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On the Application of Uncertainty Models in Copying Machine Maintenance Problem

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Abstract

The maintenance system is necessary to maintain an efficient production system and reduce the possibility of work being suspended due to breakdown of machines. For every manufacturing company the objective is to produce goods at a profit and this is only achieved by using an effective maintenance system. Maintenance is required for all types of machinery. The copying machine is one of the most important inventions of the 20th century. In this paper, we concern with the maintenance of copying machines. For this purpose, we introduce a multiobjective nonlinear programming model with uncertainty. Data in many real life engineering and economical problems suffer from inexactness. Uncertainty always exists in practical engineering problems. In order to deal with the uncertain optimization problems, fuzzy and stochastic approaches are commonly used to describe the imprecise characteristics. We will treat the problem of concern with three different approaches of uncertainty and compare among them. These approaches are fuzzy programming approach, interval number programming approach and stochastic programming approach. We introduce a numerical example to clarify the proposed method.

Keywords: Maintenance, Copying machine, Multiobjective nonlinear programming, Fuzzy programming, Stochastic programming, Interval number programming.

1. Introduction

Machine maintenance is often thought of as the repair or replacement of components after failure. Types of maintenance for growth of industry are breakdown maintenance, preventive maintenance, productive maintenance and autonomous maintenance. Maintenance techniques have changed over time from correction (breakdown) to prevention to prediction and pro-active continuous improvement [1].

Maintenance is required for all types of machinery. The copying machine is one of the most important inventions of the 20th century. The copying machines range in size, from small desktop units to large units that can produce thousands of copies each day. Copying machines are not just copy machines anymore. In this day, the copying machines are now multifunction devices which can copy, print, scan and fax documents.

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The purpose of multiobjective nonlinear programming model with uncertainty is to optimize four objective functions. These objective functions are maximizing customer's profit, maximizing maintenance capacity from company's preference, maximizing maintenance capacity from customer's preference and minimization of customer's downtime. These objective functions are subject to a set of system constraints [2].

Uncertainty is a very important concern in machine maintenance in real plants. Uncertainty appears from different sources, such as unexpected machine breakdown, staffing or operator problems, cancellation or modification of existing orders, etc. In order to deal with the uncertain optimization problems, fuzzy and stochastic approaches are commonly used to describe the imprecise characteristics. Stochastic optimization methods are optimization algorithms which incorporate probabilistic (random) elements in the problem data [3]. A nonlinear interval number programming method is suggested to transform the uncertain optimization problem to a deterministic multi-objective optimization problem based on an order relation of interval number [4]. Fuzzy programming offers a powerful means of handling optimization problems with fuzzy parameters. The concept of fuzzy set was initialized by Zadeh [5] in 1965. Fuzzy numbers, defined on a set of real numbers, will be used in this article. Also, it is assumed that they have a triangular shape [6].

In our study and for the first time, we construct three models of copying machine maintenance problem under uncertainty. These models are fuzzy programming model, interval number programming model and stochastic programming model. We find that although the fuzzy programming model, interval number programming model and stochastic programming model are substantially more complex to solve, it yields a higher valued solution and provides better insight into the dynamics of the staffing decision.

The rest of this paper is organized as follows. The problem formulation is described in section 2. In section 3 we treat the copying machine maintenance problem with random variables. In section 4 we treat the same problem with fuzzy variables. In section 5 we treat the same problem with interval number variables. Finally a case study is provided in section 6 to clarify the proposed approach.

2. Problem Formulation

In this section we introduce an optimization model for the copying machine maintenance problem. First, we introduce the notations which will be used in this formula.

2.1 Variables

x_k : The maintenance capacity (in units) for each type of copying machine *k*.

n_k : The requirements of inspection frequency.

λ_k : The failure arrival rate for a copying machine *k*.

2.2 Parameters

n : The number of inspections.

ci_k : The cost of each inspection for a copying machine *k*.

cr_k : The cost of each repair for a copying machine *k*.

$p_k(n)$: The profit per unit copy volume from operating a machine of type *k*.

$D_k(n)$: The total downtime per the all copy volume of a machine of type *k*.

μ_k : The repair arrival rate for a copying machine *k*.

i_k : The inspection arrival rate for a copying machine *k*.

$MTTI_k$: The mean time to inspect the copying machine *k*.

