Downscaling Geopotential Height Using Lapse Rate

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Abstract

In order to utilize the output from a global climate model, relevant information for the area of interest must be extracted. This is called "downscaling". In this paper, the derivation of a local geopotential height in terms of lapse rate is presented. The main assumptions are hydrostatic balance, perfect gas, constant gravity, and constant lapse rate. Two sets of data are required for this method, simulation outputs from the Education Global Climate Model (EdGCM) and the observed data at the points of interest (Chiangmai, Bangkok, Ubon Ratchathani, Phuket and Songkla). The results show that downscaling of the geopotential heights by using lapse rate dynamic equation are closer to the observed data than the geopotential heights from EdGCM.

Keywords: Downscaling, Geopotential height, Lapse rate

1. Introduction

Climate change data are simulation outputs from global climate models (GCMs). Outputs of GCMs are coarse resolution, so it must be downscaled for use in regional applications by downscaling [1]. The starting point for downscaling is a larger scale atmospheric or coupled oceanic atmospheric model run from a GCM [2]. There are two major kinds of downscaling, statistical and dynamical methods. Statistical downscaling methods use historical data and archived forecasts to produce downscaled information from large scale forecasts. Dynamical downscaling methods involve dynamical models of the atmosphere nested within the grids of the large scale forecast models [1]. The term "downscaling" refer to the use of either fine spatial scale numerical atmospheric model (dynamical downscaling), or statistical relationship (statistical downscaling) in order to achieve detailed regional and local atmospheric data [2]. This paper focuses on dynamical downscaling that used outputs data from the Education Global Climate Model (EdGCM), in which all processes of hydrostatic balance, perfect gas, constant gravity, and lapse rate are considered. A dynamic equation is derived to calculate geopotential height. Geopotential height is the height of pressure surface and it can be used to identity both speed and direction of wind [3]. Interpolation is an important method to obtain relevant data for the area of interest from known data points that cover the area [4]. The papers involved inverse distance weighted interpolation include [5-8].

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In this paper three different types of data are used for comparison. The first type is the simulation outputs from EdGCM**.** The second type is data from the dynamic equation. The third type is observed data (from meteorological stations) at the points of interest (Chiangmai, Bangkok, Ubon Ratchathani, Phuket, Songkla) as shown in Figure 1.

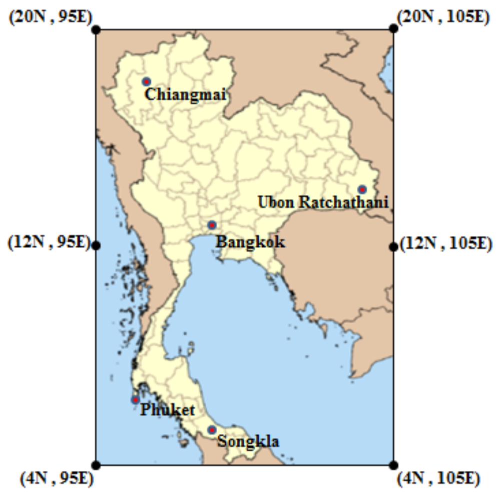

Figure 1 The points of interest, Chiangmai, Bangkok, Ubon Ratchathani, Phuket and Songkla, and the grid points of EdGCM that cover the points of interest.

The remainder of this paper is organized as follows. materials and methods are described in second section. In the third section, the results are analyzed and discussed. In the fourth section, the conclusion is presented.

2. Materials and Methods

2.1 EdGCM Software

The climate model used by the EdGCM software was developed at NASA's Goddard Institute for Space Studies (NASA/GISS). This 3 dimensional computer model is known as a grid-point GCM. A grid point GCM divides the atmosphere into a series of discrete grid cells. EdGCM's model has 7776 grid cells in the atmosphere, with each horizontal column corresponding to 8° latitude by 10° longitude and containing 9 vertical layers. The computer model numerically solves fundamental physical equations, which describe the conservation of mass, energy, momentum, and moisture in each cell, while taking into account the transport of quantities between cells [9].

2.2 Experiment Case

Locations of the points used in this research are shown in Figure 2.

