Natural Convection in a Porous Square Enclosure with Partially Cooled from Vertical Wall

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Abstract

Natural convection flow in a square enclosure filled with a fluid-saturated porous medium is investigated in this paper. The left side wall of the cavity is heated while the right side wall is partially cooled. The cooled portion is located adjacent to the top wall. The remaining walls are adiabatic. The governing equations are solved by using FlexPDE 6.14 Student Version which is based on finite element method. The different parameters in the present study are Darcy number ($10^{-5} \leq Da \leq 10^{-3}$), Grashof number ($10^{3} \leq Gr \leq 10^{5}$), Prandtl number ($0.70 \leq Pr \leq 10$) and Reynolds number ($10 \leq Pr \leq 100$). The results are illustrated in the terms of isotherms, streamlines and heatlines. It is found that the strength of fluid motion and the magnitudes of streamlines increase and temperature distribution decrease as Darcy number and Grashof number increase. In addition, a single circulation cell in clockwise direction is formed in the enclosure. The magnitudes of heatlines become larger at higher Darcy numbers, while the increasing of Grashof number has no effect for the magnitudes of heatlines.

Keywords: Finite element method; Natural convection; Partially cooled; Porous medium.

1. Introduction

Natural convection heat transfer in close cavities filled with a fluid-saturated porous medium has received a great deal of attention. This is due to a large number of applications, for example, oil extraction, fluid flow in geothermal reservoirs, separation processes in chemical industries, efficient drying process, dispersion of chemical contaminants through water saturated soil, crude production and solidification of casting etc.

Investigations of natural convection in an enclosure have been carried out by several investigators. Nawaf *et al.* [1] studied the steady natural convection in a porous square cavity with the non-Darcy model. The left vertical wall of the cavity is heated to a constant temperature, while the right wall is cooled to a constant temperature. Both the horizontal walls are adiabatic. The natural convection in an open-ended square cavity packed with porous medium has been presented by Haghshenas *et al*. [2]. The results obtained are values of Rayleigh number and porosity have considerable influence on heat transfer. Basak *et al.* [3] considered simulation of mixed convection in a square cavity filled with a porous medium with various wall thermal boundary conditions. The steady natural convection flow in a porous square cavity has been reported by Sathiyamoorthy

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et al. [4]. The boundary conditions are on the bottom wall is uniformly heated, left vertical wall is linearly heated and the right vertical wall is heated linearly or cooled while top wall is well insulated. Mahapatra *et al*. [5] introduced the influence of thermal radiation and internal heat generation on natural convection flow in a lid-driven square cavity filled with Darcy-Forchheimer porous medium. They observed that the temperature distribution decreases with the increasing in the value of Rayleigh number and the effect of increasing the thermal radiation parameter is to enhance the vertical velocity. Sompong and Witayangkurn [6] also investigated the natural convection in a porous square enclosure having two wavy vertical walls.

Studies on partially cooled cavities filled with a porous media are quite limited. Natural convection heat transfer in a partially cooled and inclined rectangular enclosure filled with saturated porous medium has been presented by Oztop [7]. The results obtained are heat transfer is increased with the increasing of Rayleigh number and dominant parameter on heat transfer and fluid flow as well as aspect ratio. Refaee *et al*. [8] studied numerical simulations of laminar natural convection in partially cooled tilted cavities which the tilt angle of the cavity is varied from 0° to 90°. The diffusion and convection heat transport for natural convection in a differentially heated square cavity has been analyzed by Mobedi *et al*. [9]. Furthermore, Varol *et al*. [10] performed the natural convection in a right-angle enclosure filled with a porous medium and which is partially cooled from the inclined wall.

This paper is the study of two-dimensional natural convection in a porous square enclosure with partially cooled from the side wall of the cavity. The cooled portion is located adjacent to the top wall. The objective of the present paper is to investigate the flow field, temperature distribution and heat flow in the enclosure. Main attention is focused on the effects of Darcy number and Grashof number. This study yields consistent performance over a wide range of parameters, Darcy number $(Da = 10^{-5} - 10^{-3})$, Grashof number $(Gr = 10^{3} - 10^{5})$, Prandtl number $(Pr = 0.7 - 10)$ and Reynolds number $(Re = 1 - 100)$.