$MTTR_k$: The mean time to repair the copying machine *k*.

w_k : The maximum speeds of each copying machine *k* according to the company's preference.

w'_k : The maximum speeds of each copying machine *k* according to the customer's preference.

af : An adjustment factor.

SE : The total number of after sales service engineers.

TM : The monthly available working time of after sales service engineers.

B : The total available budget for maintenance.

$cv_k^{\min}$: The minimum copy volume of the copying machine *k*.

LB_k : Lower bounds for x_k .

v_k : The customer profit.

I_k : The average cost of inspection per uninterrupted the all copy volume.

R_k : The average cost of repair per uninterrupted the all copy volume.

2.3 The mathematical model

The maintenance of copying machines problem is formulated as weighted multi-objective nonlinear programming problem with four weighted objectives. The proposed objectives are based on maximizing customer's profit Z_1, minimization of customer's downtime Z_2, maximizing maintenance capacity from company's preference Z_3 and maximizing maintenance capacity from customer's preference Z_4.

$$\text{Max } Z_1 = \sum_{k=1}^{4} x_k p_k(n) \tag{1}$$

$$\text{Min } Z_2 = \sum_{k=1}^{4} x_k D_k(n) \tag{2}$$

$$\text{Max } Z_3 = \sum_{k=1}^{4} w_k x_k \tag{3}$$

$$\text{Max } Z_4 = \sum_{k=1}^{4} w'_k x_k \tag{4}$$

subject to

$$\sum_{k=1}^{4} \left(\frac{n}{i_k}\right) x_k + \left[\frac{\lambda_k(n)}{\mu_k}\right] x_k \leq (af)(SE)(TM) \tag{5}$$

$$\sum_{k=1}^{4} \left[\lambda_k(n) cr_k x_k + nci_k x_k\right] \leq B \tag{6}$$

$$n_k \geq cv_k^{\min} \qquad \forall k \tag{7}$$

$$x_k \geq LB_k \qquad \forall k \tag{8}$$

$$x_k \geq 0 \qquad \forall k \tag{9}$$

where

$$D_k(n) = \frac{\lambda_k(n)}{\mu_k} + \frac{n}{i_k} \text{ and } P_k(n) = \nu_k - \nu_k \frac{\lambda_k(n)}{\mu_k} - \nu_k \frac{n}{i_k} - R_k \frac{\lambda_k(n)}{\mu_k} - I_k \frac{n}{i_k}$$

$$\frac{1}{i_k} = MTTI_k * \text{average speed of copy volume/month},$$

$$\frac{1}{\mu_k} = MTTR_k * \text{average speed of copy volume/month}.$$

We have four types of constraints. The set of constraints (5) represents the availability of working hours of after sales services engineers. The set of constraints (6) represents availability of budget. The set of constraints (7) represents minimum requirement of inspection frequency. The set of constraints (8) represents bounds due to contractual agreements.

By using weighted sum method [7], objective functions (1) - (4) are integrated to a single objective problem with weighted value assigned to each objective. The weights at an instance of optimization sum up to one. The single objective is as follows:

$$\text{Max} \sum_{k=1}^{4} \left[C_1 x_k p_k(n) - C_2 x_k D_k(n) + C_3 w_k x_k + C_4 w'_k x_k \right] \tag{10}$$

where $\sum_{i=1}^{4} C_i = 1,\ C_i \geq 0$

In our work, the uncertainty analyses concerns the changing of the available resources. These available resources are budget of maintenance and after sales service engineers. In classical machine maintenance problems, the data are generally supposed to be known and fixed. However, the reality does not check this hypothesis, not only because of variations, but also because of a lot of data that are only previsions or estimations. Optimal solutions to such machine maintenance problems which are based on fixed data and do not show the reality, will have only few chances to be applicable and will be subject to modifications. Various data inputs are modeled under interval, stochastic and fuzzy programming.

