i \sin(0)], \quad (7)$$

$$M_y = M_m \sin(\omega t - 120) [\cos(-120) + i \sin(-120)], \quad (8)$$

$$M_z = M_m \sin(\omega t + 120) [\cos(120) + i \sin(120)], \quad (9)$$

respectively. Consequently, we obtain the resultant magnetic flux density by adding the vectors from equations (7)-(9) as follows:

$$\vec{M}_r = \vec{M}_x + \vec{M}_y + \vec{M}_z. \quad (10)$$

Where  $M_r$  is the resultant magnetic flux density.

From the previous equations, we can give an example by letting  $\omega t = 0$ , then we obtain  $\vec{M}_r$  as

$$\begin{aligned}\vec{M}_r &= M_m \sin \omega t [\cos 0 + i \sin(0)] + M_m \sin(\omega t - 120) [\cos(-120) + i \sin(-120)] \\ &\quad + M_m \sin(\omega t + 120) [\cos(120) + i \sin(120)] \\ &= M_m \sin 0 [\cos(0) + i \sin(0)] + M_m \sin(-120) [\cos(-120) + i \sin(-120)] \\ &\quad + M_m \sin(120) [\cos(120) + i \sin(120)]\end{aligned}$$

$$\begin{aligned}
&= 0 + M_m \left(-\frac{\sqrt{3}}{2}\right) \left[-\frac{1}{2} + i \left(-\frac{\sqrt{3}}{2}\right)\right] + M_m \left(\frac{\sqrt{3}}{2}\right) \left[-\frac{1}{2} + i \left(\frac{\sqrt{3}}{2}\right)\right] \\
&= M_m \left[\left(-\frac{\sqrt{3}}{2}\right) \left[-\frac{1}{2} + i \left(-\frac{\sqrt{3}}{2}\right)\right] + \left(\frac{\sqrt{3}}{2}\right) \left[-\frac{1}{2} + i \left(\frac{\sqrt{3}}{2}\right)\right]\right] \\
&= M_m \frac{\sqrt{3}}{2} \left[\left[\frac{1}{2} + i \left(\frac{\sqrt{3}}{2}\right)\right] + \left[-\frac{1}{2} + i \left(\frac{\sqrt{3}}{2}\right)\right]\right] \\
&= M_m \frac{\sqrt{3}}{2} \left[i \left(\frac{\sqrt{3}}{2}\right) + i \left(\frac{\sqrt{3}}{2}\right)\right] \\
&= M_m \frac{3}{2} (i) \\
&= M_m \frac{3}{2} [\cos(90) + i \sin(90)].
\end{aligned}$$

In this example, we obtain a resultant magnetic flux equal to  $\frac{3}{2}e^{j(90)^\circ}$  at  $\omega t = 0$ . Therefore, a polar form of complex number  $e^{j(90)^\circ}$  is defined as  $M_m[\cos(90) + i \sin(90)]$ . The resultant vector and the vector addition at zero degree are shown in figure 6.

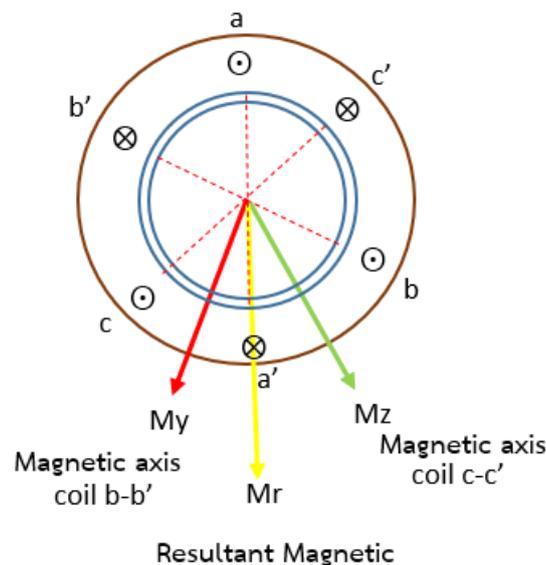


Figure 6 Illustration of the resultant vector and the vector addition at  $\omega t = 0^\circ$ .