2. Notation and Phisical Domain

Da Darcy number

g acceleration due to gravity, $\mathrm{ms^{-2}}$

K permeability, $\mathrm{m^2}$

L length of the square cavity, m

p pressure, Pa

P dimensionless pressure

Pr Prandtl number

Ra Rayleigh number

Re Reynolds number

Gr Grashof number

T temperature, K

T_h temperature of the left heated wall, K

T_c temperature of the partially cooled wall, K

u, v x and y components of velocity

U, V U and V components of dimensionless velocity

x, y distance along x and y coordinates

X, Y dimensionless distance along X and Y coordinates

Greek symbols

α thermal diffusivity, $\mathrm{m^2 s^{-1}}$

β volume expansion coefficient , $\mathrm{K^{-1}}$

γ penalty parameter

θ dimensionless temperature

υ kinematic viscosity, $\mathrm{ms^{-1}}$

ρ density, $\mathrm{kg\,m^{-3}}$

ψ streamfunction

Π heatfunction

The physical domain is shown in Figure 1. It consists of the coordinate system of two-dimensional square cavity filled with a fluid-saturated porous medium. The left vertical wall is heated with a constant temperature T_h, the right vertical wall is partially cooled with a constant temperature T_c, where $T_h > T_c$ and the partially cooled is placed adjacent to the top wall. The other walls are insulated. Length of the square cavity is L and length of the cooled portion on the right vertical wall is kept constant at L/2.

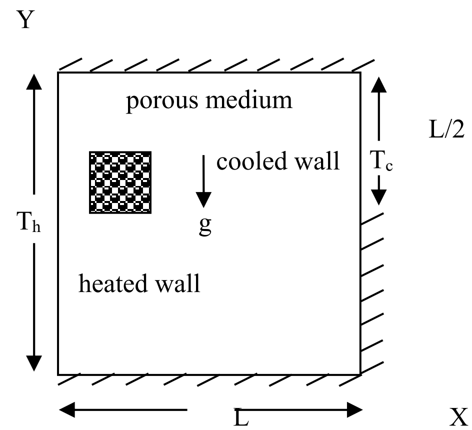


Figure 1 Physical domain with coordinates

3. Mathematical Formulation

All the physical properties are assumed to be constant except the density in buoyancy term. Change in density due to temperature variation is calculated using Boussinesq approximation. Another important assumption is that the local thermal equilibrium (LTE) is valid. The governing equations for two-dimensional natural convection flow in a porous square enclosure using conversation of mass, momentum and energy can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\upsilon}{K}u, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\upsilon}{K}v+g\beta\left(T-T_c\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

The boundary conditions are:

$$u(x,0)=u(x,L)=u(0,y)=u(L,y)=0,$$

$$v(x,0)=v(x,L)=v(0,y)=v(L,y)=0,$$

$$\frac{\partial T}{\partial y}(x,0)=\frac{\partial T}{\partial y}(x,L)=0,\ T(0,y)=T_h,$$

$$\frac{\partial T}{\partial x}(L,y)=0 \text{ when } 0\le y\le \frac{L}{2},\ T(L,y)=T_c \text{ when } \frac{L}{2}\le y\le L. \qquad (5)$$

The above governing equations are transformed to dimensionless form by using the following change of variables:

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{u}{U_0},\quad V=\frac{v}{U_0},\quad \theta=\frac{T-T_c}{T_h-T_c},\quad P=\frac{p}{\rho U_0^2},$$

$$Pr=\frac{\upsilon}{\alpha},\quad Da=\frac{K}{L^2},\quad Gr=\frac{g\beta(T_h-T_c)L^3}{\upsilon^2},\quad Re=\frac{U_0L}{\upsilon}.$$

The governing equations (1)-(4) reduce to the following non-dimensional form

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \qquad (6)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{1}{ReDa}U, \qquad (7)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{1}{ReDa}V+\frac{Gr}{Re^2}\theta, \qquad (8)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right). \qquad (9)$$