3. Copying machine maintenance problem with random variables

In this section, we assume that the budget of maintenance and the total number of after sales service engineers are random variables. The mathematical model of copying machine maintenance problem with random variables is as follows:

$$\text{Max} \sum_{k=1}^{4} \left[C_1 x_k p_k(n) - C_2 x_k D_k(n) + C_3 w_k x_k + C_4 w'_k x_k \right] \tag{11}$$

subject to

$$\sum_{k=1}^{4} \left(\frac{n}{i_k} \right) x_k + \left[\frac{\lambda_k(n)}{\mu_k} \right] x_k \leq (af)(\overline{SE})(TM) \tag{12}$$

$$\sum_{k=1}^{4}\left[\lambda_k(n)cr_k x_k + nci_k x_k\right] \le \overline{B} \tag{13}$$

$$n_k \ge cv_k^{\min} \qquad \forall k \tag{14}$$

$$x_k \ge LB_k \qquad \forall k \tag{15}$$

$$\sum_{i=1}^{4} C_i = 1 \quad , \quad C_i \ge 0 \tag{16}$$

where $\overline{B}$ and $\overline{SE}$ are considered as random variables of the budget of maintenance and the total number of after sales service engineers respectively.

3.1 Treatment of copying machine maintenance problem with random variables

According to Sakawa *et al*. approach [8], $\overline{SE}$ can be expressed as $\overline{SE} = SE_1 + \tau(a)SE_2$ where $\tau(a)$ is a random variable with mean $\tilde{\tau}$ and it is assumed to be $SE_2 \ge 0$. Then the set of constraints (12) can be written as

$$\sum_{k=1}^{4}\left(\frac{n}{i_k}\right)x_k + \left[\frac{\lambda_k(n)}{\mu_k}\right]x_k \le (af)(TM)\left(SE_1 + \tau(a)SE_2\right) \tag{17}$$

Using the chance constrained conditions, with satisfying level β_1, constraints (17) can be written as follows [9]

$$pr\left[\sum_{k=1}^{4}\left(\frac{n}{i_k}\right)x_k + \left[\frac{\lambda_k(n)}{\mu_k}\right]x_k \le (af)(TM)\left(SE_1 + \tau(a)SE_2\right)\right] \ge \beta_1 \tag{18}$$

$$pr\left[\tau(a) \ge \frac{\left[\sum_{k=1}^{4}\left(\frac{n}{i_k}\right)x_k + \left[\frac{\lambda_k(n)}{\mu_k}\right]x_k\right] - (af)(TM)(SE_1)}{(af)(TM)(SE_2)}\right] \ge \beta_1 \tag{19}$$

Let $F_\tau(g) = pr\left[\tau(a) \le g\right]$, where $F_\tau(g)$ is a distribution function of the random variable $\tau(a)$. Then we get

$$1 - F_\tau(g) = pr\left[\tau(a) \ge g\right] \tag{20}$$

$$1 - F_\tau\left[\frac{\left[\sum_{k=1}^{4}\left(\frac{n}{i_k}\right)x_k + \left[\frac{\lambda_k(n)}{\mu_k}\right]x_k\right] - (af)(TM)(SE_1)}{(af)(TM)(SE_2)}\right] \ge \beta_1 \tag{21}$$

$$F_\tau \left[\frac{\left[\sum_{k=1}^{4} \left(\frac{n}{i_k} \right) x_k + \left[\frac{\lambda_k(n)}{\mu_k} \right] x_k \right] - (af)(TM)(SE_1)}{(af)(TM)(SE_2)} \right] \leq 1 - \beta_1 \tag{22}$$

$$\left[\frac{\left[\sum_{k=1}^{4} \left(\frac{n}{i_k} \right) x_k + \left[\frac{\lambda_k(n)}{\mu_k} \right] x_k \right] - (af)(TM)(SE_1)}{(af)(TM)(SE_2)} \right] \leq F_\tau^{-1}\left(1 - \beta_1\right) \tag{23}$$

Where $F_\tau^{-1}\left(1-\beta_1\right)$ is an inverse distribution function of $\left(1-\beta_1\right)$. Then we get

$$\sum_{k=1}^{4} \left[\left(\frac{n}{i_k} \right) + \left(\frac{\lambda_k(n)}{\mu_k} \right) \right] x_k \leq (af)(TM)(SE_1) + (af)(TM)(SE_2) F_\tau^{-1}\left(1-\beta_1\right) \tag{24}$$

We treat equation (13) by the same above way, then we get the following equation:

$$\sum_{k=1}^{4} \left[\lambda_k(n) cr_k x_k + nci_k x_k \right] \leq (B_1) + (B_2) F_\tau^{-1}\left(1-\beta_2\right) \tag{25}$$

Then the set of constraints (12) and (13) in the mathematical model (11) - (16) are replaced by the set of constraints (24) and (25).

