Visualization of Conjugate Natural Convection in a Square Enclosure Divided by Conducting Partition

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Abstract

Simulation of conjugate natural convection in a square enclosure with a partition placed at the middle of the enclosure has been performed on FlexPDE 6.19 Student Version. The partition divides the enclosure into two regions and each region is filled with fluid-saturated porous media. The purpose of this study is to investigate the flow field and temperature distribution regarding the change of governing parameters, Rayleigh number (Ra), thermal conductivity ratio (k) and partition thickness (D). It is found that the intensity of circulation increases as Ra or k increases, simultaneously, the increase of k results in the lower temperature distribution along the partition. Moreover, partition thickness reduces the temperature distribution along the right-hand side of partition when the convection is obstructed.

Keywords: Conjugate natural convection, porous media, finite element method, partitioned enclosure

1. Introduction

The study of natural convection in porous media has received widely attention and can be found in many engineering applications such as geophysics, insulation of building, separation processes in industries, spreading of pollutant, etc. Many researchers have used several different configurations (for instance, square, rectangle, triangle and cylindrical) to model the natural convection system; see examples [1-4]. The natural convection also plays a role in daily life. Triangular enclosure can be used in the application of roof structure to study heat transfer under summer and winter conditions; see Asan and Namli [5-6], Hakan *et al.* [7] and Sompong and Witayangkurn [8]. In a complicated cavity, Amaresh *et al*. [9-11] studied the natural convection in a square enclosure with three flat walls and a right wavy vertical wall. To increase the complication, a square enclosure with two wavy vertical walls was studied by Sompong and Witayangkurn [12].

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In order to enhance the strength of the enclosure, a partition with finite length and finite conductivity is located inside. In addition, the partition can control heat transfer especially in electronic devices and building materials. In case where heat conduction in the partition is coupled with convection heat transfer in the fluid, it is called conjugate heat transfer. The applications of this scheme are solar collector, double pane windows, furnace design, cooling of electronic devices, and so on; see [13-15] for more applications. The conjugate natural convection in porous media has been studied using different models. Nawaf [16-17] analyzed conjugate natural convection in two-dimensional square enclosure with one and two vertical thick walls such that the horizontal walls were insulated. The Darcy model is used in mathematical formulation and it is solved by finite volume method. The interesting parameters are Rayleigh number, wall thickness and thermal conductivity ratio. Yasin *et al*. [18] also used Darcy model in mathematical formulation to study the effects of the same governing parameters on heat transfer in a triangular enclosure with thick bottom wall. They also performed investigation in a cavity saturated with cold water [19]. The cavity is heated from the thick bottom wall and cooled from ceiling. The governing equations were solved by finite-difference method. Conjugate mixed convection heat transfer for lid-driven enclosure was studied by Hakan *et al*. [20]. The physical model is a square enclosure with a driven lid and a solid partition is located at the center. Governing parameters are Richardson number and thermal conductivity ratio. Kamil [21] presented the numerical simulation of natural convection in a partitioned enclosure using polynomial-based differential quadrature (PDQ) method. Hakan *et al*. [22] investigated natural convection heat transfer in the partitioned enclosure in which the partition was used to separate the fluid and the different fluids were contained in both sides. Numerical results were carried out by using finite-difference technique.

The objective of this study is to investigate the fluid flow and temperature distribution due to the change of governing parameters on conjugate natural convection in the partitioned enclosure. The partition is located at the middle of the enclosure and values of thermal conductivity of partition are chosen covering wide range of application. The cavity is heated from the left wall and cooled from the right wall while the horizontal walls are adiabatic. Darcy model is used in mathematical formulation and FlexPDE is used to simulate the system mentioned above to obtain numerical results.

2. Materials and Methods

A configuration and boundary condition of square enclosure (L=H) with a vertical conducting partition placed at the middle is given in Figure 1. The partition separates the enclosure into two parts and each part is filled with fluid-saturated porous media. It is assumed that the left vertical wall is heated with a constant temperature T_h, while the right wall is cooled with a constant temperature T_c. The walls are impermeable in which the horizontal wall are insulated. In addition, both sides of the partition are assumed to conform with the conjugate heat transfer.