In order to solve the equations (7)-(8) by eliminating the pressure, we use the penalty finite element method with a penalty parameter [11] such that

$$P=-\gamma\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right). \qquad (10)$$

Typical values of γ that yield consistent solutions are 10^7. Substituting (10) into (7) and (8) are reduced to

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\gamma\frac{\partial}{\partial X}\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{1}{ReDa}U, \qquad (11)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=\gamma\frac{\partial}{\partial Y}\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{1}{ReDa}V+\frac{Gr}{Re^2}\theta. \qquad (12)$$

The transformed boundary conditions are:

$$U(X,0)=U(X,1)=U(0,Y)=U(1,Y)=0,$$

$$V(X,0)=V(X,1)=V(0,Y)=V(1,Y)=0,$$

$$\frac{\partial \theta}{\partial Y}(X,0)=\frac{\partial \theta}{\partial Y}(X,1)=0,\ \theta(0,Y)=1,$$

$$\frac{\partial \theta}{\partial X}(1,Y)=0 \text{ when } 0\le Y\le\frac{1}{2},\ \theta(1,Y)=0 \text{ when } \frac{1}{2}\le Y\le 1. \qquad (13)$$

The governing equations ((9), (11) and (12)) are solved by using FlexPDE 6.14 Student Version which is based on finite element method.

The streamfunction (ψ) is used to visualize the convective fluid flow in the enclosure. The dimensionless streamfunction is defined as $U=\frac{\partial \psi}{\partial Y}$ and $V=-\frac{\partial \psi}{\partial X}$. Thus, the equation (6) is changed to (14)

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}=\frac{\partial U}{\partial Y}-\frac{\partial V}{\partial X}. \tag{14}$$

The no-slip condition is valid at all boundaries as there is no cross flow. Hence, the boundaries condition for streamfunction is $\psi = 0$.

The heatfunction (Π) is used to visualize the convective heat flow in the enclosure. The dimensionless heatfunction is defined as $\frac{\partial \Pi}{\partial Y} = U\theta - \frac{\partial \theta}{\partial X}$ and $-\frac{\partial \Pi}{\partial X} = V\theta - \frac{\partial \theta}{\partial Y}$. Thus, the equation (6) is changed to (15)

$$\frac{\partial^2\Pi}{\partial X^2}+\frac{\partial^2\Pi}{\partial Y^2}=\frac{\partial}{\partial Y}\left(U\theta\right)-\frac{\partial}{\partial X}\left(V\theta\right), \tag{15}$$

with the boundary conditions:

$$\frac{\partial \Pi}{\partial X} = 1 \text{ for left heated wall,}$$

$$\frac{\partial \Pi}{\partial X} = 0 \text{ for cooled portion right wall,}$$

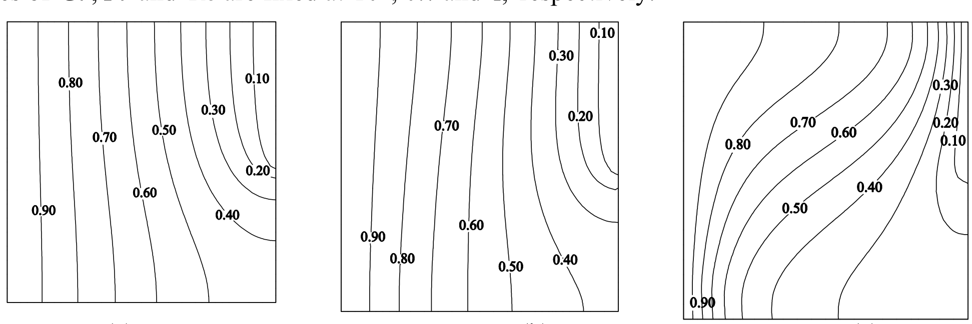
$$\Pi = 0 \text{ for adiabatic walls.} \tag{16}$$

4. Results and Discussion

The computational results for the problem of natural convection in a porous square enclosure with partially cooled right vertical wall are presented in this section. The cooled portion is located adjacent to the top wall. The procedure mentioned previously is coded into FlexPDE 6.14 Student Version which is based on finite element method. The results are carried out for different values of Darcynumber $(10^{-5} \le Da \le 10^{-3})$, Grashofnumber $(10^3 \le Gr \le 10^5)$, Prandtlnumber $(0.7 \le Pr \le 10)$ a nd Reynolds number $(1 \le Re \le 100)$. Effects of Darcy number (Da) and Grashof number (Gr) are considered. The flow, temperature and heat fields within the enclosure are shown in terms of streamlines isotherms and heatlines.