Next example if we replace  $\omega t$  with 30, we can show that

$$\vec{M}_r = M_m \frac{3}{2} [\cos(60) + i \sin(60)].$$

Using the same procedure, we also observe that the resultant magnetic flux is equal to  $\frac{3}{2}e^{j(60)^\circ}$  at  $\omega t = 30$ . A polar form of complex number  $e^{j(60)^\circ}$  is defined as  $M_m[\cos(60) + i \sin(60)]$ . The resultant vector and the vector addition at 30 degrees are illustrated in figure 7.

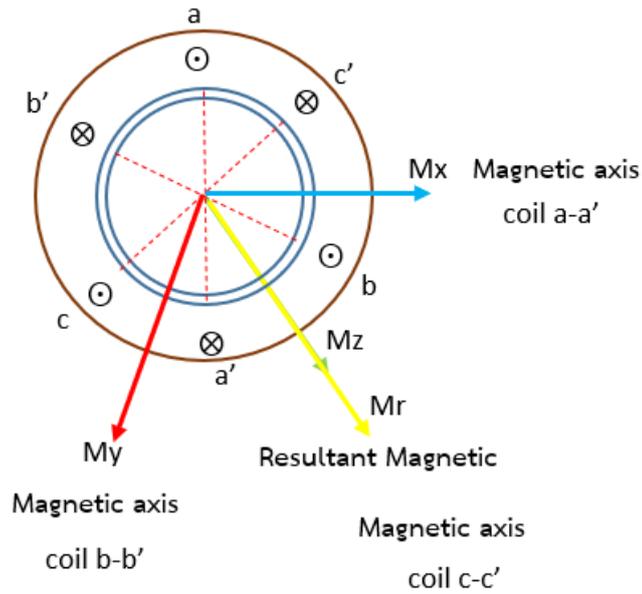


Figure 7 Illustration of the resultant vector and the vector addition at  $\omega t = 30^\circ$ .

Next example, we use  $\omega t = 60$ ,  $\vec{M}_r$  is:

$$\vec{M}_r = M_m \frac{3}{2} [\cos(30) + i \sin(30)].$$

In the following results, the resultant magnetic flux is equal to  $\frac{3}{2}e^{j(30)^\circ}$  at  $\omega t = 60$ . Here, a polar of complex number  $e^{j(30)^\circ}$  is defined as  $M_m[\cos(30) + i \sin(30)]$ . The figure also reveals that the resultant vector and the vector addition are at 60 degrees.

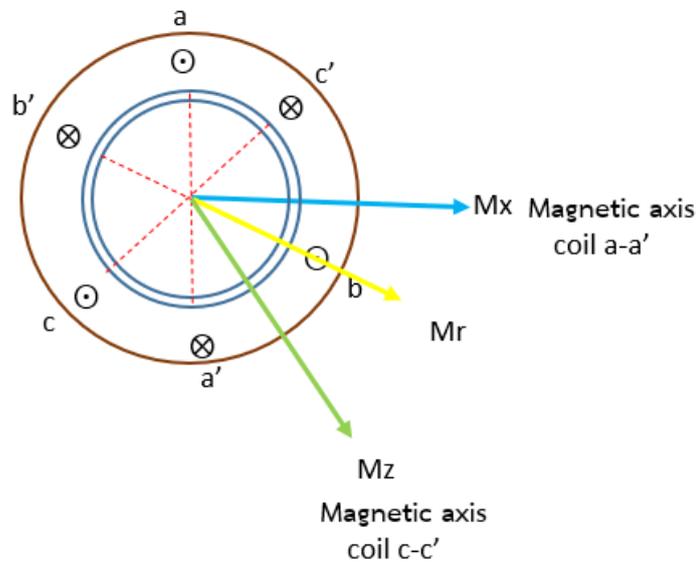


Figure 8 Illustration of the resultant vector and the vector addition at  $\omega t = 60^\circ$ .

