

Incorporation of Shrinkage into Two Layer Diffusion Model for Air Drying

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Abstract

Shrinkage was modelled using a simple moisture volume displacement and incorporated into an existing two layer diffusion drying model. The resulting combined model was tested using shrinkage and drying data for banana slices (approximately 5 to 9 mm thickness) from three sets of conditions measured in replicate, over a range of conditions from 28-38°C and 12-30% relative humidity, using a heat pump dryer. Experiments for carrot were conducted at the same drying conditions for three different slice thicknesses: 3, 5 and 8 mm. The shrinkage model provided better agreement for carrot than for banana, but both were considered adequate fits and so the model was included in the drying rate equation. Parameters for the combined model were estimated by regression. It was found that on average, the prediction of the shape of the moisture-time drying curves was improved by inclusion of shrinkage in the two layer model, as the R^2 values of the model for banana and carrot increased by 9% and 14% respectively. The results confirmed that product shrinkage affects moisture transfer, and should be included in predictive model development.

Keywords: Shrinkage, Drying, Moisture Diffusion

1. Introduction

Drying is the most important food preservation technique and has been used for centuries to improve food stability. Drying involves simultaneous coupled and multiphase heat and mass transfer phenomena [1]. Loss of water during drying may cause a change in dimensions and affect other physical properties. Many foodstuffs are known to undergo volumetric changes due to water loss which is called shrinkage [2]. Shrinkage is a process that influences the volume and shape of the dried product, but may also affect the heat and mass transport properties. In particular the heat and mass diffusivities may change, the product surface area for evaporation may decrease, and the distance for molecular diffusion may decrease. Thus it should be expected that shrinkage influences the overall drying rate [3].

Many researchers have developed shrinkage models for different products, determining the mathematical relationship with the product moisture content of the product during drying. Linear models were developed for several fruits and vegetables, including apple [4], carrot [5] and potato [6]. This gives a simple empirical correlation. Similarly, simple parametric models were developed for apple, carrot, potato and banana to correlate product properties of the product (bulk density, particle density, porosity, specific volume) with moisture content [7, 8].

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Although linear models have proved adequate to describe shrinkage, some products were found to vary from linear behaviour. This may be due to decreased solid elasticity at low moisture content and the effect of increased internal porosity. Non-linear empirical models have therefore been developed more recently to describe the behaviour of such food products [5, 9].

Theoretical models have been developed for product shrinkage, based on mass balances and assuming additivity of the volumes of the different components of the system [10, 11]. The model developed by Perez and Calvelo [11] was used in this study to model shrinkage as a function of moisture content of carrot and banana, assuming that no air is present for the initial wet condition of the product. Then the initial volume is the sum of the volumes of water and dry solids,

$$V_0 = V_{t_0} = V_{s_0} + V_{w_0} \quad (1)$$

Porosity (ϵ) is defined as the volume ratio of internal pores to the total volume of the product. If the product becomes porous on drying, the relation between the product shrinkage and moisture content was derived as below,

$$\text{Shrinkage, } S = \frac{V}{V_0} = \frac{1}{(1-\epsilon)} \left[1 - \frac{\rho_{a_0}}{\rho_w} \frac{M_0 - M}{(1+M_0)} \right] \quad (2)$$

$$S = \left[1 - \frac{\rho_{a_0}}{\rho_w} \frac{M_0 - M}{(1+M_0)} \right] \quad (3)$$

This shrinkage model predicts shrinkage at different moisture contents and can be included in a drying model.

Thin layer drying equations are important tools for mathematical modelling of the drying process. The first term approximation to Crank's solutions for regular shapes leads to Newton's Law of Cooling (as applied to mass diffusion), and historically, this single term model has been used extensively to model thin layer drying. Realising that this often gives poor fit to an actual drying curve, many investigators have developed modifications to the single term model, in an attempt to retain model simplicity but improve accuracy, for example the two term model has been used for describing the drying behaviour of some products [12]. The two term model [1] was first proposed by Henderson using the first two terms of the general series solution of Fick's second law of diffusion giving,

$$MR = \frac{M - M_e}{M_i - M_e} = A_0 \exp(-k_0 t) + A_1 \exp(-k_1 t) \quad (4)$$

A variation on the two term model is the two layer model. This differs from two-term models in that it was expressly developed to solve the limitations of models based on Crank's solutions, specifically:

- Surface evaporation is now included.
- The model is valid under changing aeration conditions and for deep bed drying.
- The model can be used for a varying initial moisture state within a product.

The model was successful in describing the drying of ginger under changing aeration conditions [13]. The model assumes that two layers can approximate the whole drying process and each layer is isotropic in its properties.