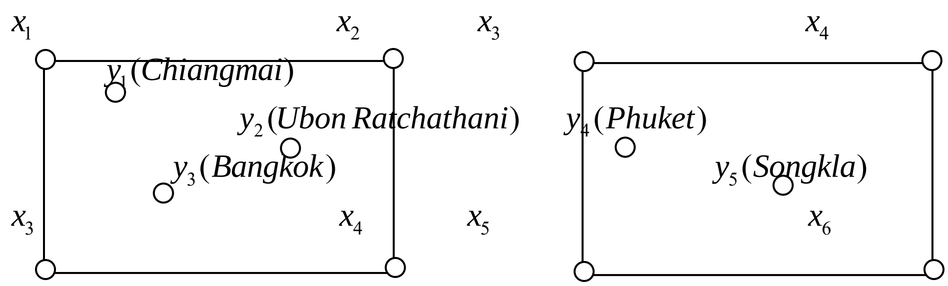

Figure 2 Locations of the data points from EdGCM and the points ofinterest (y_1 , y_2 , y_3 , y_4 and y_5)

Figure 2 shows the data positions x_1, x_2, x_3, x_4, x_5 and x_6 from EdGCM at the latitudes and longitudes, (20N,95E), (20N,105E), (12N,95E), (12N,105E), (4N,95E) and (4N,105E), respectively, which cover the points of interest y_1 , y_2 , y_3 , y_4 and y_5 at the latitudes and longitudes (18.47N,98.59E), (15.15N,104.53E), (13.44N,100.3E), (8.8N,95.19E) and (7.11N,100.37), respectively. Two sets of data are required for this method. The first set consists of surface pressure, temperature, topography and 850 hPa geopotential height of August monthly mean between 2000-2009 from EdGCM with the resolution of 8° latitude and 10° longitude. The second set consists of observed 850 hPa geopotential height of August monthly mean between 2000-2009 at the points of interest (Chiangmai, Bangkok, Ubon Ratchathani, Phuket, Songkla).

2.3 Experiment Design

The steps for the experiment are shown in Figure 3.