In addition, if  $\omega t$  is converted into 90, then the resultant magnetic flux density  $\vec{M}_r$  is expressed as:

$$\vec{M}_r = M_m \frac{3}{2} [1 + i(0)].$$

As shown in the above examples, the resultant magnetic flux is equal to  $\frac{3}{2}e^{j(0)^\circ}$  at  $\omega t = 90^\circ$ . Here, a polar form of complex number  $e^{j(0)^\circ}$  is defined as  $M_m[\cos(0) + i\sin(0)]$ . The following figure indicates the resultant vector and the vector addition at 90 degrees.

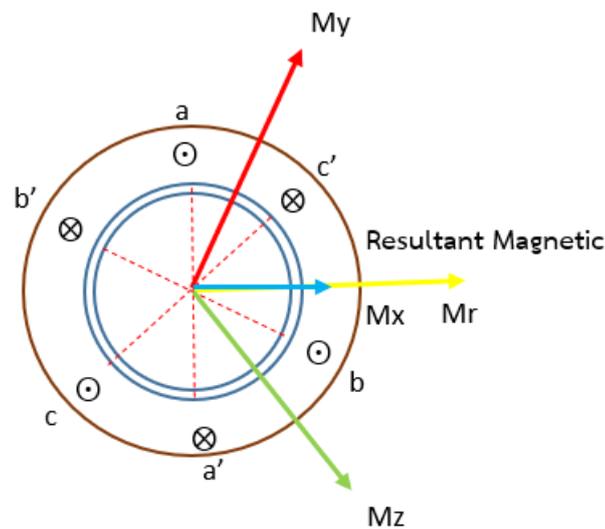


Figure 9 Illustration of the resultant vector and the vector addition at  $\omega t = 90^\circ$ .

## Results and Discussion

### Scilab Simulation

Scilab is a freeware tool for explaining the vector rotation in electric magnetic fields topic, which is used in most mathematics and engineering teaching software. In this section, we will briefly introduce the basic code for running Scilab to perform the calculations of  $M_r$  at different degrees of  $\omega t$ . The details are as follows:

