The Simulation of Effect of Fin on Natural Convection in Porous Non-uniformly Heated Triangular Cavities

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Abstract

The effect of fin and without fin on natural convection flowing through a fluid-saturated porous medium in a triangular cavity with non-uniformly temperature on the bottom wall was simulated. The governing equations are written using Darcy model. All simulations are computed using FlexPDE Student Version 6.19. Temperatures of both inclined walls are zero while the bottom wall is non-uniform and fin is solid adiabatic. The study is performed for different Darcy-modified Rayleigh number from 100 to 1000, the position of fin from 0.4 to 0.6 and the height of fin from 0.15 to 0.3. The results are presented in terms of streamlines, isotherms and heatlines. It is observed that the fin can be used as a passive control element for flow field, temperature distribution and heat transfer.

Keywords: Thin fin, Natural convection, Non-uniform temperature, Porous medium

1. Introduction

Natural convection heat transfer in fluid-saturated porous medium is an important problem in engineering. Applications of porous media can be found in geophysic problems, solar collectors, heat exchangers, grain storage, nuclear reactor and so on. Detailed reviews of subject of porous media can be found in the recent books of Nield and Bejan [1], and Ingham and Pop [2].

The studies on natural convection in different shaped cavities filled with porous media under different temperature are available in literature. Based on knowledge for the application of engineering in flow field, temperature distribution and heat transfer can be used to design high efficient thermal systems. In the past, most researchers focused on investigation of natural convection in cavity, which can be classified into three groups: (a) triangular cavities [3-8], (b) rectangular/square cavities [9-11] and (c) the other cavities [12-14]. Nevertheless, studies on control of natural convection are very limited for cavities filled with porous media.

Varol and Oztop [3] studied the effect of thin fin on natural convection in porous triangular enclosure, where the vertical wall of the enclosure was insulated while the bottom and the inclined walls were isothermal. Later in that, they also investigated the effects of fin location onto the bottom wall of triangular enclosure filled with porous media whose height base ratio was 1 [4]. The parameters involved in the investigation are the location, height and width of fin. Mobedi, Varol, Oztop and Pop [5] performed visualization of natural convection heat transport using heatline method in porous non-isothermally heated from the bottom wall of triangular cavity,

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which was analyzed using finite difference technique. Basak *et al.* [6-7] studied simulation of natural convection flow in isosceles triangular enclosure filled with porous medium. Note that on both reviews, both inclined walls were not similar. Khansila and Witayangurn [8] studied numerical modeling of natural convection for steady flows in porous media heated from the bottom wall of triangular cavity, which was computed using FlexPDE (Student Version). Baytas and Pop [9] analyzed free convection in square porous cavity using a thermal nonequilibruim model. Bilgen [10] investigated natural convection in cavities with a thin fin while the bottom wall of the cavity was heated. Varol *et al.* [11] studied natural convection in diagonally divided square cavity filled porous media, where a finite difference scheme was used to solve the dimensionless governing partial differential equations along with the corresponding boundary conditions. Pop *et al.* [12] investigated numerical analysis of natural convection in trapezoidal enclosure heated and cooled from inclined walls. Dalal and Das [13] and Sompong and Witayangurn [14] investigated heatline method for the visualization of natural convection in a complicated cavity.

In this study, an adiabatic thin solid fin will be considered as a control parameter for flow field, temperature distribution and heat transfer in a triangular cavity filled with porous medium. Two cases will be considered, the effects of fin and no fin on natural convection in porous non-uniformly heated triangular cavity. In case of the cavity without fin, different Darcy-modified Rayleigh number will be studied, while the height of fin and the width of fin at $Ra = 1000$, will be studied in the case of the cavity with fin when the location of fin.