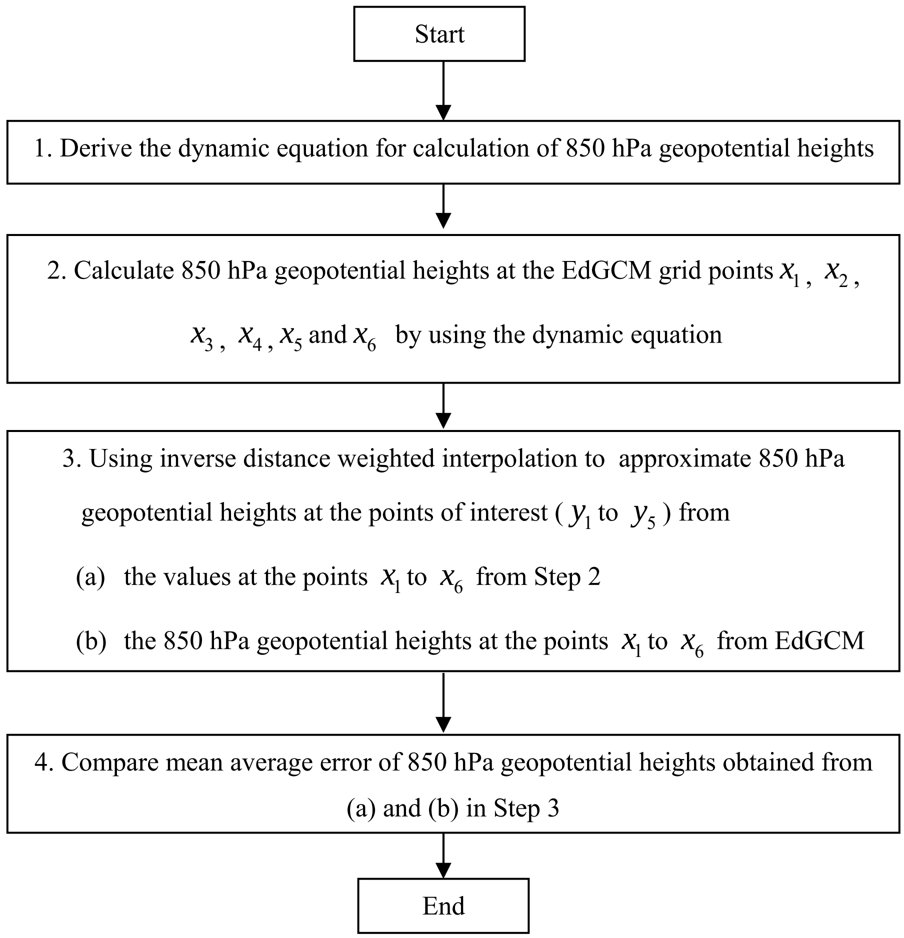

Figure 3 The sequence for estimation of the geopotential heights at the points of interest

The experiment design is started by deriving the dynamic equation using the main assumptions which are hydrostatic balance, perfect gas, constant gravity, and constant lapse rate. Next, calculate geopotential heights at the EdGCM grid points x_1 to x_6 using the dynamic equation obtained from the first step. The variables used as inputs in the dynamical equation consist of surface pressure, surface temperature, topography and 850 hPa geopotential height from outputs of the EdGCM. Next, apply the inverse distance weighted interpolation for the points of interest using the results from the dynamic equation and the outputs for EdGCM. Finally, compare mean average error of geopotential heights obtained from the dynamic equation and EdGCM outputs.

2.4 Dynamic Equation

The dynamic equation is derived from a fluid layer. The fluid motion is determined by the vertical momentum equation using the hydrostatic equation, perfect gas, constant gravity, and constant lapse rate. For fluid layer in Figure 4, η is the height of the free surface, η_b is the height of the topography, T is temperature, T_o is sea level temperature, T_s is surface air temperature, p is pressure, p_o is sea level pressure, p_s is surface pressure.

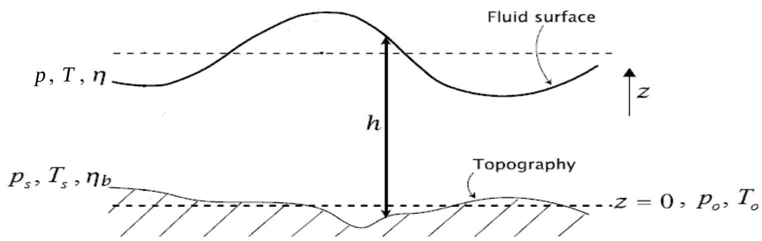

Figure 4 The dynamic equation system [10]

In the hydrostatic equation, the change in pressure with altitude must oppose the gravitational force on the air, that is

$$\frac{dp}{dz} = -\rho g \tag{1}$$

where p is pressure
z is altitude
ρ is air density
g is gravitational acceleration

The ideal gas law states that

$$p = \rho RT \tag{2}$$

where R is gas constant for dry air,
T is temperature

Combining Equations (1) and (2), ρ can be eliminated

$$\frac{dp}{dz} = -\frac{P}{RT}g \tag{3}$$

$$\frac{1}{p}dp = -\frac{g}{R}\cdot\frac{1}{T}dz$$

$$\int_{p_s}^{p}\frac{1}{p}dp = \int_{0}^{\eta_b}\frac{-g}{RT}dz$$

$$\ln p - \ln p_s = -\frac{g}{R}\int_{\eta_b}^{\eta}\frac{1}{T}dz \tag{4}$$

Consider $\int_{\eta_b}^{\eta}\frac{1}{T}dz$ in Eq. (4)

$$\int_{\eta_b}^{\eta} \frac{1}{T}dz = \int_{\eta_b}^{\eta} \frac{1}{T_o + \gamma z}dz$$
$$= \frac{1}{\gamma}\ln\frac{T_o + \gamma\eta}{T_o + \gamma\eta_b} \tag{5}$$

