Robust Control with Finite Time Convergence for Flexible Spacecraft Attitude Tracking

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Abstract

The problem of attitude tracking for a flexible spacecraft is studied in this paper. A finite-time sliding mode controller is applied to quaternion-based attitude control for tracking maneuvers with external disturbances. The proposed sliding mode control law is developed by using a terminal sliding mode control algorithm which is able to guarantee finite time reachability of given desired attitude motion of a flexible spacecraft. By using the second method of Lyapunov and terminal sliding mode control concepts, stability of the closed-loop system can be achieved in finite time. An example of multiaxial attitude maneuvers is presented. Simulation results are included to demonstrate and verify the usefulness of the developed controller.

Keywords: Attitude tracking control, flexible spacecraft, terminal sliding mode, finite time convergence

1. Introduction

Attitude control has been a popular research area during the last few decades. In practical situations, the model parameters of the spacecraft may not be exactly known and the spacecraft is always subject to external disturbances. Thus, the attitude control problem with external disturbance has also attracted a great deal of attention. Without the presence of flexibility, the spacecraft control is reduced to the well-known rigid body control problem. Various nonlinear robust control approaches have been proposed for solving the attitude tracking control problem including adaptive control [1-2], sliding mode control [3-5], robust H_∞ control [6-7]. For flexible spacecraft the effect of motion of the elastic appendages makes the control problem more complicated. The modal-independent proportional-derivative (PD) controller proposed in [8] can achieve asymptotical stability of both the attitude and angular velocity. Sliding mode control (SMC) has been shown to be a potential approach when applied to a system with disturbances which satisfy the matched uncertainty condition [9]. Robust attitude controllers based on the SMC scheme have been proposed in [10-12]. These control laws can achieve global asymptotic stability and provide good tracking results. However, these controllers were designed based on an asymptotic stability analysis which implies that the system trajectories converge to the equilibrium with infinite settling time. It is well known that finite time stabilization of dynamical systems may

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provide a faster disturbance attenuation besides giving faster convergence to the required orientation. The terminal sliding mode (TSM) method [13,14] can be used to design a mode controller that will guarantee a finite time convergence to the origin. In [15] and [16] the attitude stabilization for rigid spacecraft has been studied and the TSM method was used to design finite time controllers.

In this research, the proposed controller based on TSM concepts is designed so that the attitude of flexible spacecraft will converge to the origin in finite time. The control law is developed by the TSM control algorithm. Although applications of finite time control schemes to attitude control systems are not recent, we believe that much research remains to be done in this area. Since these algorithms have rarely been studied for applications to flexible spacecraft, we hope that this paper will contribute to the popularity of the area and will enhance future development.

This paper is organized as follows. In Section 2 the dynamic and kinematic equations governing the attitude model [17-18] and the control design problem that we consider is formulated. Section 3 presents a basic control algorithm for a flexible spacecraft. The sliding manifold is chosen and the sliding control law is studied and a proof of finite time convergence of this controller is given. A numerical example of spacecraft tracking maneuvers is presented in Section 4 to verify the usefulness of the proposed controller. In Section 5 we present conclusions.

2. Preliminaries

2.1 Mathematical Model of Flexible Spacecraft

We define here the quaternion $Q = \begin{bmatrix} q_0 & q^T \end{bmatrix}^T$ with $q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T$. Consider the first time derivative of Q. The kinematic equations are given by [17],

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q^T \\ \left(q_0 I_3 + q^{\times}\right) \end{bmatrix} \omega , \tag{2.1}$$

where I_3 is a 3×3 identity matrix, and $q^{\times}$ is a skew-symmetric matrix

$$q^{\times} = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}.$$

The equation governing a flexible spacecraft is expressed as [18]

$$\dot{\omega} = J^{-1}{}_{mb} \left[-\omega^{\times} \left(J_{mb} \omega + H \vartheta \right) + L \vartheta - M \omega + \tau + T_d \right] \tag{2.2}$$