For the internal layer,

$$\frac{dM_1}{dt} = -k_1 \mu (M_1 - M_2) \quad (5)$$

For the surface layer,

$$\frac{dM_2}{dt} = -k_1(M_2 - M_1) - k_e(M_2 - M_e) \quad (6)$$

The average moisture content of the product at any time is:

$$M = \frac{M_1 + \mu M_2}{1 + \mu} \quad (7)$$

The model equations are expressed and used in differential form so that changes during the drying process are correctly modelled. However this model does not include product shrinkage, and so the immediate goal was to develop the simplest variation of this model which would incorporate changes in product shape during drying. Data for both carrot and banana drying was used to determine any changes in drying rate prediction.

2. Materials and Methods

2.1 Materials and Equipment

Ripe bananas (Cavendish) from a local supermarket (Sydney, Australia) were prepared on the day of the experiment by slicing manually to a thickness of approximately 6 and 9 mm. Carrots were purchased from a local market (Sydney, Australia) and sliced by a vegetable shredder to approximately the required sample thickness (3, 5 and 8 mm), which was then measured precisely. The carrot samples were bought and prepared and vacuum packed on the day before the experiment.

2.2 Drying Experiment

The experiments on drying of bananas were performed at three different drying conditions with the slice thicknesses of approximately 6 mm. In addition, two experiments were conducted with a slice thickness of close to 9 mm at 38°C and 33°C to provide more data on the effect of slice thickness on the drying rate. The banana slices were dried in mesh trays at 28°C, 33°C and 38°C in a heat pump dryer for around 24 to 26 hours. For carrot drying, the experiments were conducted at the same drying conditions with three different slice thicknesses (approximately 3, 5 and 8 mm). Three of the run conditions were replicated in order to check the reproducibility of the experiments. The carrot slices were dried until the product mass reached 40% of its initial mass, at which point the moisture ratio was 0.33±0.03. The drying time for all experiments ranged from 1 to 9 hours depending on the drying conditions and slice thickness.

Initial and Final Moisture content measurement

Initial and final moisture content of the product were determined by drying in a vacuum oven for around 24 hours respectively at 70±1°C and 100 to 140 mm Hg pressure. The drying condition of the vacuum oven was maintained according to the standard AOAC method [14]. The time required to attain constant weight in the vacuum oven for the product was predetermined experimentally by testing for constant weight over two hour time intervals.

2.3 Drying condition and moisture measurement

The selected drying condition was set up in the dryer a minimum of 12 hours prior to the experiment start time, so as to achieve the desired drying temperature and relative humidity during drying. A Tinytag® data logger was kept in the dryer to record air temperature and relative humidity during the process. The mass of the product was digitally recorded each minute using an analytical balance during drying.

2.4 Shrinkage measurement

Before starting each experimental run, 40 slices of banana were prepared. These slices were put in eight different mesh trays, each containing 5 slices. Since the carrots dried faster, less sample was required for moisture determination, so only 30 slices of carrots were prepared for shrinkage measurement, arranged into 6 mesh trays. The diameter and height of each product slice was measured by digital Vernier callipers before drying. During drying, the mesh trays with samples were taken out randomly at different moisture contents and the dried product dimensions determined by digital Vernier callipers, allowing volumes to be calculated. As shrinkage is the ratio of final volume and initial volume of the product, the shrinkage of samples in the mesh tray can be determined. The trays extracted from the dryer were not returned to the drying chamber, since the measurement time was significant and may have affected the drying profile.

3. Results and Discussion

3.1 Drying kinetics of banana and carrot

From the temperature and relative humidity measurements of the drying air, the average drying conditions were determined to be 27°C with 32% RH, 32°C with 19% RH and 38°C with 20% RH. The moisture content of the product was determined from the reduction of mass of the product in the dryer. The initial and final moisture content were evaluated separately to calculate the moisture change of the product during the process. The average banana moisture content was 78% wet basis. The final moisture content of the banana slices varied from 8% to 25% (wet basis) depending on slice thickness and drying condition.

Figure 1 shows the change in moisture ratio (MR) of the banana slices with time at the different temperatures. The moisture reduction pattern indicates that most of the drying of the banana slices took place in the falling rate period, so moisture diffusion within the product was the dominant factor. The equilibrium moisture content was evaluated using the modified Oswin model [15] which is dependent on the air temperature and relative humidity of the product. The average equilibrium moisture contents of banana at the three conditions were found to be 2.65%, 2.67% and 5.8% (dry basis) respectively. The moisture diffusivities of banana were determined for each drying condition using the experimental data. The values were calculated by modelling the slices as a thin sheet, and estimated from the slope of $\ln(MR)$ vs. drying time and Crank's solution of Fick's second law. The average moisture diffusivity of banana slices at 38°C, 33°C and 28°C were found to be 2.1×10^{-10} m²/s, 1.8×10^{-10} m²/s and 1.0×10^{-10} m²/s, respectively. These values support the findings of Kechaou and Maalej [14] who reported the moisture diffusivity values of banana to be within 3.0×10^{-11} to 2.1×10^{-10} m²/s for the temperature range 20°C-40°C.