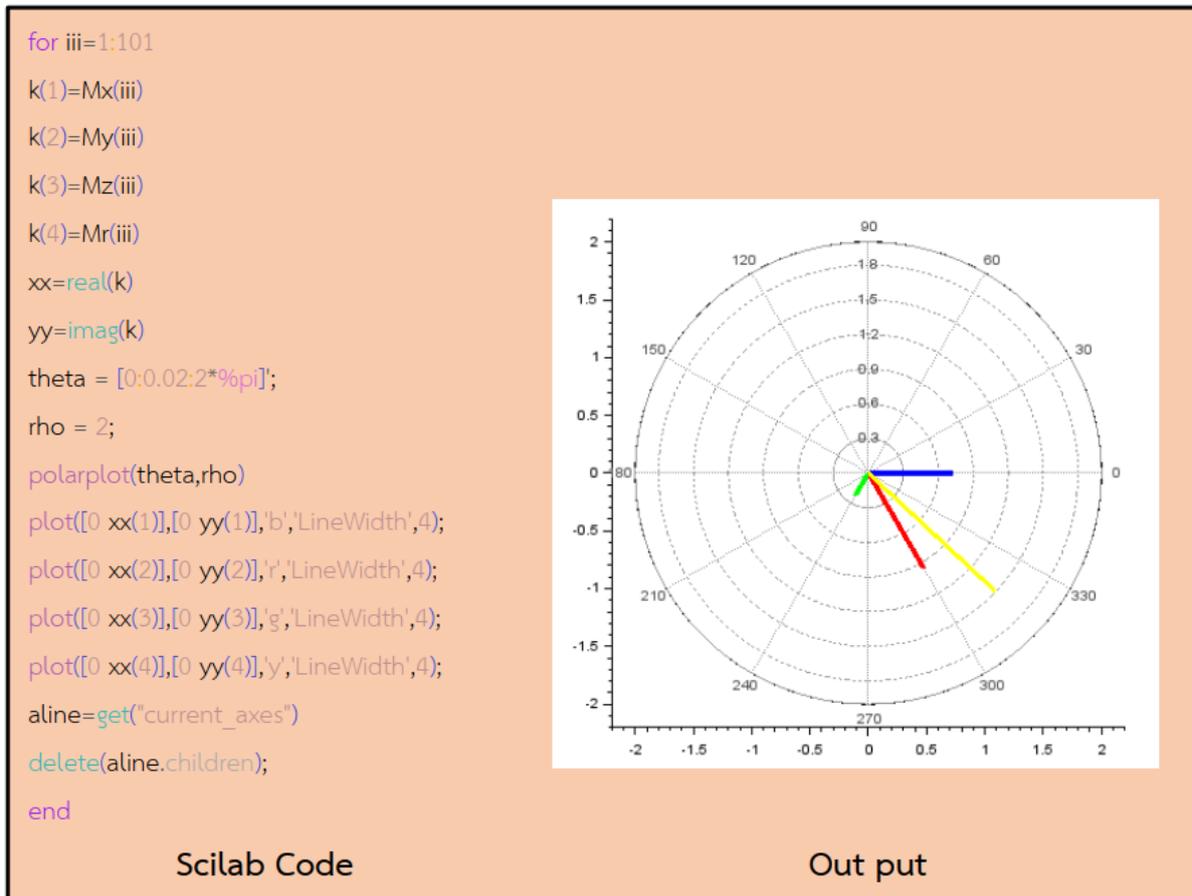
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j = sqrt(-1)
pi = 3.14159
Mx = sin(pi/2)*(cos(0) + j*sin(0))
My = sin(pi/2 + 2*pi/3)*(cos(-2*pi/3) + j*sin(-2*pi/3))
Mz = sin(pi/2 - 2*pi/3)*(cos(2*pi/3) + j*sin(2*pi/3))
Mr = Mx + My + Mz

```

Figure 10 Illustration of the code of Scilab to calculate the value of  $M_x$ ,  $M_y$ ,  $M_z$ , and  $M_r$ . Here Omega is time multiply by angular velocity.

From the above expression, we obtain the plots of  $M_x$ ,  $M_y$ ,  $M_z$ , and  $M_r$  as shown in figure 11.



**Figure 11** Illustration of the plot of  $M_x$ ,  $M_y$ ,  $M_z$ , and  $M_r$ . Here blue, red, green, and yellow lines represent  $M_x$ ,  $M_y$ ,  $M_z$ , and  $M_r$ , respectively.

Next, we will give an example of output of the vector rotation in electric magnetic fields at various angles of  $\omega t$ .