In order to investigate the fluid flow and temperature distribution due to the natural convection inside a cavity filled with fluid-saturated porous media, the continuity, Darcy and energy equations for steady two-dimensional porous enclosure are used to describe the flow inside it [16], which can be stated as follows:

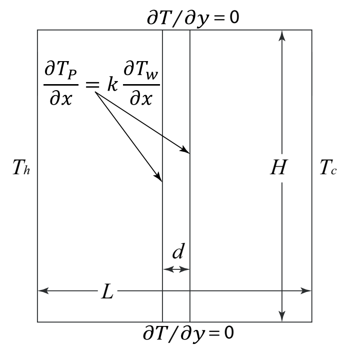

Figure 1 Configuration of square enclosure that has a partition located in the middle

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=-\frac{g\beta K}{\upsilon}\frac{\partial T_p}{\partial x}\,, \tag{2}$$

$$u\frac{\partial T_p}{\partial x}+v\frac{\partial T_p}{\partial y}=\alpha\left(\frac{\partial^2 T_p}{\partial x^2}+\frac{\partial^2 T_p}{\partial y^2}\right), \tag{3}$$

$$\frac{\partial^2 T_w}{\partial x^2}+\frac{\partial^2 T_w}{\partial y^2}=0\,, \tag{4}$$

where (4) is the energy equation of partition. By applying the definition of stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ together with non-dimensional variable expressed by

$$X=\frac{x}{L},\;\; Y=\frac{y}{L},\quad D=\frac{d}{L}, \tag{5a}$$

$$\theta_p=\frac{T_p-T_c}{T_H-T_c},\qquad \theta_w=\frac{T_w-T_c}{T_H-T_c},\qquad \Psi=\frac{\psi}{\alpha}, \tag{5b}$$

the governing equation (1)-(4) are transformed into dimensionless form and can be written as

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-Ra\frac{\partial\theta_p}{\partial X}\,, \tag{6}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\theta_p}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\theta_p}{\partial Y}=\frac{\partial^2\theta_p}{\partial X^2}+\frac{\partial^2\theta_p}{\partial Y^2}\,, \tag{7}$$

$$\frac{\partial^2\theta_w}{\partial X^2}+\frac{\partial^2\theta_w}{\partial Y^2}=0\,. \tag{8}$$

The non-dimensional boundary conditions are

$$\Psi(X,0)=\Psi(X,1)=0\,,\;\; \frac{\partial\theta_p}{\partial Y}(X,0)=\frac{\partial\theta_p}{\partial Y}(X,1)=0 \tag{9}$$

$$\Psi(0,Y)=\Psi(1,Y)=0\,,\qquad \theta_p(0,Y)=1\,,\qquad \theta_p(1,Y)=0 \tag{10}$$

$$\theta_p(c \pm \frac{D}{2}) = \theta_w(c \pm \frac{D}{2}), \quad \frac{\partial \theta_p}{\partial X}(c \pm \frac{D}{2}) = k \frac{\partial \theta_w}{\partial X}(c \pm \frac{D}{2}), \tag{11}$$

where $k = k_w/k_f$ is the thermal conductivity ratio.

The partial differential equations mentioned above are implemented by FlexPDE 6.19 Student Version. FlexPDE is a software package which performs the operations necessary to turn a description of a partial differential equation system into a finite element model. It then solves the system and presents graphical and tabular outputs of the results.

3. Results and Discussion

In this study, the effects of Rayleigh number (*Ra*), thickness of the partition (*D*) and thermal conductivity ratio (*k*) are observed. The results obtained from FlexPDE are displayed by streamlines and isotherms in which they are modified to show numerical values. Furthermore, the vertical temperature distributions along the partition are presented.

Computations are carried out for different values of *Ra*, *D* and *k*. The range of *Ra* is given within 100-1000 and three different values of partition thickness, $D = 0.1 - 0.3$ are selected. The ratios of thermal conductivity are chosen to cover wide range of application, that is epoxy-water ($k = 0.44$), epoxy-air ($k = 9.9$) and steel-water ($k = 23.8$) mentioned in [18]. The location of partition is kept constant at $c = 0.5$. Figure 2 shows the effects of *Ra* on streamlines and isotherms for $D = 0.1$ and $k = 9.9$. Since the fluid near the hot wall moves upwards and impinges to the top of the enclosure while the fluid near the left-hand side of partition moves downwards, the fluid motion rotates in clockwise direction and stream function values are negative. The intensity of flow circulation increases as *Ra* increases. As shown in Figure 2(a)-2(c), maximum value of stream function is represented with $|\Psi_{max}| = 2.1$ for $Ra = 100$ and with $|\Psi_{max}| = 9.0$ for $Ra = 1000$, respectively. These values also illustrate higher intensity of convection for higher *Ra*. The shapes of streamlines are almost elliptical at the core for all *Ra* and are dense near the left bottom and right top corner when *Ra* increases. Isotherms pattern shown in Figure 2(d)-2(f) illustrate that the temperature distributes uniformly in the partition. At low *Ra*, temperature lines are almost parallel to the conducting partition due to the weak flow, which indicates the conduction domination heat transfer in the cavity. Isotherms are distorted because the increase of *Ra*, which implies that the convection plays a role in both sides of the cavity.