The moisture diffusivity of the product decreases with decreasing temperature in accordance with the Arrhenius equation. Figure 2 shows as expected that thin banana slices dried more quickly than the thick slices [17, 18]. For the carrot experiments, all three slice thicknesses (3, 5 and 8 mm) were used at each of drying conditions. Drying in all experiments continued until the product mass reduced 40% of its original mass. The initial moisture content of carrot range from 87% to 91% (wet basis). The average final moisture content of the dried carrot samples for all experiments was 72% (wet basis).

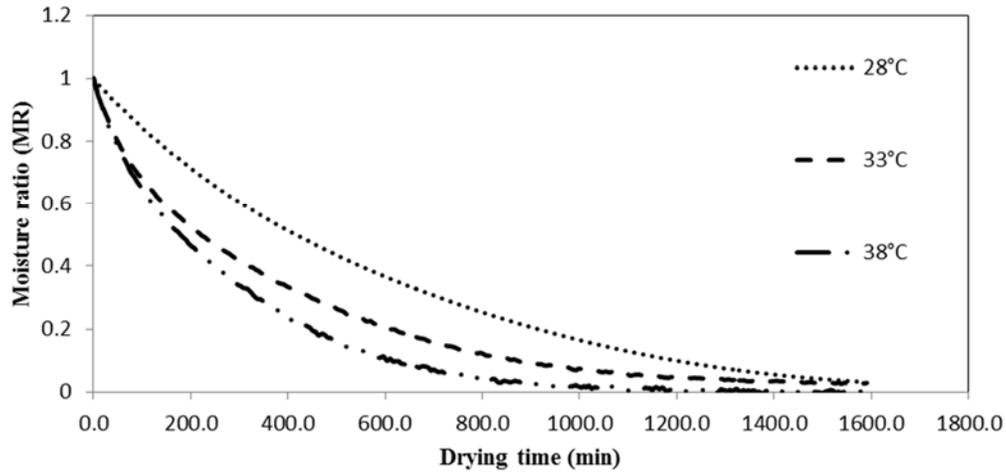


Figure 1. Variation in moisture ratio (MR) at three different drying conditions for banana with slice thickness of 5-6 mm

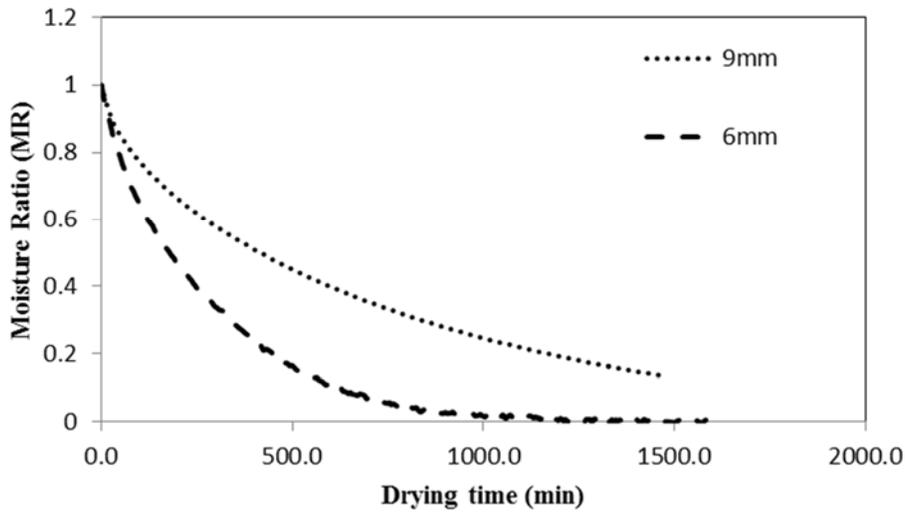


Figure 2. Variation in pattern of moisture ratio at 38°C and relative humidity 17% with two different slice thicknesses of banana

Figure 3 shows the thin layer drying profile of carrot slices at three different drying conditions. The falling rate period was not obvious for these experiments due to incomplete drying. The estimated average moisture diffusivities of carrot slices at 38°C and 33°C were both close to $3 \times 10^{-10} \text{ m}^2/\text{s}$ and at 28°C was approximately $1 \times 10^{-10} \text{ m}^2/\text{s}$. These values are close to the reported moisture diffusivities values in the literature [19]. For each chosen air condition, the

equilibrium moisture content was calculated using the modified Oswin model [20]. The average equilibrium moisture content at 38°C, 33°C and 28°C were around 10%,12% and 19% (dry basis) respectively. The effect of different slice thicknesses on the drying rate can be seen in Figure 4, which shows that the final moisture ratio for each thickness was close to 0.3 at the same drying condition, but the drying times varied from 50 min to 250 min as slice thickness increased.