$$\dot{\vartheta} = A \vartheta + B \omega \tag{2.3}$$

with $\vartheta = \begin{bmatrix} \eta^T & \left(\dot{\eta} + \delta\omega\right)^T \end{bmatrix}^T$, and where η is the modal displacement, and δ is the coupling matrix between the central rigid body and the flexible attachments. $\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T$ represents the angular velocity vector and $\omega^{\times}$ is a skew-symmetric matrix with a formula similar to $q^{\times}$. $\tau = \begin{bmatrix} \tau_1 & \tau_2 & \tau_3 \end{bmatrix}^T$ is the control input and $T_d = \begin{bmatrix} T_{d1} & T_{d2} & T_{d3} \end{bmatrix}^T$ represents the external disturbance torque. Here, the matrices J_{mb}, H, L, M, A and B are given by

$$J_{mb} = J - \delta^T \delta, \; H = \begin{bmatrix} 0 & \delta^T \end{bmatrix}, \; L = \begin{bmatrix} \delta^T K & \delta^T C \end{bmatrix}, \; M = \delta^T C \delta,$$

$$A = \begin{bmatrix} 0_{4\times4} & I_4 \\ -K & -C \end{bmatrix}, \; B = \begin{bmatrix} -\delta \\ C\delta \end{bmatrix},$$

where $J = J^T$ is the total inertia matrix of the spacecraft, K and C denote the stiffness and damping matrices, respectively, which are defined as

$$K = diag\left(\omega_{ni}^2,\ i = 1,\ 2,\ ...,\ N\right) \text{ and } C = diag\left(2\varsigma_i\omega_{ni},\ i = 1,\ 2,\ ...,\ N\right) \tag{2.4}$$

with damping ς_i and natural frequency ω_{ni}.

2.2 Control Design Problem

In order to apply the proposed design method, we define a state variable $x = \begin{bmatrix} q^T & \omega^T \end{bmatrix}^T$ and a controlled output $y = q$. The satellite motion (2.1) – (2.3) can be modeled in the form of an uncertain nonlinear system

$$\dot{x} = f(x) + g(x)\tau + g(x)T_d \tag{2.5}$$

$$y = h(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} x, \tag{2.6}$$

where

$$f(x) = \begin{bmatrix} \frac{1}{2}\left(q^\times + q_0 I\right)\omega \\ J_{mb}^{-1}\left[-\omega^\times\left(J_{mb}\omega + H\vartheta\right) + L\vartheta - M\omega\right] \end{bmatrix},\quad g(x) = \begin{bmatrix} 0 \\ J_{mb}^{-1} \end{bmatrix},$$

$u = \left[u_1, u_2, ..., u_m\right]^T$, $y = \left[y_1, y_2, ..., y_m\right]^T$, and $m = 3$.

The control objective is to design a control law which causes the output to track a desired trajectory in the presence of a bounded disturbance.

From (2.6) we have the output is

$$y = q$$

and its time derivative is

$$\dot{y} = \dot{q} = \frac{1}{2}\left(q^\times + q_0 I_3\right)\omega. \tag{2.7}$$

Let the matrix $P \in R^{3\times 3}$ be defined as $P = q^\times + q_0 I_3$. Using (2.1), one obtains $\omega = 2P^{-1}\dot{q}$. After taking the time derivative of (2.7) and premultiplying both sides of the resulting expression by $P^{-T} J_{mb} P^{-1}$, we obtain the following

$$J_p\ddot{q} = \frac{1}{2}J_p\dot{P}\omega + \frac{1}{2}P^{-T}J_{mb}\dot{\omega}, \tag{2.8}$$

where $J_p = P^{-T}J_{mb}P^{-1}$. Substituting (2.2) into (2.8), we get

$$J_p\ddot{q} = \frac{1}{2}J_p\dot{P}\omega + \frac{1}{2}P^{-T}\left[-\omega^\times\left(J_{mb}\omega + H\vartheta\right) + L\vartheta - M\omega + \tau + T_d\right], \tag{2.9}$$

Substituting (2.7) into (2.9), yields

$$\begin{aligned} \ddot{q} = {} & J_p^{-1}J_p\dot{P}\left(P^{-1}\dot{q}\right) - 2J_p^{-1}P^{-T}\left(P^{-1}\dot{q}\right)^\times J_{mb}\left(P^{-1}\dot{q}\right) - J_p^{-1}P^{-T}\left(P^{-1}\dot{q}\right)^\times H\vartheta + \frac{1}{2}J_p^{-1}P^{-T}L\vartheta \\ & - J_p^{-1}P^{-T}M\left(P^{-1}\dot{q}\right) + \frac{1}{2}J_p^{-1}P^{-T}\tau + \frac{1}{2}J_p^{-1}P^{-T}T_d. \end{aligned} \tag{2.10}$$