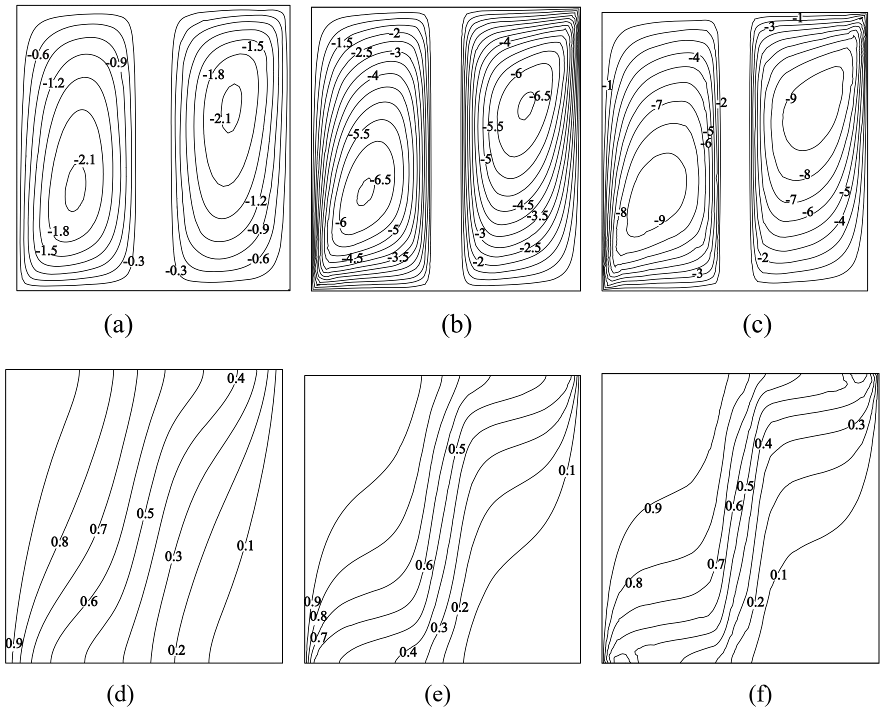

Figure 2 Streamlines (top) and isotherms (bottom) for various *Ra* with D = 0.1, k = 9.9

The streamlines and isotherm characteristics for various thermal conductivity ratios for Ra = 500 and D = 0.1 are shown in Figure 3. The intensity of fluid circulation becomes stronger with increasing values of k. Maximum value of stream function is $|\Psi_{max}|$ = 6.5 for k = 0.44 while it is $|\Psi_{\max}|$ = 8.0 for k = 23.8. It is also found that the increase of k causes the eye of cell moves to the center of two adjacent enclosures and the region of temperature θ = 0.7 expands, which means that heat transfer through the enclosure increases. There is no difference of flow circulation and temperature distribution in the case of k = 0.44 and k = 9.9.

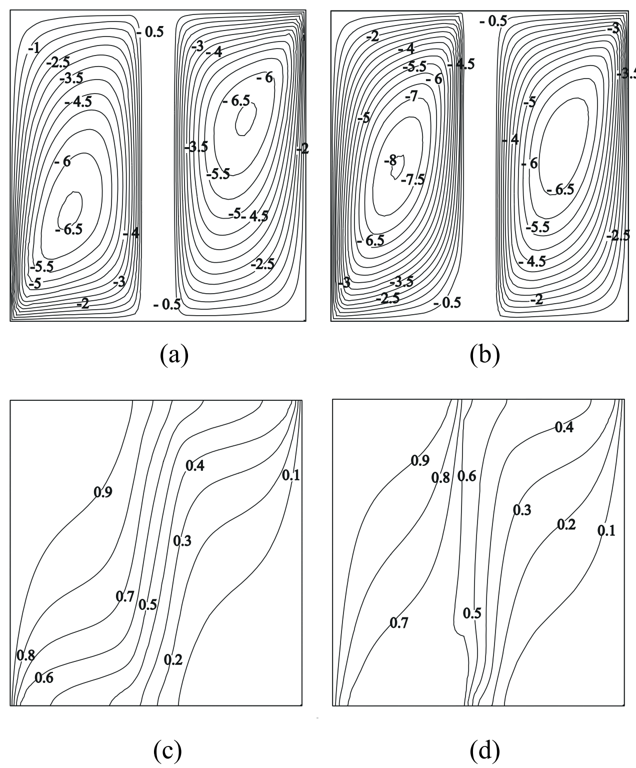

Figure 3 Streamlines (top) and isotherms (bottom) for various *k* with *D* = 0.1, *Ra* = 500