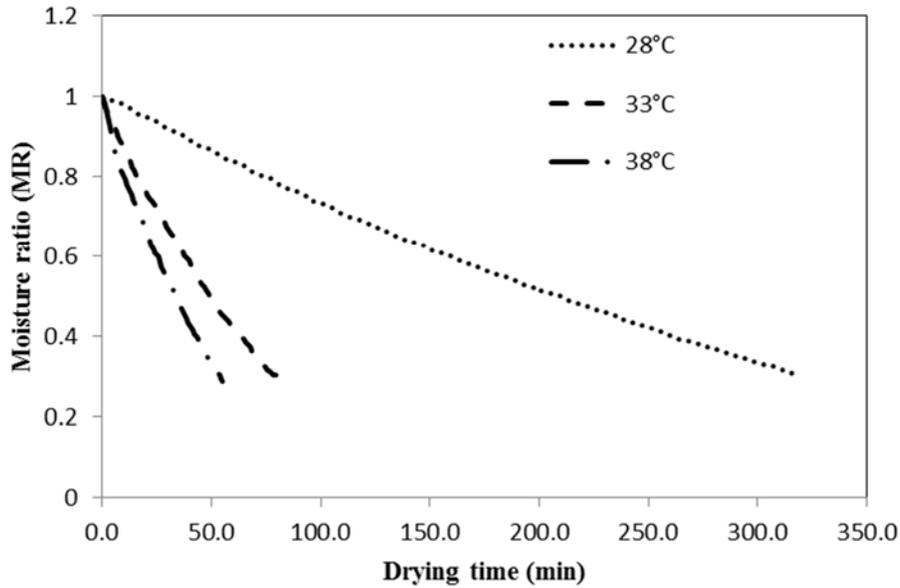


Figure 3. Variation in moisture ratio (MR) at three different drying conditions for carrot with slice thickness of 3 mm

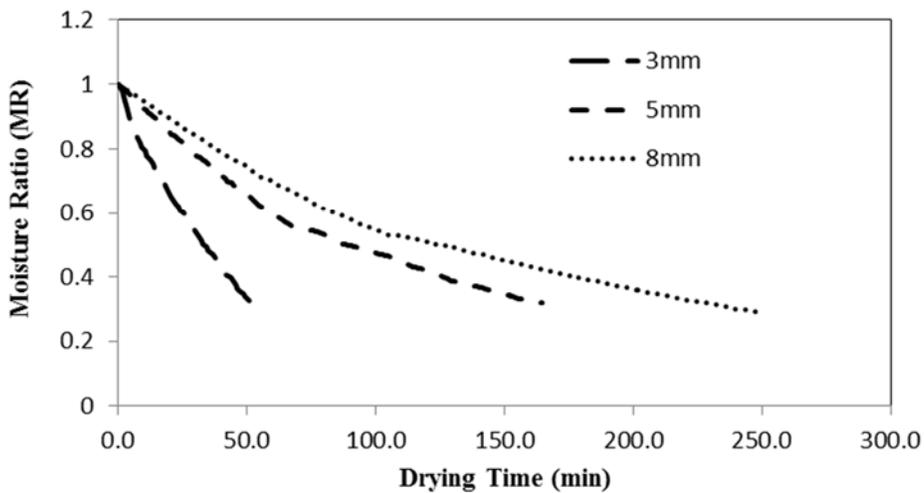


Figure 4. Variation in pattern of moisture ratio at 38°C and relative humidity 17% with three different slice thicknesses of carrot

3.2 Modelling the pattern of shrinkage for banana and carrot

Banana shrinkage at the two different thicknesses are shown in Figure 5. The effect of slice thickness on shrinkage was found to be insignificant, so shrinkage was dependent on the moisture content of the product only. Below moisture contents of 50% wet basis, non linearity was observed at all temperatures. This suggests that at high moisture contents, when the material is in a rubbery state, shrinkage almost entirely compensates for the moisture loss, resulting in a linear reduction in volume with moisture content. When the moisture content is very low, the phase of the material transforms to a glassy state, the rate of shrinkage is reduced and non-linear behaviour is observed for the product [21]. The region of non-linear shrinkage was small compared to the whole moisture reduction period during drying.