4.1 Effect of Darcy number

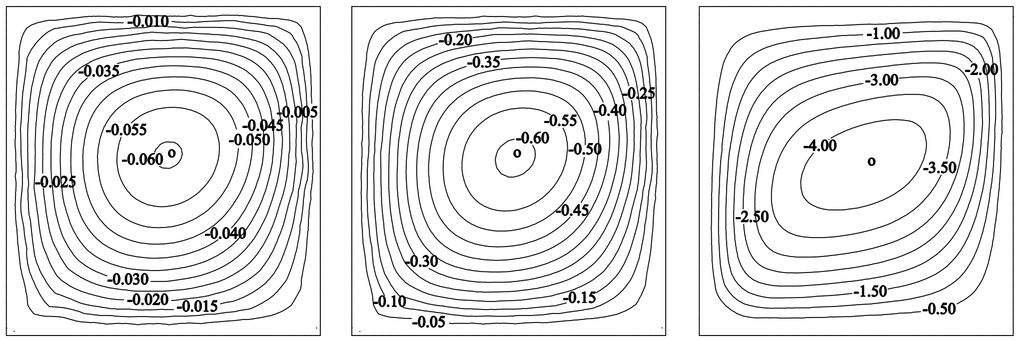
Figures 2–4 illustrate the isotherms, streamlines and heatlines inside a partially cooled square cavity filled with a porous medium. Three different Da are choosen as 10^{-5}, 10^{-4} and 10^{-3}. The values of Gr, Pr and Re are fixed at 10^5, 0.7 and 1, respectively.



(a) (b) (c)

Figure 2 Isotherms with $Gr = 10^5$, $Pr = 0.7$, $Re = 1$ for (a) $Da = 10^{-5}$ (b) $Da = 10^{-4}$ and (c) $Da = 10^{-3}$.

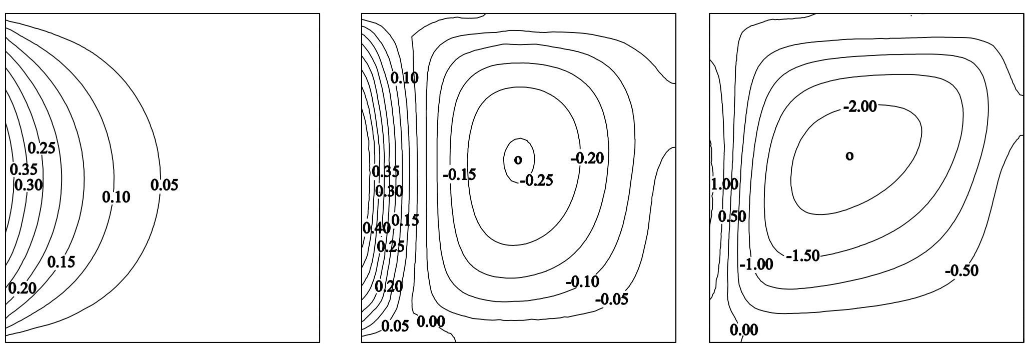
It is observed that isotherms are smooth. At low Darcy number ($Da = 10^{-5}$), isotherms are almost parallel to the side walls. The isotherm moves downward to the bottom wall for $\theta = 0.40$ (Figure 2(b)). As Da is increased to 10^{-3}, values of isotherms with $\theta = 0.30 - 0.90$ move toward to the left side wall [Figure 2(c)].



(a) (b) (c)

Figure 3 Streamlines with $Gr = 10^5$, Pr = 0.7, $Re = 1$ for (a) $Da = 10^{-5}$ (b) $Da = 10^{-4}$ and (c) $Da = 10^{-3}$.