4. Copying machine maintenance problem with fuzzy variables

In this section, we assume that the budget of maintenance and the total number of after sales service engineers are fuzzy variables. The fuzzy model for copying machine maintenance problem is as follows:

$$\text{Max} \sum_{k=1}^{4} \left[C_1 x_k p_k(n) - C_2 x_k D_k(n) + C_3 w_k x_k + C_4 w'_k x_k \right] \tag{26}$$

subject to

$$\sum_{k=1}^{4} \left(\frac{n}{i_k} \right) x_k + \left[\frac{\lambda_k(n)}{\mu_k} \right] x_k \leq (af)(\tilde{SE})(TM) \tag{27}$$

$$\sum_{k=1}^{4} \left[\lambda_k(n) cr_k x_k + nci_k x_k \right] \leq \tilde{B} \tag{28}$$

$$n_k \geq cv_k^{\min} \qquad \forall k \tag{29}$$

$$x_k \geq LB_k \qquad \forall k \tag{30}$$

$$\sum_{i=1}^{4} C_i = 1 \ , C_i \geq 0 \tag{31}$$

where $\tilde{B}$ and $\tilde{SE}$ are considered as triangular fuzzy numbers of the budget of maintenance and the total number of after sales service engineers respectively.

4.1 Treatment of copying machine maintenance problem with fuzzy variables

A triangular fuzzy number $\tilde{SE}$ can be expressed as $\tilde{SE} = (SE_1, SE_2, SE_3)$ where SE_1, SE_2 and SE_3 are real numbers [10]. Also, the α - level set of a triangular fuzzy number $\tilde{SE} = (SE_1, SE_2, SE_3)$ is as follows [11]:

$$SE_{\alpha 1} = \left[SE_1 + \alpha 1(SE_2 - SE_1),\ SE_3 - \alpha 1(SE_3 - SE_2)\right] \tag{32}$$

Similarly, the α cut of a triangular fuzzy number $\tilde{B} = (B_1, B_2, B_3)$ is as follows:

$$B_{\alpha 2} = \left[B_1 + \alpha 2(B_2 - B_1),\ B_3 - \alpha 2(B_3 - B_2)\right] \tag{33}$$

By substitute in equation (12) and (13), we get

$$\sum_{k=1}^{4} \left(\frac{n}{i_k}\right) x_k + \left[\frac{\lambda_k(n)}{\mu_k}\right] x_k \leq (af)(TM)\left[SE_1 + \alpha 1(SE_2 - SE_1), SE_3 - \alpha 1(SE_3 - SE_2)\right] \tag{34}$$

$$\sum_{k=1}^{4} \left[\lambda_k(n) cr_k x_k + nci_k x_k\right] \leq \left[B_1 + \alpha 2(B_2 - B_1), B_3 - \alpha 2(B_3 - B_2)\right] \tag{35}$$

Then the set of constraints (12) and (13) in the mathematical model (11) - (16) are replaced by the set of constraints (34) and (35).

5. Copying machine maintenance problem with interval number variables

In this section, we assume that the budget of maintenance and the total number of after sales service engineers are interval number variables. The interval number model for copying machine maintenance problem is as follows:

$$\text{Max } \sum_{k=1}^{4} \left[C_1 x_k p_k(n) - C_2 x_k D_k(n) + C_3 w_k x_k + C_4 w'_k x_k\right] \tag{36}$$

subject to

$$\sum_{k=1}^{4} \left(\frac{n}{i_k}\right) x_k + \left[\frac{\lambda_k(n)}{\mu_k}\right] x_k \leq (af)(TM)\left[SE^L, SE^R\right] \tag{37}$$

$$\sum_{k=1}^{4} \left[\lambda_k(n) cr_k x_k + nci_k x_k\right] \leq \left[B^L, B^R\right] \tag{38}$$

$$n_k \geq cv_k^{\min} \qquad \forall k \tag{39}$$

$$x_k \geq LB_k \qquad \forall k \tag{40}$$

$$\sum_{i=1}^{4} C_i = 1, C_i \geq 0 \quad i = 1,2,3,4 \tag{41}$$

where B^L and B^R are considered as the lower and upper bounds of the budget of maintenance respectively. SE^L and SE^R are the lower and upper bounds of the total number of after sales service engineers respectively.