2. Definition of the Physical Model

The physical model of the two-dimensional triangular cavity filled with porous medium and a solid adiabatic fin attached at the bottom wall is shown in Fig. 1. In this study, the bottom wall is heated by non-uniformly temperature while the boundary conditions of the inclined walls are both zero. The temperature of the bottom wall is given by

$$T(x) = \frac{1}{2}\left(T_C + (T_H - T_C)\sin\left(\frac{\pi x}{L}\right)\right). \tag{1}$$

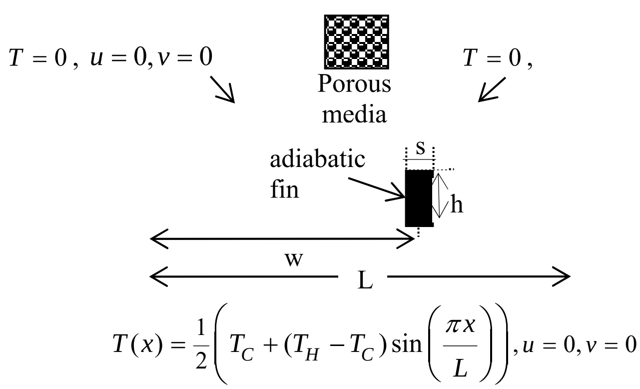

Figure 1 Physical model, coordinates and boundary conditions

3. Governing Equations and Boudary Conditions

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2}$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=-\frac{g\beta K}{\upsilon}\frac{\partial T}{\partial x}, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

The equation (2) is written in terms of the stream function defined by

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}. \tag{5}$$

Using the following change of parameters:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad \theta=\frac{T-T_C}{T_H-T_C},\quad \Psi=\frac{\psi}{\alpha},\quad Ra=\frac{g\beta K(T_H-T_C)L}{\upsilon\alpha}. \tag{6}$$

The governing equations (2)-(4) can be reduce to following non-dimensional forms:

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-Ra\frac{\partial\theta}{\partial X}, \tag{7}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}. \tag{8}$$

Heat function for a dimensional convection problem can be defined as

$$-\frac{\partial h}{\partial x}=\rho c_p v(T-T_C)-k\frac{\partial T}{\partial y}, \tag{9}$$

$$\frac{\partial h}{\partial y}=\rho c_p u(T-T_C)-k\frac{\partial T}{\partial x}. \tag{10}$$

By relations (6), equations (9) and (10) can be written in non-dimensional form as

$$-\frac{\partial\Phi}{\partial X}=V\theta-\frac{\partial\theta}{\partial Y}, \tag{11}$$

$$\frac{\partial\Phi}{\partial Y}=U\theta-\frac{\partial\theta}{\partial X}. \tag{12}$$

Where H is the non-dimensional heat function and given by

$$\Phi=\frac{h}{(T_H-T_C)k}. \tag{13}$$

Assuming that H is a continuous function to its second order derivatives, it yields the following differential equation for heat function:

$$\frac{\partial^2\Phi}{\partial X^2}+\frac{\partial^2\Phi}{\partial Y^2}=\frac{\partial(U\theta)}{\partial Y}-\frac{\partial(V\theta)}{\partial X}. \tag{14}$$

3.1 Boundary conditions

The following boundary conditions of the system are also shown in Figure 1. Note that velocities of u and v are both zero for all solid boundaries.

On the bottom wall:

$$0 < X < 1,\ \theta = \frac{\sin(\pi X)}{2}. \tag{15}$$

On the inclined wall:

$$0 \le X \le 0.5,\ Y = X,\ \theta = 0 \text{ and}$$
$$0.5 \le X \le 1,\ Y = 1 - X,\ \theta = 0. \tag{16}$$

On the fin:

$$\frac{\partial \theta}{\partial n} = 0. \tag{17}$$

4. Results and Discussion

A numerical study has been performed to investigate the natural convection in triangular cavity filled with porous media and effects of thin fin on cavity while the bottom of triangular cavity is heated. A parametric study has been carried out to determine the influence of Darcy-modified Rayliegh number $\left(100 \le Ra \le 1000\right)$ and portion of thin fin. The results of flow fields, temperature distributions and heat transfer are shown in the terms of streamlines, isotherms and heatlines, respectively.

4.1 Results of natural convection in triangular cavity filled with porous media without fin

Figure 2 illustrates the streamlines (the first and second rows), isotherms (the third and fourth rows) and heatlines (the fifth and sixth rows) at different modified Rayliegh numbers. The Figure shows the maximum and minimum values of streamlines, isotherms and heatlines.