Let $u = \frac{1}{2}J_p^{-1}P^{-T}\tau$ and $d = \frac{1}{2}J_p^{-1}P^{-T}T_d$. Then (2.10) becomes

$$\ddot{q} = \dot{P}\left(P^{-1}\dot{q}\right) - 2J_p^{-1}P^{-T}\left(P^{-1}\dot{q}\right)^\times J_{mb}\left(P^{-1}\dot{q}\right) - J_p^{-1}P^{-T}\left(P^{-1}\dot{q}\right)^\times H\vartheta + \frac{1}{2}J_p^{-1}P^{-T}L\vartheta$$

$$- J_p^{-1} P^{-T} M\left(P^{-1}\dot{q}\right) + u + d \tag{2.11}$$

We let

$$u = -\dot{P}\left(P^{-1}\dot{q}\right) + 2J_p^{-1}P^{-T}\left(P^{-1}\dot{q}\right)^{\times} J_{mb}\left(P^{-1}\dot{q}\right) + J_p^{-1}P^{-T}\left(P^{-1}\dot{q}\right)^{\times} H\vartheta - \frac{1}{2}J_p^{-1}P^{-T}L\vartheta$$
$$+ J_p^{-1}P^{-T}M\left(P^{-1}\dot{q}\right) + v, \tag{2.12}$$

where v will be defined later. Putting (2.12) in (2.11), one can obtain

$$\ddot{q} = v + d. \tag{2.13}$$

Let us define $z_1 \in R^3$, $z_2 \in R^3$ as $z_1 = q$ and $z_2 = \dot{q}$. Then (2.13) can be written as

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = v + d \tag{2.14}$$

Also we assume that the disturbance d is bounded by $\|d\| \le \bar{D}$, where $\bar{D}$ is a positive constant. z_d, $\dot{z}_d$ and $\ddot{z}_d$ are the desired attitude response, first time derivative and second time derivative, respectively. Also it is assumed that $\|z_d\| \le \bar{z}_1$, $\|\dot{z}\| < \bar{z}_2$ and $\|\ddot{z}_d\| < \bar{z}_3$, where $\bar{z}_1, \bar{z}_2, \bar{z}_3$ are positive constants. Next the error dynamic can be derived. Let $e_1 = z_1 - z_d$ and $e_2 = z_2 - \dot{z}_d$, then the system can be transformed to

$$\dot{e}_1 = e_2$$
$$\dot{e}_2 = v + d - \ddot{z}_d. \tag{2.15}$$

when using controller (2.12), we can reduce the complicated spacecraft model in (2.5) to the simple model in (2.14) and the error dynamic system in (2.15) is obtained.

3. Sliding Mode Attitude Controller

This section examines a feedback sliding model controller to solve the attitude tracking problem. The controller strategy is constructed by applying the control law (2.12) and TSM concepts. Consider the following controller

$$u = -\dot{P}\left(P^{-1}z_2\right) + 2J_p^{-1}P^{-T}\left(P^{-1}z_2\right)^{\times} J_{mb}\left(P^{-1}z_2\right) + J_p^{-1}P^{-T}\left(P^{-1}z_2\right)^{\times} H\vartheta - \frac{1}{2}J_p^{-1}P^{-T}L\vartheta$$
$$+ J_p^{-1}P^{-T}M\left(P^{-1}z_2\right) + v \tag{3.1}$$

and $v = -k_P e_1 - k_d e_2 - \rho\, sign(s)^r + \ddot{z}_d$,

where k_p, k_d are positive constants and $\rho = diag\begin{bmatrix}\rho_1 & \rho_2 & \rho_3\end{bmatrix}$ with $\rho_i > 0$, $0 < r < 1$ and $\dot{P} = \dot{q}_0 I_3 + \dot{q}^{\times}$.