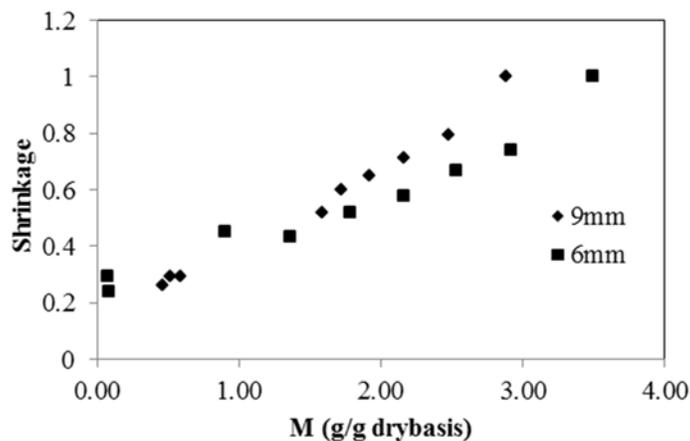


Figure 5. Shrinkage patterns of banana for two different thicknesses at drying condition 38°C and relative humidity 17%

Shrinkage of banana was modelled using the theoretical shrinkage model of equation 6 (Figure 6). The coefficient of determination was close to 0.91 and mean square error was 0.007 (Table1). The shrinkage model gave a reasonable prediction over the linear shrinkage region.

Figure 7 shows the shrinkage behaviour of carrot at 38°C for three different thicknesses. As carrot drying was stopped when the moisture content reached 67% wet basis, the shrinkage pattern for all conditions appeared to be linear throughout the drying process. The proposed shrinkage model fitted well for carrot slices (Figure 8). The shrinkage model was thus shown to be adequate for both test products, so was accepted for inclusion in the two layer drying model.

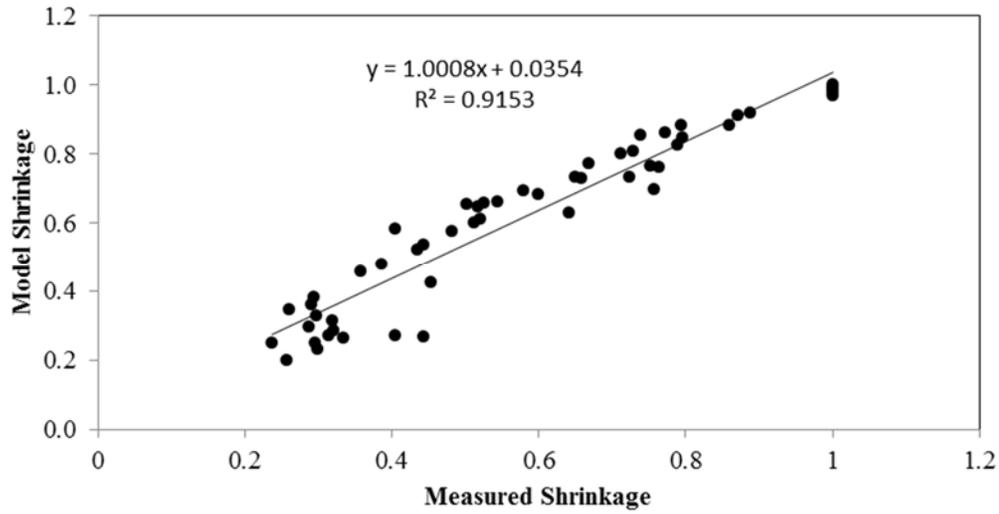


Figure 6. Comparison of model with experimental shrinkage values for all banana drying experiments

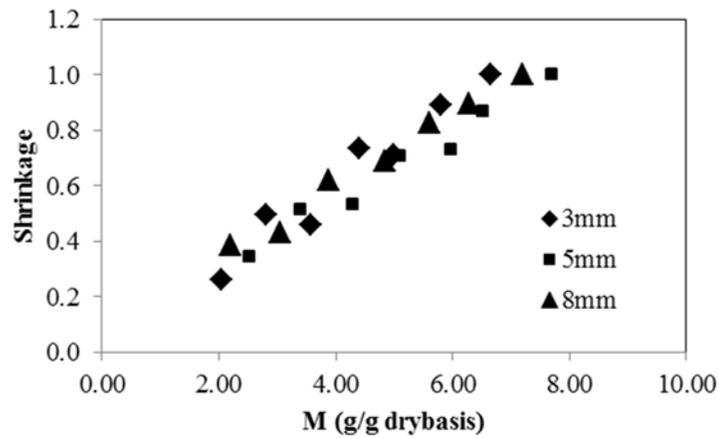


Figure 7. Shrinkage of carrot for three different thicknesses at drying condition 38°C and relative humidity 17%