It is noted that a single circulation cell of streamlines is formed in clockwise rotating direction for all values of Da. The flow of streamlines is weak as seen from the maximum absolute value of streamfunction is 0.060 (Figure 3(a)). As Da is increased to 10^{-4}, the flow characteristic is similar to the case of $Da = 10^{-5}$ but the intensity of fluid flow is stronger. The flow circulation cell is found to be changed with the main vortex expands in size. The maximum absolute value of streamfunction is 4.00 [Figure 3(c)].



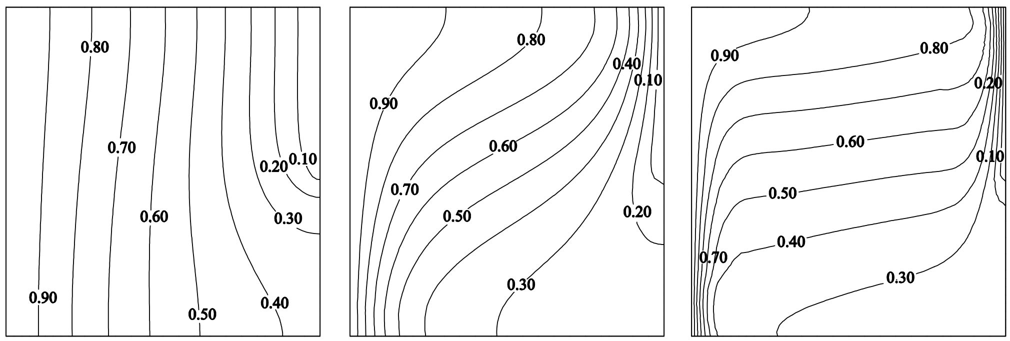
(a) (b) (c)

Figure 4 Heatlines with $Gr = 10^5$, Pr = 0.7, $Re = 1$ for (a) $Da = 10^{-5}$ (b) $Da = 10^{-4}$ and (c) $Da = 10^{-3}$.

The heatlines are smooth and these heatlines disperse near the right vertical wall (Figure 4(a)). As Da is increased to 10^{-4}, the main heat flow rotates in clockwise direction with the maximum absolute value of heatfunction is 0.25. In addition the dense heatlines appear near heated wall. The less dense heatlines are also observed near heated wall compared to the case of $Da = 10^{-4}$ and the strength of heatlines increases for $Da = 10^{-3}$ (Figure 4(c)).

4.2 Effect of Grashof number

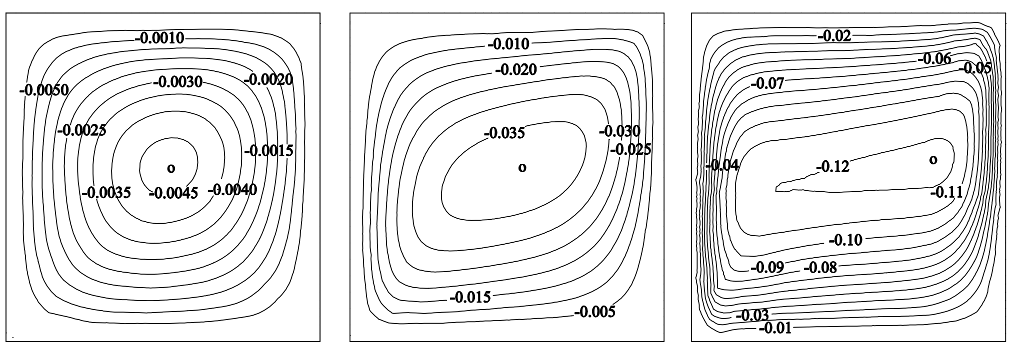
Isotherms, streamlines and heatlines inside a partially cooled square enclosure filled with a porous medium are plotted in Figsures 5–7. Three different Gr are choosen as 10^3, 10^4 and 10^5. The values of Da, Pr and Re are fixed at 10^{-3}, 10 and 10, respectively.