The variation of the temperature along the partition for different governing parameters is shown in Figure 4. The horizontal axis represents the height of the partition and the vertical axis represents the temperature along the partition. It is observed that the top of the partition tends to be hotter than the bottom. The temperature lines in Figure 4(a) are the results for *k* = 9.9 and *D* = 0.1 which illustrate that higher temperature is formed at the highest *Ra* according to the influence of natural convection heat transfer. Figure 4(b) shows the results for different *k* with *Ra* = 500 and *D* = 0.1. As value of *k* increases, the conductivity of the partition is small resulting in the lower temperature distribution along the partition. This satisfies the isotherms shown in Figure 3(d), in which there are several isothermal lines across the partition. To test the effect on partition thickness, values of *Ra* and *k* are set to 500 and 9.9, respectively. To obtain the different results, the partition is located at *c* = 0.75 and the temperature distribution is measured on the right-hand side of the partition. As seen from Figure 4(c), the temperature decreases when partition thickness increases due to the heat transfer from the left chest to the right chest is obstructed by larger partition.

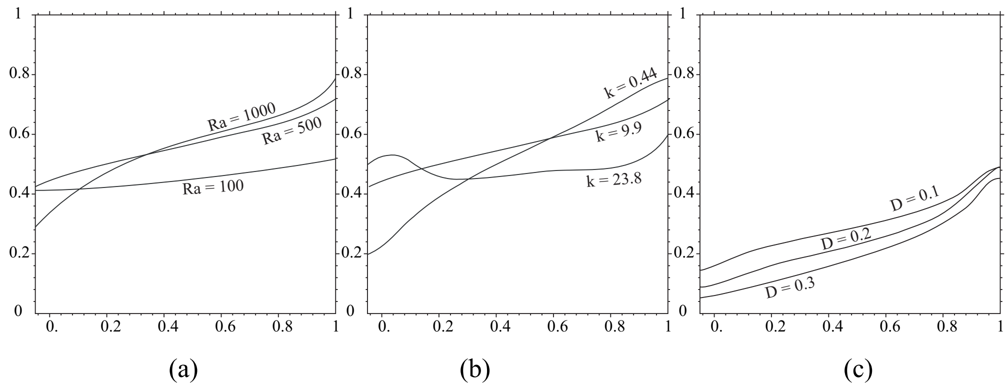

Figure 4 Temperature distribution along the partition for (a) Ra = 100 - 1000, (b) k = 0.44 - 23.8, (c) D = 0.1 - 0.3

4. Conclusions

The present work visualizes the natural convection in a square enclosure that has a vertical conducting partition placed at the middle of the enclosure in which the fluid-saturated porous media is contained. The main purpose of this work is to study the flow field, temperature distribution and effects of Rayleigh number, thermal conductivity ratio and partition thickness on natural convection in the enclosure. To analyze the performance, FlexPDE 6.19 Student Version is used to simulate the flow. The interesting results are obtained and displayed by streamlines and isotherm. It is found that the change of governing parameters affects the fluid flow and temperature distribution. The increase of Rayleigh number enhances the strength of fluid flow, due to increasing of domination convection, and the conduction in the partition becomes higher. The partition located in the enclosure plays a barrier role on heat transfer. The increase of thermal conductivity of partition leads to the decrease of thermal resistance of the overall system and the intensity of circulation increases as a result of higher heat transfer. Moreover, the thermal interaction between the regions decreases resulting in the lower temperature along the partition. Nevertheless, increasing the partition thickness reduces the temperature along the right-hand side of the partition when the convection is obstructed.

5. Acknowledgements

This research is partially supported by Centre of Excellence in Mathematics, the Commission on Higher Education, Thailand. The authors would like to thank Department of Mathematics, Faculty of Science, Khon Kaen University, Thailand, for necessary computational resources for this work.

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Notation

d	partition thickness (m)	x, y	Cartesian coordinates (m)
D	dimensionless partition thickness	X,Y	dimensionless Cartesian coordinates
H	height of enclosure (m)	u,v	velocity components along xy-axis (m s^{-1})
L	length of the bottom wall (m)	α	effective thermal diffusivity (m^2 s^{-1})
T	temperature (K)	β	coefficient of thermal expansion (K^{-1})
g	gravitational acceleration (m s^{-2})	θ	non-dimensional temperature
K	permeability of the porous medium (m^2)	υ	kinematic viscosity (m^2 s^{-1})
k	thermal conductivity ratio, k_w/k_p	Ψ	non-dimensional stream function
k_w	thermal conductivity of partition (W m^{-1} K^{-1})		
k_p	effective thermal conductivity of porous medium (W m^{-1} K^{-1})		
Ra	Rayleigh number for porous medium, $Ra = g\beta K(T_h - T_c)L/\upsilon\alpha$		