The sliding vector s is given by

$$s = e_2 + \lambda e_1 \tag{3.2}$$

with a positive constant λ. The function $sign(s)^r$ is defined as

$$sign(s)^r = \left[|s_1|^r\, sign(s_1) \quad |s_2|^r\, sign(s_2) \quad \dots \quad |s_m|^r\, sign(s_m)\right]^T.$$

It is necessary to prove the ultimate boundedness of the state of systems (2.15) and (2.3) under the action of controller (3.1).

Theorem 3.1. For suitable k_p, k_d, λ and ρ_i, the controller (3.1) ensures the ultimate boundedness of the trajectories for attitude systems (2.15) and (2.3).

Proof. Consider the following Lyapunov function:

$$V_1 = \frac{k_p}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \lambda e_1^T e_2 + \vartheta^T P_1 \vartheta , \tag{3.3}$$

where $P_1 > 0$ is a solution of the Lyapunov equation $A^T P_1 + P_1 A = -Q$ with a positive-definite Q. V_1 can be bounded as

$$\frac{1}{2}\sigma_{\min}\left(\Lambda_1\right)\|\alpha\|^2 \le V_1 \le \frac{1}{2}\sigma_{\max}\left(\Lambda_2\right)\|\alpha\|^2 , \tag{3.4}$$

where $\sigma_{\min}\left(\Lambda_1\right)$ and $\sigma_{\max}\left(\Lambda_2\right)$ denote the minimum and maximum singular values of the matrix Λ_1; α and Λ_2 are given as

$$\alpha = \left[\|e_1\| \quad \|e_2\| \quad \|\vartheta\|\right]^T \text{ and } \Lambda_1 = \begin{bmatrix} \frac{k_p}{2} & \frac{\lambda}{2} & 0 \\ \frac{\lambda}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \sigma_{\min}\left(P_1\right) \end{bmatrix} \text{ and } \Lambda_2 = \begin{bmatrix} \frac{k_p}{2} & \frac{\lambda}{2} & 0 \\ \frac{\lambda}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \sigma_{\max}\left(P_1\right) \end{bmatrix},$$

Obviously, for suitable parameters k_p and λ, V_1 is positive definite.

Calculating the time derivative of V_1 gives

$$\dot{V}_1 = k_p e_1^T \dot{e}_1 + e_2^T \dot{e}_2 + \lambda e_1^T \dot{e}_2 + \vartheta^T P_1 \dot{\vartheta} \tag{3.5}$$

Substituting (2.3), (2.15) and (3.1) into (3.5), we obtain

$$\begin{aligned}\dot{V}_1 &= k_p e_1^T e_2 + \left(e_2^T + \lambda e_1^T\right)\left[-k_p e_1 - k_d e_2 - \rho sign\left(s\right)^r + d\right] \\ &\quad + \vartheta^T P_1\left(A\vartheta + 2BPe_2\right) \\ &= k_p e_1^T e_2 - k_p e_2^T e_1 - k_p \lambda e_1^T e_1 - k_d e_2^T e_2 - \lambda k_d e_1^T e_2 \\ &\quad - s^T\left[\rho sign\left(s\right)^r - d\right] - \vartheta^T Q \vartheta + 2\vartheta^T P_1 B P^{-1}\left(e_2 + \dot{z}_d\right),\end{aligned}$$

which can be further written as

$$\begin{aligned}\dot{V}_1 &\le -k_p \lambda \|e_1\|^2 - k_d \|e_2\|^2 - k_d \lambda \|e_1\|\|e_2\| - \sum_{i=1}^{3}\left(\rho |s_i|^{r+1} - s_i d_i\right) \\ &\quad - \sigma_{\min}\left(Q\right)\|\vartheta\|^2 + 2\|P_1 B\|\|\vartheta\|\|e_2\| + 2\|P_1 B\|\|\vartheta\|\bar{z}_2 .\end{aligned}$$

If we choose $\rho_i > \bar{D}$, then

$$\dot{V}_1 \le -\alpha^T \prod \alpha + 2\|P_1\|\|B\|\|\vartheta\|\bar{z}_2 ,$$

where

$$\prod = \begin{bmatrix} k_p \lambda & \frac{1}{2}k_d \lambda & 0 \\ \frac{1}{2}k_d \lambda & k_d & -\|P_1 B\| \\ 0 & -\|P_1 B\| & \sigma_{\min}\left(\prod\right) \end{bmatrix}.$$