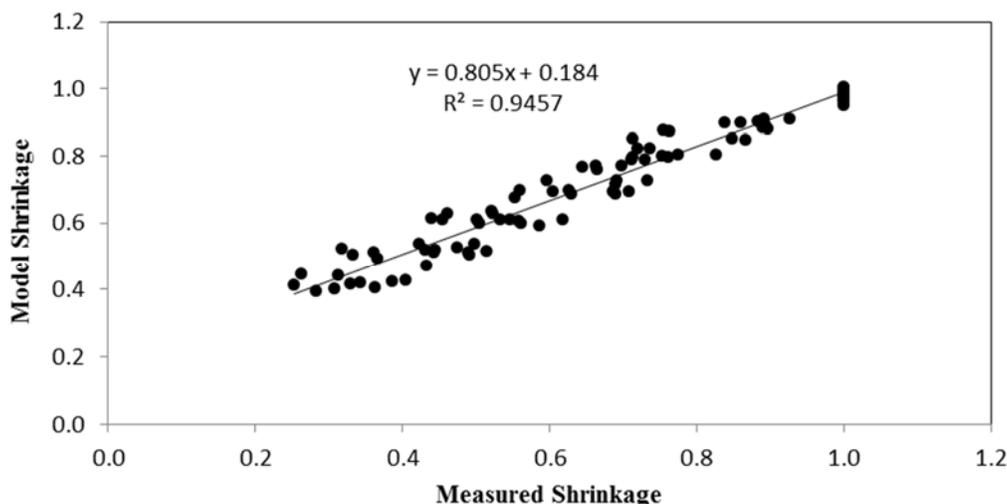


Figure 8. Comparison of model with experimental shrinkage values for all carrot drying experiments

Table 1 Comparison of models for banana drying experiments

Phenomena	Model	Coefficient of Determination, R ²	Mean square error
Shrinkage	Theoretical model	0.915	0.007
Drying	Two term model	0.895	0.076
	Two layer model	0.901	0.073
	Two layer model with Shrinkage	0.982	0.013

3.3 Model prediction of moisture content of banana and carrot

The drying data for all banana runs and all carrot runs were modelled separately by regression. The models tested in this study were, a) the two term model [1], b) the two layer diffusion model and c) the two layer diffusion model with shrinkage. The model performance was evaluated by comparing the coefficient of determination R² and mean square error [17, 22].

Since constant aeration conditions were used for all experiments, no substantial difference was observed between the two term model and the two layer model.

Figure 9 shows that all three models for banana slices were under predicting the moisture content in the initial stage of drying. After the initial transient effect, the two layer model with shrinkage gave the best fit with experimental values. A comparison of R² values between the models showed that the R² value for both products was highest for the two layer model with shrinkage, improving from 0.90 for the two layer model to 0.98 (Table 1). The mean square error also substantially decreased. This showed that the two layer model with shrinkage was a better predictor of the product drying behaviour than the other two models.

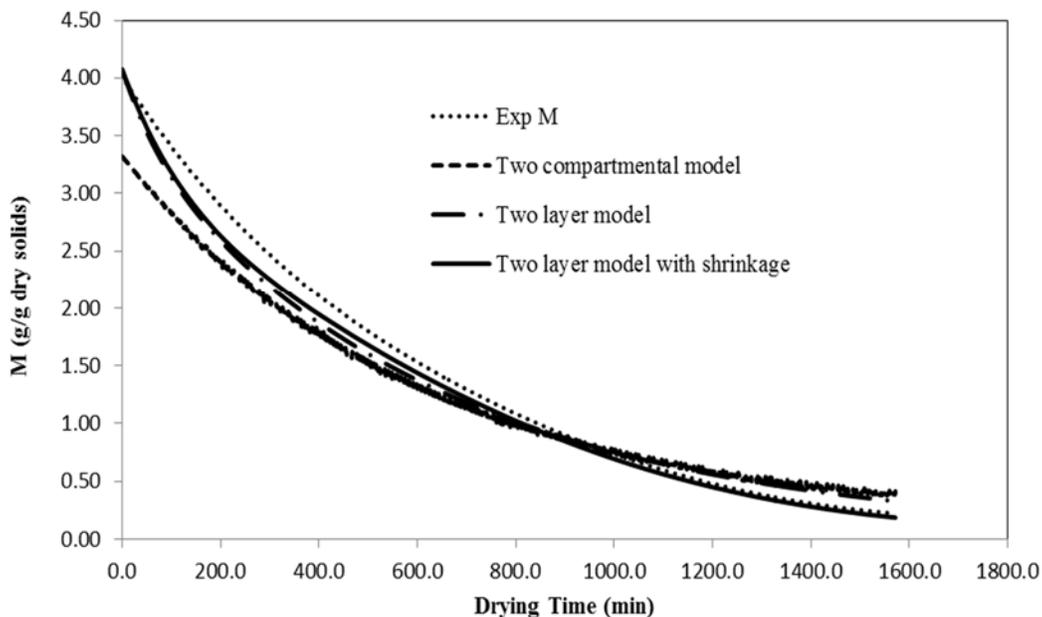


Figure 9. Example of comparison of three models with experimental moisture values for banana (at 28°C and 30 %RH)