For streamlines, it can be seen that double circulation cells are formed in different rotating directions: the cell of left half rotates in counter clockwise circulation whereas the cell of right half rotates in clockwise circulation. In addition, it can be seen that streamlines are symmetric at $\Psi = 0$. Values of streamlines increase as modified Rayleigh number increases and the magnitudes of cells expand close to the height of cavity. The contours of isotherms are smooth and monotonic. The contour near the inclined portion of cavity is cooler than the contour near the bottom wall. As Ra increases, it is observed that isotherms are more compressed towards the center of the bottom wall and top portion of the side walls. For $Ra = 100$, contours of heatlines are smooth cruves and the contour near the bottom wall is hotter than the contour near the inclined wall. In addition, the contours of heatline are double cells, where the right half cell is smaller than the left one and the forms of rotating direction are similar to those of streamlines.

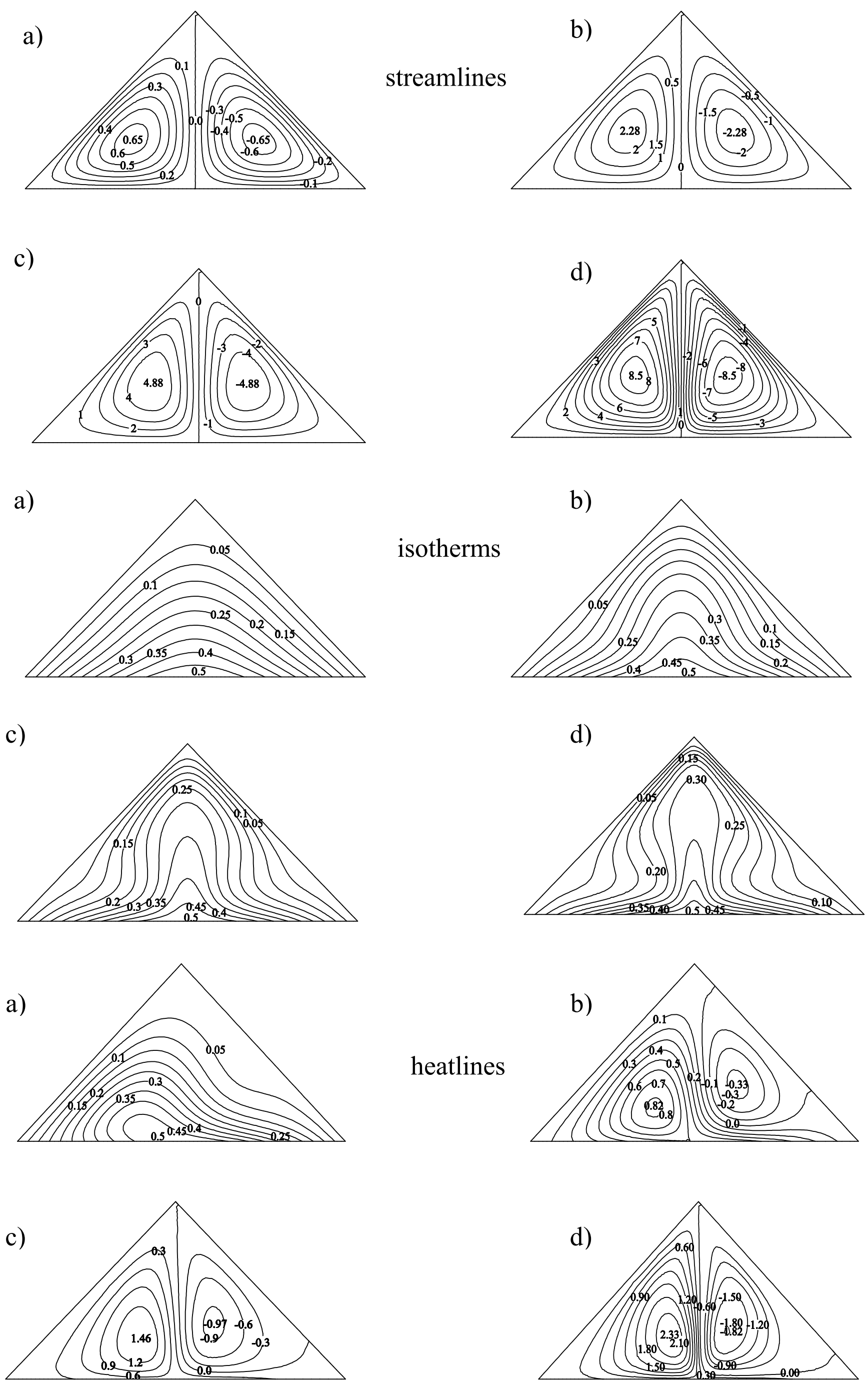

Figure 2 Streamlines, isotherms and heatlines for different Rayleigh number heated triangular cavity without fin (a) $Ra = 100$, b) $Ra = 250$, c) $Ra = 500$ and d) $Ra = 1000$).

4.2 Results of natural convection in triangular cavity filled with porous media within fin