Substituting Eq. (4) by Eq. (5)

$$\ln\frac{p}{p_s} = -\frac{g}{R\gamma}\ln\frac{T_o + \gamma\eta}{T_o + \gamma\eta_b}$$

$$\frac{p}{p_s} = \left(\frac{T_o + \gamma\eta}{T_o + \gamma\eta_b}\right)^{-\frac{g}{R\gamma}}$$

$$\left(\frac{p}{p_s}\right)^{-\frac{R\gamma}{g}} = \frac{T_o + \gamma\eta}{T_o + \gamma\eta_b}$$

$$T_o + \gamma\eta = \left(T_o + \gamma\eta_b\right)\left(\frac{p}{p_s}\right)^{-\frac{R\gamma}{g}}$$

The dynamic equation for geopotential height is

$$\eta = \frac{1}{\gamma}\left[\left(T_o + \gamma\eta_b\right)\left(\frac{p}{p_s}\right)^{-\frac{R\gamma}{g}} - T_o\right] \tag{6}$$

where

$$\gamma = -\frac{dT}{dZ}\ ,\ T_o = \frac{\eta_b \cdot T - z \cdot T_s}{\eta_b - z}\ ,\ p_s = \rho_s R T_s\ ,\ \rho_s = \frac{P_o}{RT_s}\left(1 + \frac{\gamma\eta_b}{T_o}\right)^{\frac{-g_o}{R\gamma}}$$

γ is lapse rate

η is 850 hPa geopotential height

η_b is topography

z is altitude

g is gravitational acceleration

R is gas constant

T is temperature

T_o is sea level temperature

T_s is surface air temperature

p is pressure

p_o is sea level pressure

p_s is surface pressure

2.5 Inverse Distance Weighted Interpolation

An example of 850 hPa geopotential height approximation at the point y_1 in Figure 2 is shown in Figure 5.

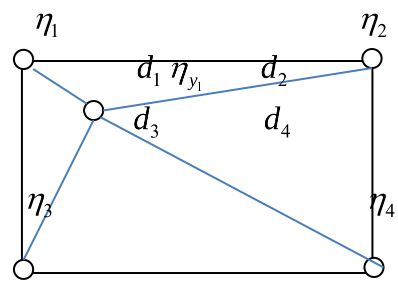

Figure 5 η_1, η_2, η_3 and η_4 are 850 hPa geopotential heights at the points x_1, x_2, x_3 and x_4 in Figure 2 that are used to approximate geopotential heights η_{y_1} at the point of interest y_1 in Figure 2.

Inverse distance weighted is a method for interpolation, a process of assigning values to interest points by using values from a set of known points. The importance of observations is represented by the non-negative numerical coefficient and the sum of weitght is equal to one. In this research, weighting is determined from distance between the EdGCM grid points and the points of interest which is covered by the EdGCM grid points. That is, the inverse distance weighted interpolation is defined as

$$\eta_{y_1} = \frac{\sum_{i=1}^{4} \eta_i / d_i}{\sum_{i=1}^{4} 1 / d_i}$$

where

η_{y_1} is the geopotential height at the point of interest y_1.

d_i is distance between the point of interest and the EdGCM grid point x_i.

$$d_i = \sqrt{\left(lat_{y_1} - lat_{x_i}\right)^2 + \left(lon_{y_1} - lon_{x_i}\right)^2}$$

η_i is the 850 hPa geopotential height at the EdGCM grid point x_i.

3. Results and Discussion

Table 1 shows distance and weight number of the points of interest. The minimal distance has the maximum weight because the nearest observed point has the most influence.

Table 1 Distance (km) and weights for Chiangmai, Bangkok, Ubon Ratchathani, Phuket, Songkla.