We know that $\|\vartheta\| \le \|\alpha\|$ and obtain

$$\begin{aligned}\dot{V}_1 &\le -\sigma_{\min}\left(\prod\right)\|\alpha\|^2 + 2\|P_1 B\|\|\alpha\|\bar{z}_2 . \qquad (3.6) \\ &\le -\sigma_{\min}\left(\prod\right)\left(1-\theta\right)\|\alpha\|^2 - \sigma_{\min}\left(\prod\right)\theta\|\alpha\|^2 + 2\|P_1 B\|\|\alpha\|\bar{z}_2 \\ &\le -\sigma_{\min}\left(\prod\right)\left(1-\theta\right)\|\alpha\|^2 - \sigma_{\min}\left(\prod\right)\theta\|\alpha\|\left(\|\alpha\| - \frac{2\|P_1 B\|\bar{z}_2}{\sigma_{\min}\left(\prod\right)\theta}\right)\end{aligned}$$

for $\theta \in (0,1)$. Let $\mu = \dfrac{2\|P_1 B\|\bar{z}_2}{\sigma_{\min}(\Pi)\theta}$

following the standard step provided in Khalil [19], the ultimate bounded of system (2.15) and (2.3) states can be provided as

$$\|\alpha\| \le \sqrt{\frac{\sigma_{\max}(\Lambda_2)}{\sigma_{\min}(\Lambda_1)}}\mu .$$

Next, using the ultimate boundedness result, analyze the finite time stability of the closed-loop system. To establish the stability proof, the following lemmas are required.

Lemma 3.2. If $p \in (0,1)$, then the following inequality holds [13-14]:

$$\sum_{i=1}^{3} |x_i|^{1+p} \ge \left(\sum_{i=1}^{3} |x_i|^2\right)^{(1+p)/2}$$

Lemma 3.3. For any real numbers, an extended Lyapunov condition of finite-time stability can be given in the form of fast terminal sliding mode (TSM) as [13-14]:

$$\dot{V}(x) + \lambda_1 V(x) + \lambda_2 V^{\alpha} \le 0 , \tag{3.11}$$

where the settling time can be estimated by

$$T_r \le \frac{1}{\lambda_1(1-\alpha)} \ln \frac{\lambda_1 v^{1-\alpha}(x_0) + \lambda_2}{\lambda_2} . \tag{3.12}$$

Theorem 3.4. Assume that parameters $k_p > 0,\ k_d > 0,\ \lambda > 0,\ \rho_i\ (i = 1,2,3)$ of the controller (3.1) is chosen such that conditions $k_d > \lambda$ and $\rho_i > \bar{D}$, are satisfied. Under the controller (3.1) the trajectories of the closed-loop systems (2.15) and (2.3) can be driven onto the sliding surface in a finite time.

To prove the theorem above we select another proper Lyapunov function. With the ultimate boundedness result in Theorem 3.1, the finite time stability of the states e_1 and e_2 can be guaranteed.

Proof. Consider the following Lyapunov function

$$V_2 = \frac{1}{2} s^T s \tag{3.13}$$

The derivative of V_2 can be written as

$$\dot{V}_2 = s^T \left\{ v + d - \ddot{z}_d + \lambda e_2 \right\} \tag{3.14}$$

Substituting v into (3.14) we have

$$\begin{aligned} \dot{V}_2 &= s^T \left(-k_P e_1 - k_d e_2 - \rho sign(s)^r + d + \lambda e_2 \right) \\ &= s^T \left(-k_P e_1 - (k_d - \lambda) e_2 - \rho sign(s)^r + d \right) \end{aligned}$$

Letting $k_f = k_d - \lambda$ and choosing $\gamma = k_p = \lambda k_f$, one has

$$\begin{aligned} \dot{V}_2 &= s^T \left(-k_f s - \rho sign(s)^r + d \right) \\ &= -\sum_{i=1}^{3} \left(-\gamma_i s_i^{\,2} - \rho_i |s_i|^{r+1} - d_i |s_i| \right) . \end{aligned}$$