Figure 3 illustrates the effect of streamlines with fin at different position, height and width of fin. In the first column, the fin is located on the left-hand side and the cell in the left-hand side is squeezed by fin. It is observed that the cell in the left-hand side is smaller than the cell in the right-hand side when fin is elevated, it can be seen that streamlines have three cells, where the cell in the left-hand side is divided into two cells and the value of streamline decreases. For the fin located at the center, streamlines are symmetric. The cells are compressed and become smaller when fin is higher and elevated. For fin located the right-hand side (the third column), the cell on the right-hand side is squeezed by fin becomes smaller. We can see that the cell on left-hand side is more than the cell on the right-hand side when the fin is higher, the cell on the right-hand side is divided into two cells and the cell on the left-hand side is squeezed by cell in upper half. Values of streamlines decrease as fin is higher. The value of cavity without fin is higher than the value of cavity with fin. Moreover, the cell on the left-hand side rotates in counter-clockwise circulation, while the cell on the right-hand side rotates in clockwise circulation. As the cell is divided, it rotates in the same direction.

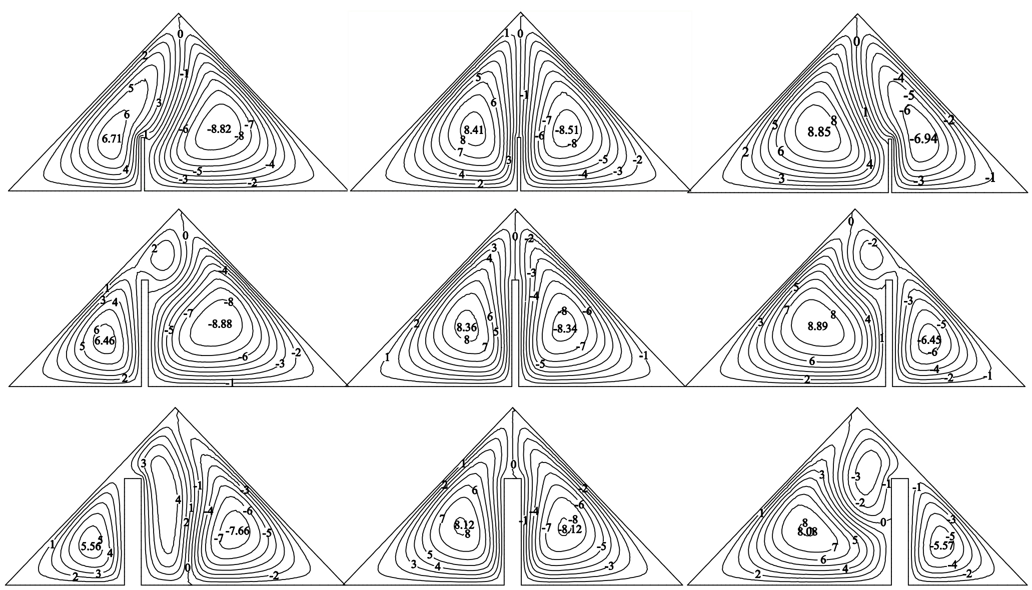

Figure 3 Streamlines in the cavity for different fins at $Ra = 1000$ (the first row: $h = 0.15$, $s = 0.01$, the second row: $h = 0.3$, $s = 0.02$, the third row: $h = 0.3$, $s = 0.05$, the first column: $w = 0.4$, the second column: $w = 0.5$, the third column: $w = 0.6$).