Chiangmai		Bangkok		Ubon Ratchathani		Phuket		Songkla	
Distance	Weight	Distance	Weight	Distance	Weight	Distance	Weight	Distance	Weight
659.007	0.232	806.992	0.197	487.272	0.288	1036.627	0.136	673.417	0.237
390.244	0.392	843.348	0.189	1069.315	0.131	320.306	0.441	726.285	0.220
739.926	0.207	549.214	0.290	1003.710	0.140	480.204	0.294	620.556	0.257
910.763	0.168	491.565	0.324	318.487	0.441	1096.629	0.129	557.754	0.286

Tables 2-6 compare the errors of geopotential heights at 850 hPa obtained from the dynamic equation and EdGCM for Chiangmai, Bangkok, Ubon Ratchathani, Phuket and Songkla. At Chiangmai, Bangkok and Ubon Ratchathani the errors by the dynamic equation are much less than the errors by EdGCM. For Phuket and Songkla, the errors by the dynamic equation are only slightly less than the errors by EdGCM. This may be because Phuket and Songkla are surrounded by the sea which can results in difference lapse rates from Chiangmai, Bangkok and Ubon Ratchathani.

Table 2 Comparison of geopotential height (m) at 850 hPa by the dynamic equation and EdGCM for Chiangmai.

Chiangmai	Dynamic Equation	EdGCM	Observed	Dynamic Eq. error	EdGCM error
2000	1434	1377	1463	29	86
2001	1431	1383	1460	29	77
2002	1435	1380	1468	33	88
2003	1444	1387	1468	24	81
2004	1438	1376	1462	24	86
2006	1435	1384	1462	27	78
2007	1438	1376	1464	26	88
2009	1441	1385	1483	42	98

Table 3 Comparison of geopotential height (m) at 850 hPa by the dynamic equation and EdGCM for Bangkok.

Bangkok	Dynamic Equation	EdGCM	Observed	Dynamic Eq. error	EdGCM error
2000	1441	1420	1486	45	66
2001	1442	1419	1487	45	68
2002	1447	1421	1502	55	81
2003	1455	1430	1498	43	68
2004	1451	1423	1494	43	71
2006	1449	1423	1477	28	54
2007	1448	1423	1480	32	57
2008	1446	1426	1490	44	64
2009	1441	1420	1490	49	70

Table 4 Comparison of geopotential height (m) at 850 hPa by the dynamic equation andEdGCM for Ubon Ratchthani.

Ubon Ratchathani	Dynamic Equation	EdGCM	Observed	Dynamic Eq. error	EdGCM error
2000	1438	1414	1476	38	62
2001	1438	1416	1475	37	59
2002	1445	1416	1487	42	71
2003	1452	1428	1487	35	59
2004	1449	1422	1482	33	60
2006	1446	1421	1474	28	53
2007	1445	1418	1517	72	99
2009	1443	1417	1472	29	55

Table 5 Comparison of geopotential height (m) at 850 hPa by the dynamic equation and EdGCM for Phuket.

Phuket	Dynamic Equation	EdGCM	Observed	Dynamic Eq. error	EdGCM error
2001	1434	1432	1516	82	84
2002	1437	1435	1512	75	77
2003	1447	1445	1505	58	60

Table 6 Comparison of geopotential height (m) at 850 hPa by the dynamic equation andEdGCM of Songkla.

Songkla	Dynamic Equation	EdGCM	Observed	Dynamic Eq. error	EdGCM error
2000	1439	1437	1496	57	59
2001	1439	1437	1494	55	57
2002	1444	1443	1499	55	56
2004	1448	1446	1504	56	58
2006	1445	1444	1499	54	55
2007	1445	1443	1508	63	65
2009	1444	1442	1487	43	45

4. Conclusions

This research is aimed to obtain downscaled geopotential heights by using a dynamic equation for Chiangmai, Bangkok, Ubon Ratchathani, Phuket and Songkla. The dynamic equation is derived from the main assumptions of hydrostatic balance, perfect gas, constant gravity, and lapse rate. The inverse distance weighted interpolation is used for interpolation from the EdGCM grid points to the points of interest. The mean absolute error of geopotential heights from the dynamical equation approximation for all points of interest are less than that of EdGCM. However, for Phuket and Songkla there are less differences between the dynamic equation and EdGCM values. This could be because these two locations are surrounded by the sea which result is difference lapse rates from other locations which are in land.

5. Acknowledgements

The first author would like to thanks the "Earth System Science" of King Mongkut's University of Technology Thonburi for the financial support and the Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi for computer facilities.

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