Problem 1: Case  $\omega t = 0^\circ$

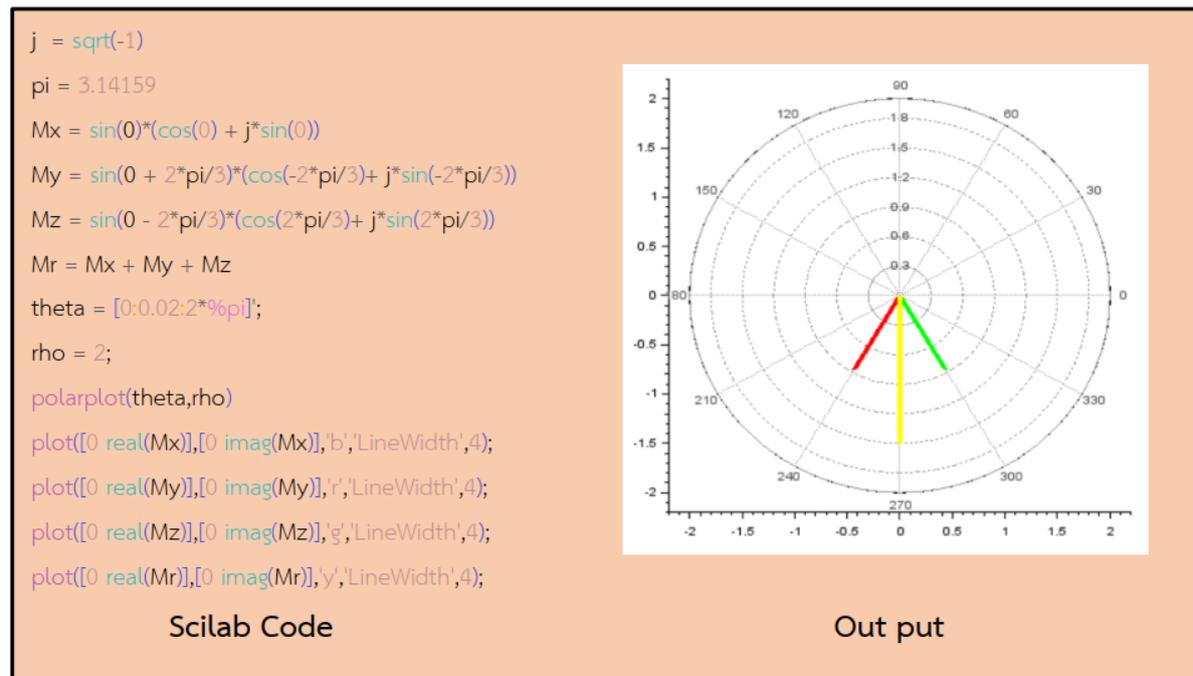


Figure 12 Illustration of Scilab - codes to calculate  $M_x$ ,  $M_y$ ,  $M_z$ , and  $M_r$  at  $\omega t = 0^\circ$ . Here red, green and yellow lines represent  $M_y$ ,  $M_z$ , and  $M_r$ , respectively.

Problem 2: Case  $\omega t = 30^\circ$  or  $\omega t = \frac{\pi}{6}$

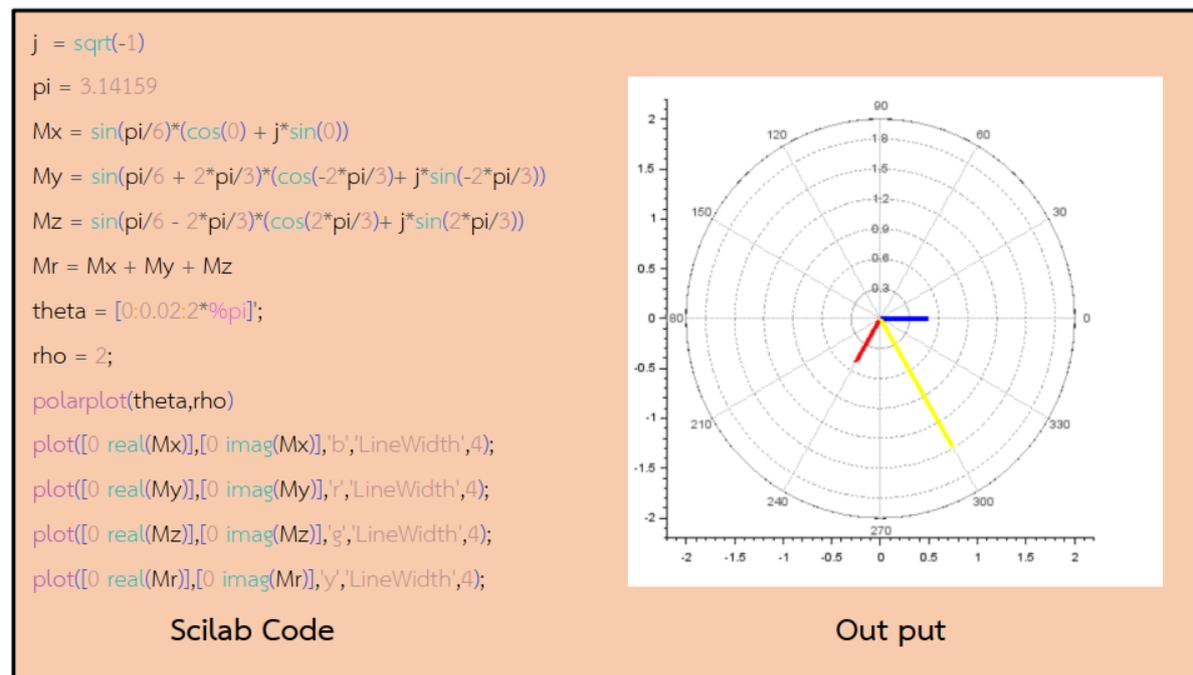


Figure 13 Illustration of Scilab - codes to calculate  $M_x$ ,  $M_y$ ,  $M_z$ , and  $M_r$  for  $\omega t = 30^\circ$ . Here blue, red, and yellow lines represent  $M_x$ ,  $M_y$  and  $M_r$ , respectively.

Problem 3: Case  $\omega t = 60^\circ$  or  $\omega t = \frac{\pi}{3}$

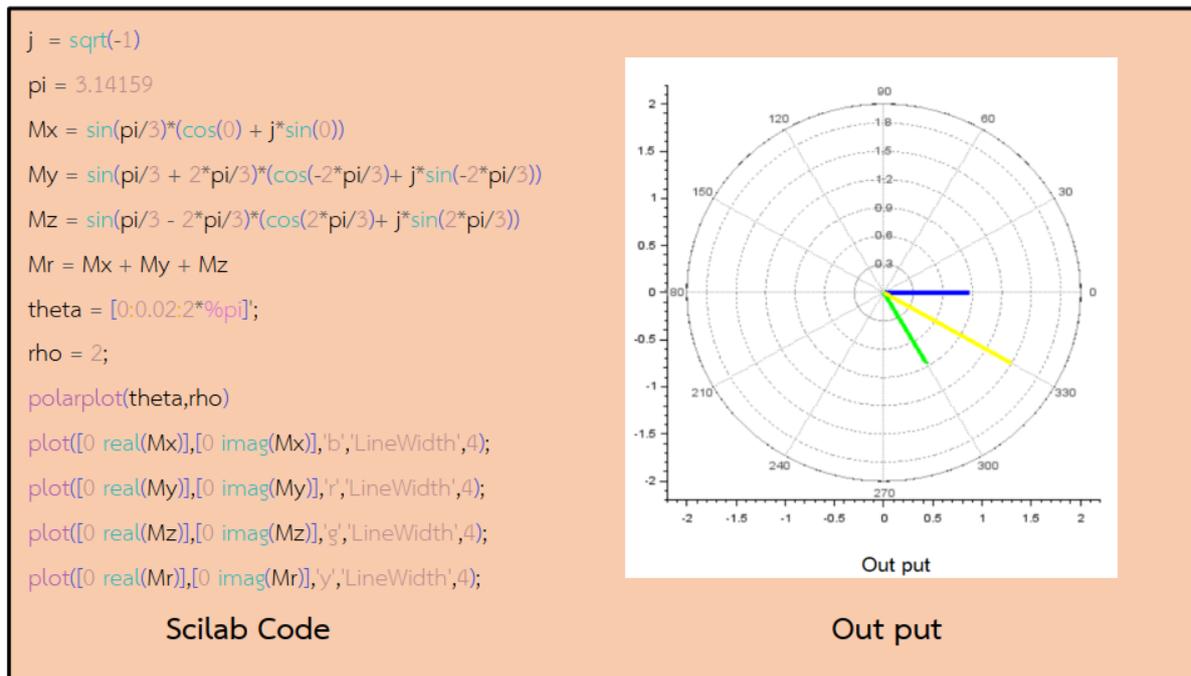