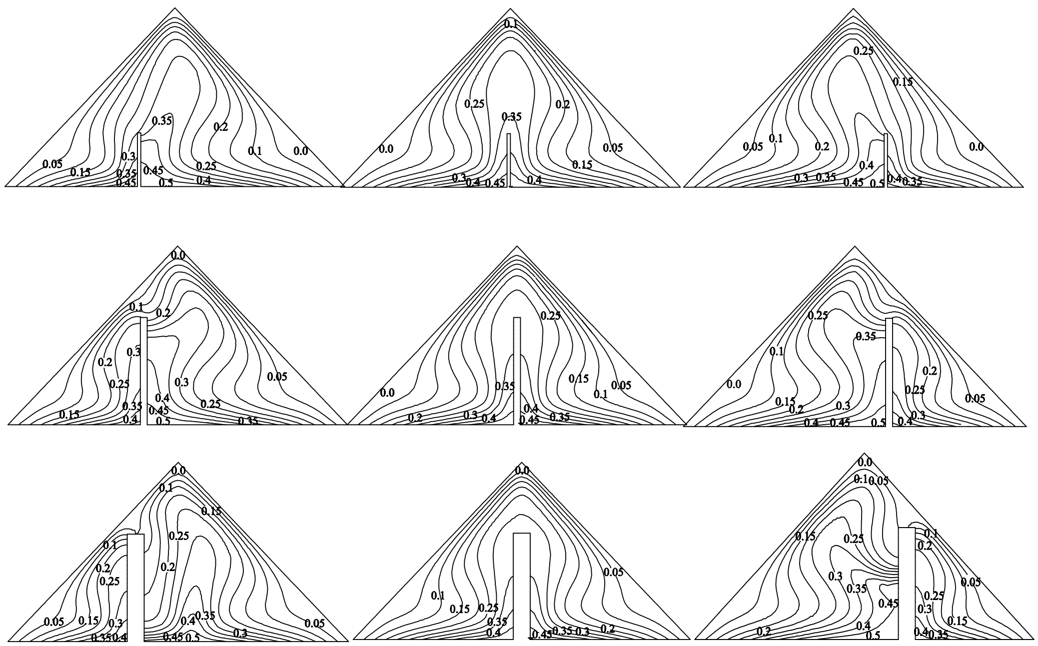

Figure 4 Isotherms in the cavity for different fins at $Ra = 1000$ (the first row: $h = 0.15$, $s = 0.01$, the second row: $h = 0.3$, $s = 0.02$, the third row: $h = 0.3$, $s = 0.05$, the first column: $w = 0.4$, the second column: $w = 0.5$, the third column: $w = 0.6$).

Figure 4 illustrates the effect of isotherms within fin at $Ra = 1000$ for different position, height and width of fin. The sides of fin are insulated in the locations where fin is attached to hot wall. When the fin is located on the right-hand side (the first column), the center of contours are shifted to the left by fin and are compressed to slope gradient along on the right-hand side of the inclined wall. When the fin is positioned at in the center, it can be seen that the contours of isotherms are pushed towards the central portion of the bottom wall. The isotherms are symmetric while the fin is higher and wider. When the fin is positioned on the right-hand side (the third column), the center of contours are shifted to the right by fin and are compressd to slope gradient along the left-hand side of the inclined wall. Moreover, the contours near bottom walls are hotter than the contours near inclined walls. The values of maximum and minimum are similar to the cavity without fin. In any cases, the appearance of isotherms increases from 0 to 0.5.