Since none of the models are designed for initial transient region, the period during drying was not considered as a criterion for assessing the model (Figures 9 and 10). Figure 10 shows a comparison of the three models with experimental values for the carrot drying experiments. The two layer model was a good fit to the experimental data. The R^2 value increased from 0.78 to 0.89 due to inclusion of shrinkage in the two layer model (Table 2). The drying period for all banana experiments was longer than the carrot, and giving more complete drying curve, and this may have improved the model correlation.

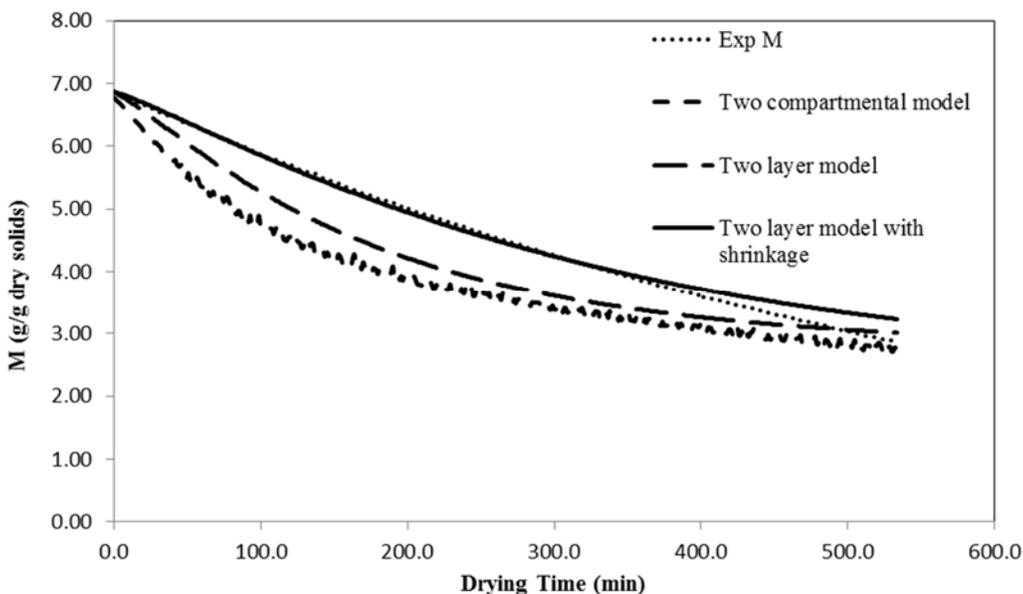


Figure 10. Experimental and model predicted value of moisture content during drying at 28°C and relative humidity with Carrot. (same corrections)

Table 2. Comparison of models for carrot drying experiments

Phenomena	Model	Coefficient of Determination, R ²	Mean square error
Shrinkage	Theoretical model	0.946	0.008
Drying	Two term model	0.728	0.698
	Two layer model	0.778	0.573
	Two layer model with Shrinkage	0.885	0.294

Inclusion of shrinkage for carrot reduced the error mean square from 0.5 to 0.2. The results clearly prove that shrinkage during drying has a significant effect on the drying rate and inclusion of this effect in the model improved the prediction of drying rates for both products.

4. Conclusions

The moisture reduction behaviour of both banana and carrot during hot air drying were observed in this study. The drying of both products took place mostly in the falling rate period. The shrinkage of both products was predicted well, by considering a theoretical relationship between product volume and moisture content during the process. The comparison of three different drying models showed that the model with shrinkage was a better predictor of the drying behavior of the two products than the other two models. This indicates that shrinkage of a product during drying affects the drying rate and should be considered in the drying model development.