With $0 < r < 1$ it follows from $|s_i|^{r+1} > |s_i|$ that

$$\dot{V}_2 \le -\sum_{i=1}^{3} \left(\gamma_i s_i^{\,2} + \rho_i |s_i|^{r+1} - \bar{D} |s_i|^{r+1} \right)$$

$$\leq -\sum_{i=1}^{3}\left(\gamma_i s_i^{\ 2} + \beta_i \left|s_i\right|^{r+1}\right), \tag{3.15}$$

where $\beta_i = \rho_i - \bar{D}$. Using (3.15) and Lemma (3.31) we obtain $\dot{V}_2 + \lambda_3 V_2 + \lambda_3 V_2^{\alpha_1} \leq 0$,

where $\alpha_1 = \dfrac{r+1}{2}$, $\lambda_3 = 2\min\left(\gamma_i\right)$ and $\lambda_4 = 2^{\alpha_1+1}\min\left(\beta_1\right)$.6

The trajectory of closed-loop system will be driven onto the sliding surface s in the finite time

$$t_r \leq \frac{1}{\lambda_3\left(1-r\right)}\ln\frac{\lambda_3 V^{1-\alpha_1}\left(s_0\right)+\lambda_4}{\lambda_4},$$

where $V_2\left(s_0\right)$ is the initial value of $V_2\left(s\right)$

4. Simulation Results

In this section, simulation results are presented to demonstrate the performance of the developed controller. For this simulation, the model parameters for the flexible spacecraft are chosen as [18]

$$J_{mb} = \begin{pmatrix} 800 & 12 & 5 \\ 12 & 400 & 1.5 \\ 5 & 1.5 & 600 \end{pmatrix} \text{ kgm}^2,\ \delta = \begin{pmatrix} 10 & 0.5 & 0.2 \\ 0.5 & 2 & 0 \\ 0.1 & 10.9 & 0.8 \\ 1 & 0.5 & 0.5 \end{pmatrix} \text{kg}^{1/2}\text{m}$$

$$\omega_{n1} = 1.9,\ \ \omega_{n2} = 4.1,\ \ \omega_{n3} = 5.8,\ \ \omega_{n4} = 6 \text{ rad/s}$$

$$\varsigma_1 = 0.05,\ \ \varsigma_2 = 0.04,\ \ \varsigma_3 = 0.16,\ \ \varsigma_4 = 0.005$$

We assume that the external disturbance vector is of the form

$$d = \begin{pmatrix} 0.5\sin\left(t\right) & 0.5\sin\left(t\right) & 0.5\sin\left(t\right) \end{pmatrix}^T \text{ Nm},$$

and the desired tracking trajectory is given by

$$q_d = \begin{pmatrix} 0.5\cos\left(\dfrac{\pi t}{50}\right) & 0.5\sin\left(\dfrac{\pi t}{50}\right) & -0.5\sin\left(\dfrac{\pi t}{50}\right) \end{pmatrix}^T.$$

The initial conditions are $q\left(0\right) = \begin{bmatrix} 0.4618 & 0.1915 & 0.7999 \end{bmatrix}^T$, $\dot{q}\left(0\right) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$,

and $\vartheta\left(0\right) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$. The choices of control parameter ε, k_d, k_p γ and ρ_i are given by $\varepsilon = 0.01$, $k_d = 1.0$, $k_p = 0.5$ and $\rho_i = 0.85\ \left(i = 1,2,3\right)$ respectively. The sliding manifold is chosen as (3.2) with $\lambda = 0.5$, $\gamma = 0.6667$.

Simulation results are presented in Figures 1-5. The proposed controller provides good responses of attitude tracking and the time derivative of tracking errors. As shown in Figures 1 and 2 the trajectories are forced to be zero after 15 seconds. Obviously, the effect of external disturbances on attitude tracking responses and its time derivative is totally removed. Figure 4 depicts the control torque responses. The responses of the modal variables are presented in Figure 5 in which low vibration level is illustrated. In view of these simulation results, the proposed sliding mode controller works well for flexible spacecraft attitude tracking control problem.