(a) (b) (c)

Figure 5 Isotherms with $Da = 10^{-3}$, $Pr = 10$, $Re = 10$ for (a) $Gr = 10^3$ (b) $Gr = 10^4$ and (c) $Gr = 10^5$.

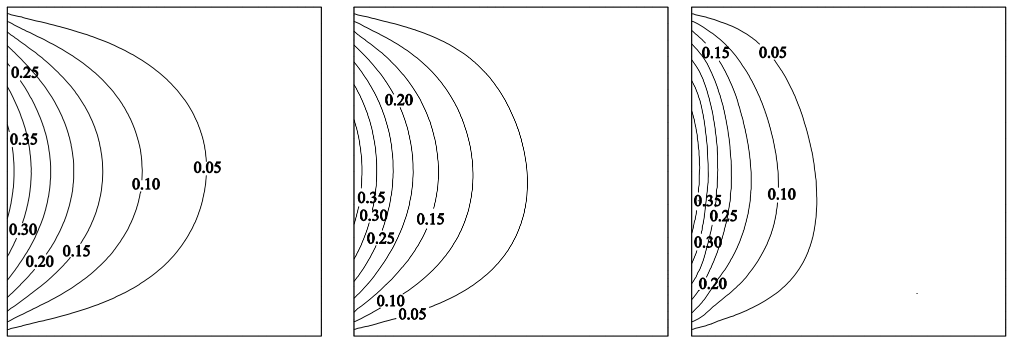
It may be noted that isotherms are smooth. The isotherms are almost parallel to the side walls (Figure 5(a)). As Gr is increased to 10^4, isotherms move toward the heated wall with $\theta = 0.30 - 0.90$. Furthermore, isotherms with $\theta \geq 0.30$ are almost parallel to the horizontal walls (Figure 5(c)).



(a) (b) (c)

Figure 6 Streamlines with $Da = 10^{-3}$, $Pr = 10$, $Re = 10$ for (a) $Gr = 10^3$ (b) $Gr = 10^4$ and (c) $Gr = 10^5$.

The convection streamfunction takes negative values due to the clockwise rotation of convective fluid flow (Figures 6(a)-6(c)). Weak streamlines circulation cell with the maximum absolute value of streamfunction is 0.0045 for $Gr = 10^3$. Single flow circulation cell is changed with the main vortex expands in size (Figure 6(b)). As Gr is increased to 10^5, the flow circulation cell elongates in the enclosure and the intensity of fluid motion is stronger with $|\psi|_{max} = 0.12$ (Fig. 6(c)).



(a) (b) (c)

Figure 7 Heatlines with $Da = 10^{-3}$, $Pr = 10$, $Re = 10$ for (a) $Gr = 10^3$ (b) $Gr = 10^4$ and (c) $Gr = 10^5$.

At low Grashof number ($Gr = 10^3$), the heatlines are smooth and these heatlines disperse near the heated side wall (Figure 7(a)). Moreover, the heatlines are shifted toward to the heated side wall as Gr are increased to 10^4 and 10^5 (Figures 7(b)-7(c)). The magnitudes of heatlines no change for all values of Gr .

5. Conclusions

This study investigates the natural convection flow in a porous square enclosure with partially cooled. The right vertical wall is partially cooled while the left vertical wall is heated and the remaining walls are adiabatic. The governing equations are solved by using a software package FlexPDE 6.14 Student Version. The main parameters of interest are Darcy number $(10^{-5} \le Da \le 10^{-3})$, Grashof number $(10^3 \le Gr \le 10^5)$, Prandtl number $(0.7 \le Pr \le 10)$ and Reynolds number $(1 \le Re \le 100)$. The effects of parameters such as Darcy number and Grashof number are examined.

From the study results, the temperature distribution decrease as values of Da and Gr increase. With both of these parameters, a single circulation cell occurred in clockwise rotating direction. In addition, the magnitudes of streamlines and heatlines become larger with high Da . In the case of increasing Gr , the main vortex of circulation cell expands in size and the strength of buoyant convection flow is enhanced. Furthermore, the magnitudes of heatlines no change with increasing of value of Gr .

6. Acknowledgements

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