Figure 5 illustrates the effect of heatlines with fin at $Ra = 1000$ for different position, height and width of fin. This time, the location of fin on the left-hand side, it can be seen that the heatlines have two or three cells. Two cells rotate in different directions : the cell on the right-hand side rotates in clockwise direction and the cell on the left-hand side rotates in counter-clockwise direction. In case of three cells, we can see that cell is divided into direction as well. Consequently, two of the three cells rotate in the same direction. In case of fin being made higher, it is observed that the value of heatlines increases. As fin is wider, the value of heatlines decreases. For the fin located at the center, we can see that heatlines have double cells, where the contours of cell on the left-hand side are more than the contours of cell on the right-hand side and rotate in different directions. In case of fin becoming higher or wider, the value of maximum decreases. The location of fin on the right-hand side is similar to the location of fin on the left. The forms of heatlines are similar to streamlines but the contours decrease.

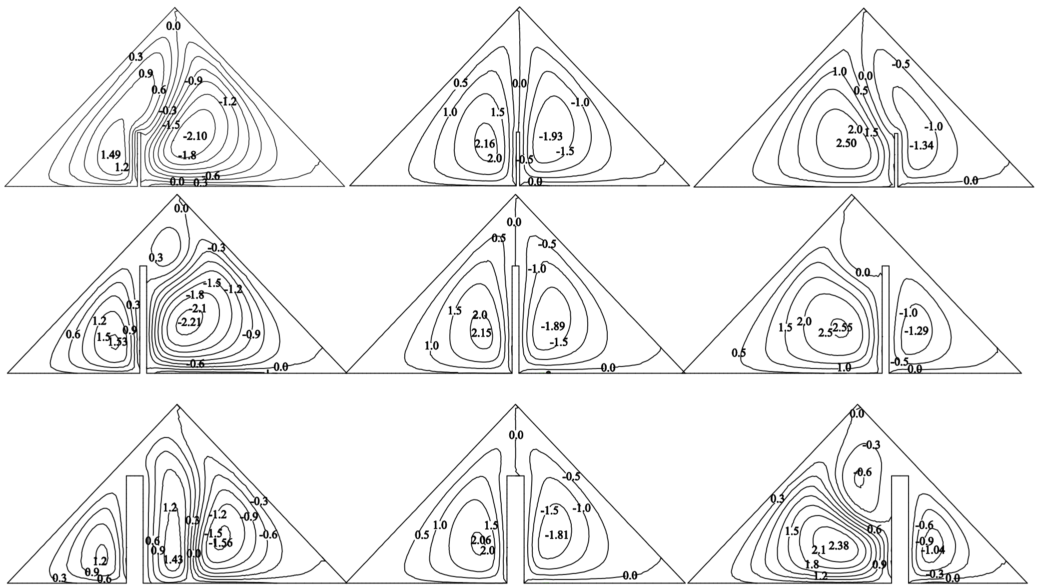

Figure 5 Heatlines in the cavity for different fins at $Ra = 1000$ (the first row: $h = 0.15$, $s = 0.01$, the second row: $h = 0.3$, $s = 0.02$, the third row: $h = 0.3$, $s = 0.05$, the first column: $w = 0.4$, the second column: $w = 0.5$, the third column: $w = 0.6$).

5. Conclusions

We study the effect of natural convection in triangular cavity filled with porous media by non-uniformly temperature in two cases, namely the cavity without fin and with fin. The location of the fin ranges from 0.4 to 0.6, the height of the fin ranges from 0.15 to 0.3 and Darcy-modified Rayleigh number ranges from 100 to 1000. Some important observations from the results are concluded as follows:

(1) In all cases, the values of streamlines and heatlines increase as Rayleigh number increases.

(2) The value of maximum for all simulation of triangular cavity without fin is smaller than the triangular cavity with fin.

(3) The triangular cavities without fin are smaller than the one of triangular cavity with fin.

(4) For all streamlines, it is observed that double cells are rotated in different direction, where the cell on the left-hand side rotates in clockwise circulation and the cell on the right-hand side rotates in counter-clockwise circulation.

(5) The direction of heatlines is similar to the direction of streamlines.

(6) For isotherms, the contours are smooth and pushed from the bottom wall towards

the inclined walls.

(7) When the fin is made higher, the maximum value of streamlines decreases while the maximum value of heatlines increases.