Figure 14 Illustration of Scilab - codes to calculate  $M_x, M_y, M_z,$  and  $M_r$  for  $\omega t = 60^\circ$ . Here blue, green and yellow lines represent  $M_x, M_z,$  and  $M_r,$  respectively.

Problem 4: Case  $\omega t = 90^\circ$  or  $\omega t = \frac{\pi}{2}$

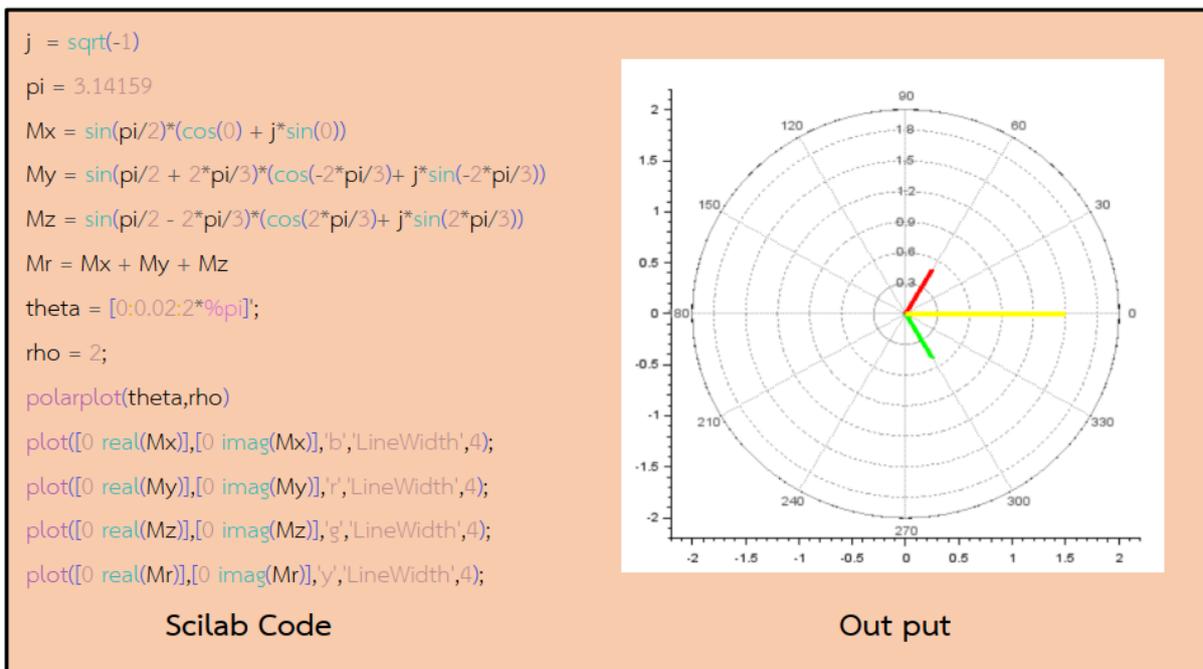


Figure 15 Illustration of Scilab - codes to calculate  $M_x, M_y, M_z,$  and  $M_r$  for  $\omega t = 90^\circ$ . Here red, green and yellow lines represent  $M_y, M_z,$  and  $M_r,$  respectively.

Scilab have shown to be the beneficial free software of simulation techniques in the rotating magnetic field for vector analysis. By illustrating the applications of vector rotation in electric magnetic field in several situations, students can gain further insight into the magnitude of the resultant flux and how it can be constructed and summarized in practical studies. We have also found that Scilab is an efficient tool in teaching of vector analysis.

## Conclusion

According to the results of our research, Scilab is a powerful simulation package. In this work, we attempted to incorporate this work into our energy conversion curriculum, as can be seen, that the output of the numerical simulation part is quite clear. The students can understand this problem through pictures. Using a vector to analyze this simulation with Scilab motivated learners to continue studying our electric machines and look forward to performing simulations that corresponded to their homework or concepts explained in class. For further studies we will consider research on rotating phasors for synchronous machines operating at various power factors.

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