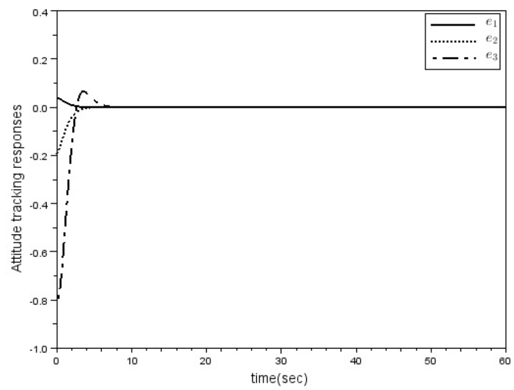

Figure 1 Attitude tracking responses.

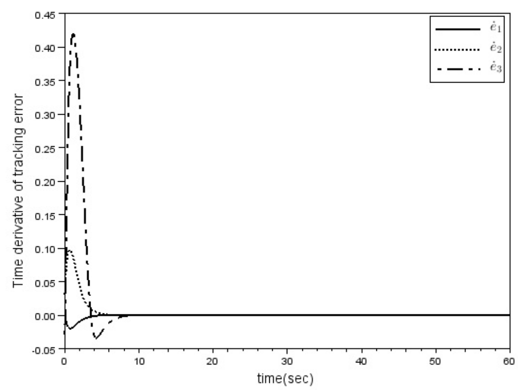

Figure 2 Responses of time derivative of tracking error.

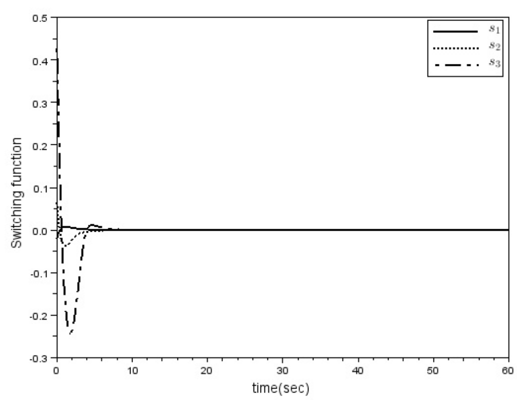

Figure 3 Switching function.

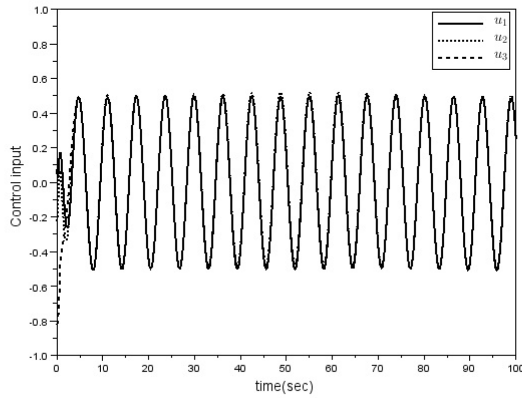

Figure 4 Control input.

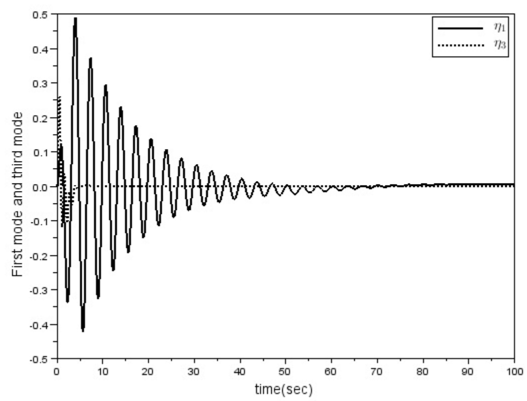

Figure 5 Responses of the modal coordinates.

5. Conclusions

In this research, a finite time sliding mode controller has been derived for flexible spacecraft attitude tracking in the presence of external disturbances. Based on TSM concepts, the presented controller has been proposed to force the state variables of the closed-loop system to converge to the desired state. The Lyapunov function is chosen to ensure the ultimate boundedness of the state and modal variables. Also another Lyapunov function has been selected to ensure the finite time convergence of the resulting closed-loop system. An example of multiaxial attitude maneuvers is presented and simulation results are included to verify the usefulness of the developed controller.

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