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References

[1] Nield, D.A. and Bejan, A. **2006**. *Convection in Porous Media*. Second ed., New York, Springer.

[2] Ingham, D.B. and Pop, I. **2005**. *Transport Phenomena in Porous Media*. Oxford, Pergamon.

[3] Varol, Y., Oztop, H.F. and Varol, A. **2007**. Effects of thin fin on natural convection in porous triangular enclosure, *International Journal of Thermal Sciences,* 46, 1033-1045.

[4] Varol, Y., Oztop, H.F. and Varol, A. **2007**. Natural convection in porous triangular enclosures with a solid adiabatic fin attached to the horizontal wall, *International Communications in Heat and Mass Transfer,* 34, 19-27.

[5] Varol, Y., Oztop, H.F., Mobedi, M. and Pop, I. **2008**. Visualization of natural convection heat transport using heatline method in porous non-isothermally heated triangular cavity, *International Journal of Heat and Mass Transfer,* 51, 5040-5051.

[6] Basak, T., Roy, S., Babu, S.K. and Balakrishnan, A.R. **2008**. Finite element analysis of natural convection flow in a isosceles triangular enclosure due to uniform and non-uniform heating at the side walls, *International Journal of Heat and Mass Transfer,* 51, 4496-4505.

[7] Basak, T., Roy, S. and Babu, S.K. **2009**. Natural convection and flow simulation in differentially heated isosceles triangular enclosures filled with porous medium, *Chemical Engineering Science,* 48, 1405-1415.

[8] Khansila, P. and Witayangurn, S. **2011**. Numerical modeling of natural convection for steady flows in porous media heated triangular cavity. *Proceedings 16th Annual Meeting in Mathematics*, Khon Kaen, Thailand, pp.183-194.

[9] Baytas, A.C. and Pop, I. **2002**. Free convection in a square porous cavity using a thermal nonequilibruim model, *International Journal of Thermal Sciences,* 41, 861-870.

[10] Bilgen, E. **2005**. Natural convection in cavities with a thin fin on the hot wall, *International Journal of Heat and Mass Transfer,* 48, 3493-3505.

[11] Varol, Y., Oztop, H.F. and Pop, I. **2009**. Natural convection in a diagonally divided square cavity filled with a porous medium, *International Journal of Thermal Sciences,* 48, 1405-1415.

[12] Varol, Y., Oztop, H.F. and Pop, I. **2008**. Numerical analysis of natural convection in an inclined trapezoidal enclosure filled with a porous medium, *International Journal of Thermal Sciences,* 47, 1316-1331.

[13] Dalal, A. and Das, M.K. **2008**. Heatline method for the visualization of natural convection in a complicated cavity, *International Journal of Heat and Mass Transfer,* 51, 263-272.

[14] Sompong, P and Witayangurn, S. **2012**. Simulation of natural convection in a complication enclosure with two wavy vertical walls, *Applied Mathematical Sciences,* 6, 2833-2842.

Notation

AR	cavity aspect ratio ($AR = H / L$)	x, y	dimensional coordinates (m)
c_p	heat capacity ($Jkg^{-1}K^{-1}$)	X, Y	dimensionless coordinates
g	acceleration due to gravity (ms^{-2})		*Greek symbols*
h	dimensional heat function, height of fin (m)	α	thermal diffusivity (m^2s^{-1})
k	thermal conductivity ($Wm^{-1}K^{-1}$)	β	thermal expansion coefficient (K^{-1})
K	permeability of the porous medium (m^2)	υ	kinematic viscosity (ms^{-1})
L	cavity width (m)	θ	dimensionless temperature
Ra	Darcy-modified Rayleigh number	ρ	fluid density (kgm^{-3})
T	temperature (K)	ψ	dimensional stream function (m^2s^{-1})
T_H	temperature of hot wall	Ψ	dimensionless stream function
T_C	temperature of cool wall	Φ	dimensionless heat function
s	width of fin (m)		*Subscripts*
u, v	x and y component of fluid velocities (ms^{-1})	C	cold
U, V	dimensionless fluid velocities	H	hot
w	dimensionless fin position		