Notation

V_{t_0}	the sum of volume of water and dry solids at initial condition
V_t	the sum of volume of water and dry solids at any time
V_0	the initial volume
V	the volume of dried sample at any time
V_a	the volume of air
ρ_{a_0}	apparent density of product at initial condition
ρ_w	density of water
m_{w_0}	initial mass of water
M_0	initial moisture content (db)
m_w	mass of water after drying and
M	final moisture content (db)
m_s	the dry mass of the sample
S	product shrinkage
MR	the moisture ratio
M_t	the moisture content at any time of the product
M_0	the initial moisture content
M_e	the equilibrium moisture content
A_0, A_1	the model constant for two term model
k_0, k_1	the drying constant for two term model
M_1	the moisture contents in the internal layer
M_2	the moisture contents in the surface layer
M_e	the equilibrium moisture content respectively.
k_1 and k_e	drying constants
μ	the dry mass ratio of the surface layer and second layer

References

- [1] Erbay, Z. and Icier, F., **2010**. A review of thin layer drying of foods: theory, modeling, and experimental results. *Critical Reviews in Food Science and Nutrition*, 50(5), 441-464.
- [2] Hatamipour, M. and Mowla, D., **2002**. Shrinkage of carrots during drying in an inert medium fluidized bed. *Journal of Food Engineering*, 55(3), 247-252.
- [3] Suarez, C. and Viollaz, P.E., **1991**. Shrinkage effect on drying behavior of potato slabs. *Journal of Food Engineering*, 13(2), 103-114.
- [4] Lozano, J., Rotstein, E. and Urbicain, M., **1980**. Total porosity and open-pore porosity in the drying of fruits. *Journal of Food Science*, 45(5), 1403-1407.
- [5] Ratti, C., **1994**. Shrinkage during drying of foodstuffs. *Journal of Food Engineering*, 23(1), 91-105.
- [6] Wang, N. and Brennan, J., **1995**. Changes in structure, density and porosity of potato during dehydration. *Journal of Food Engineering*, 24(1), 61-76.
- [7] Krokida, M.K. and Maroulis, Z.B., **1997**. Effect of drying method on shrinkage and porosity. *Drying Technology*, 15(10), 2441-2458.
- [8] Zogzas, N.P., Maroulis, Z.B. and Marinou-Kouris, D., **1994**. Densities, shrinkage and porosity of some vegetables during air drying. *Drying Technology*, 12(7), p. 1653-1666.
- [9] Lozano, J., Rotstein, E. and Urbicain, M., **1983**. Shrinkage, porosity and bulk density of foodstuffs at changing moisture contents. *Journal of Food Science*, 48(5), 1497-1502.
- [10] Shafiur Rahman, M. and Lal Potluri, P., **1990**. Shrinkage and density of squid flesh during air drying. *Journal of Food Engineering*, 12(2), 133-143.

- [11] Perez, M. and Calvelo, A., **1984**. Modeling the thermal conductivity of cooked meat. *Journal of Food Science*, 49(1), 152-156.
- [12] Palipane, K.B. and Driscoll, R.H., **1994**. The thin-layer drying characteristics of macadamia in-shell nuts and kernels. *Journal of Food Engineering*, 23(2), 129-144.
- [13] Phoungchandang, S. and Woods, J., **2000**. Moisture diffusion and desorption isotherms for banana. *Journal of Food Science*, 65(4), 651-657.
- [14] Kechaou, N. and Maalej, M., **1994**. Evaluation of diffusion coefficient in the case of banana drying. *Drying Technology*, 94, 841-848.
- [15] Madamba, P., Driscoll, R. and Buckle, K., **1994**. Shrinkage, density and porosity of garlic during drying. *Journal of Food Engineering*, 23(3), 309-319.
- [16] Zogzas, N., Maroulis, Z. and Marinos-Kouris, D., **1996**. Moisture diffusivity data compilation in foodstuffs. *Drying Technology*, 14(10), 2225-2253.
- [17] Phoungchandang, S. and Wongwatanyoo, J., **2010**. Desorption isotherms and drying characteristics of carrot using tray and heat pump-assisted dehumidified drying. *KKU Research Journal*, 15(3), 171-186.
- [18] Mayor, L. and Sereno, A., **2004**. Modelling shrinkage during convective drying of food materials: A review. *Journal of Food Engineering*, 61(3), 373-386.
- [19] Helrich, K., **1990**. Official methods of Analysis of the AOAC. Volume 2. Association of Official Analytical Chemists Inc.
- [20] Wu, X., Srzednicki, G. and Driscoll, R., **2012**. *Thin layer drying of ginger*. in *Southeast Asia Symposium on Quality Management in Postharvest Systems and Asia Pacific Symposium on Postharvest Quality* 989.
- [21] Sahari, Y. and Driscoll, R., **2013**. *Thin layer drying of agricultural products: A review*. in *Third Malaysian Postgraduate Conference (MPC) 2013*.
- [22] Noomhorm, A. and Verma, L.R., **1986**. Generalized single-layer rice drying models. *Transactions of the ASAE*, 29(2), 587-591.