5.1 Treatment of copying machine maintenance problem with interval number

Based on the approach of Jiang *et al.* [4] for treating interval number variables, we can treat the uncertainty of model (36) - (41). The possibility degree of interval number variables represents certain degree that one interval number is larger or smaller than another [12].

$$P_{a \geq SE} = \begin{cases} 0 \\ \dfrac{a - SE^L}{SE^R - SE^L} \\ 1 \end{cases} \Rightarrow P_{-a \leq -SE} = \begin{cases} 0 \\ \dfrac{-a + SE^L}{-SE^R + SE^L} \\ 1 \end{cases} \tag{42}$$

where $a = \dfrac{\sum_{k=1}^{4}\left[\left(\dfrac{n}{i_k}\right)x_k + \left(\dfrac{\lambda_k(n)}{\mu_k}\right)x_k\right]}{(af)(TM)}$ and

$P_{-a \leq -\left[SE^L, SE^R\right]} \geq \theta_1$ is the possibility degree of the constraint (37). $0 \leq \theta_1 \leq 1$ is a predetermined possibility degree level. By substitute in equation (37), we get

$$\frac{-\dfrac{\sum_{k=1}^{4}\left[\left(\dfrac{n}{i_k}\right)x_k + \left(\dfrac{\lambda_k(n)}{\mu_k}\right)x_k\right]}{(af)(TM)} + \left(SE^L\right)}{\left(-SE^R + SE^L\right)} \geq \theta_1 \tag{43}$$

$$-\left[\sum_{k=1}^{4}\left[\left(\frac{n}{i_k}\right)x_k + \left(\frac{\lambda_k(n)}{\mu_k}\right)x_k\right] - (af)(TM)\left(SE^L\right)\right] \geq -(af)(TM)\left(SE^R - SE^L\right)(\theta_1) \tag{44}$$

$$\sum_{k=1}^{4}\left[\left(\frac{n}{i_k}\right)x_k + \left(\frac{\lambda_k(n)}{\mu_k}\right)x_k\right] - (af)(TM)\left(SE^L\right) \leq (af)(TM)\left(SE^R - SE^L\right)(\theta_1) \tag{45}$$

$$\sum_{k=1}^{4}\left[\left(\frac{n}{i_k}\right)x_k + \left(\frac{\lambda_k(n)}{\mu_k}\right)x_k\right] \leq (af)(TM)\left[\left(SE^L\right) + \left(SE^R - SE^L\right)(\theta_1)\right] \tag{46}$$

When we treat equation (38) by the same way, then we get the following equation:

$$\sum_{k=1}^{4}\left[\lambda_k(n) cr_k x_k + nci_k x_k\right] \leq \left[B^L + \left(B^R - B^L\right)(\theta_2)\right] \tag{47}$$

The set of constraints (37) and (38) in the mathematical model (36) - (41) are replaced by the set of deterministic constraints (46) and (47).

6. Case study

A company vending copying machines has four types of copiers. These types are low speed machines m_1, medium speed machines m_2, medium/high speed machines m_3 and high speed machines m_4. The average speeds of copy volume are 4000 copy volume/month (equivalent to 0.5 page/minute), 10000 copy volume/month (equivalent to 1.2 page/minute), 30000 copy volume /month (equivalent to 3.6 page/minute) and 75000 copy volume/month (equivalent to 8.9 page/minute) for m_1, m_2, m_3 and m_4 respectively. Table 1 contains the data of our case study.

Table 1 Data of case study

$ci_k = \$16 \ \ \forall k$	$cr_k = \$22 \ \ \forall k$	$af = 0.4$	$TM = 8$ hours/day
$w_1 = 1$	$w_2 = 3$	$w_3 = 6$	$w_4 = 9$
υ_1=\$4000	υ_2=\$4000	υ_3=\$4000	υ_4=\$4000
$w'_1 = 51$	$w'_2 = 39.2$	$w'_3 = 9.7$	$w'_4 = 0.1$
$cv_1^{\min} = 0.2$ copy volume	$cv_2^{\min} = 0.15$ copy volume	$cv_3^{\min} = 0.1$ copy volume	$cv_4^{\min} = 0.05$ copy volume
$\lambda_1 = \dfrac{0.48}{n_1}$	$\lambda_2 = \dfrac{0.46}{n_2}$	$\lambda_3 = \dfrac{0.27}{n_3}$	$\lambda_4 = \dfrac{0.27}{n_4}$
$LB_1 = 4000$	$LB_2 = 2000$	$LB_3 = 500$	$LB_4 = 10$
$MTTI_1 =$ 37 min.	$MTTI_2 =$ 49 min.	$MTTI_3 =$ 68 min.	$MTTI_4 =$ 98 min.
$MTTR_1 =$ 72 min.	$MTTR_2 =$ 80 min.	$MTTR_3 =$ 107 min.	$MTTR_4 =$ 147 min.

We have three cases of uncertainty as follows:
First: Copying machine maintenance problem with random variables

Let $C_1 = C_2 = C_3 = C_4 = 0.25$, $\theta_1 = 0.9$, $\theta_2 = 0.8$, $\beta_1 = \beta_2 = 0.1$, $B_1 = 800000\$$, $B_2 = 1300000\$$, $SE_1 = 300$ persons, $SE_2 = 550$ persons and $F_\tau^{-1}(0.99) = 2.3263$. The copying machine maintenance problem with random variables is as follows:

$$\max \begin{pmatrix} 38x_1 - 0.50545\dfrac{x_1}{n_1} - 4.81175x_1n_1 + 35.575x_2 - 1.291675\dfrac{x_2}{n_2} - 14.68625x_2n_2 \\ +28.925x_3 - 3.048425\dfrac{x_3}{n_3} - 62.04625x_3n_3 + 27.275x_4 - 9.18215\dfrac{x_4}{n_4} - 213.3x_4n_4 \end{pmatrix}$$

subject to

$$86.4\frac{x_1}{n_1} - 92.5x_1n_1 + 36.8\frac{x_2}{n_2} - 49x_2n_2 + 9.63\frac{x_3}{n_3} + 22.67x_3n_3$$
$$+4.704\frac{x_4}{n_4} + 13.067x_4n_4 \le 5760\overline{SE}$$
$$660\frac{x_1}{n_1} - 1000x_1n_1 + 253\frac{x_2}{n_2} - 400x_2n_2 + 49.5\frac{x_3}{n_3} + 133.3x_3n_3 + 17.6\frac{x_4}{n_4} + 53.3x_4n_4 \le \overline{B}$$
$$x_1 \ge 4000$$
$$x_2 \ge 2000$$
$$x_3 \ge 500$$
$$x_4 \ge 10$$
$$n_1 \ge 0.2$$
$$n_2 \ge 0.15$$
$$n_3 \ge 0.1$$
$$n_4 \ge 0.05$$

By applying the mathematical technique which was described in subsection 3.1, we get the following final deterministic form of the concerned problem as follows:

$$\max\left(\begin{array}{l} 38x_1 - 0.50545\frac{x_1}{n_1} - 4.81175x_1n_1 + 35.575x_2 - 1.291675\frac{x_2}{n_2} - 14.68625x_2n_2 \\ +28.925x_3 - 3.048425\frac{x_3}{n_3} - 62.04625x_3n_3 + 27.275x_4 - 9.18215\frac{x_4}{n_4} - 213.3x_4n_4 \end{array}\right)$$

subject to

$$86.4\frac{x_1}{n_1} - 92.5x_1n_1 + 36.8\frac{x_2}{n_2} - 49x_2n_2 + 9.63\frac{x_3}{n_3} + 22.67x_3n_3$$
$$+4.704\frac{x_4}{n_4} + 13.067x_4n_4 \le 6417820.8$$
$$660\frac{x_1}{n_1} - 1000x_1n_1 + 253\frac{x_2}{n_2} - 400x_2n_2 + 49.5\frac{x_3}{n_3} + 133.3x_3n_3 + 17.6\frac{x_4}{n_4} + 53.3x_4n_4$$
$$\le 2463497$$

By using WinQSB package, we get the following results:

$x_1 = 4000, x_2 = 2000, x_3 = 500, x_4 = 10, n_1 = 0.8107, n_2 = 0.8039,$
$n_3 = 0.6115,\ n_4 = 0.7760,\ Z_1 = 643065.68,\ Z_2 = 266813.429,\ Z_3 = 13090,$
$Z_4 = 287451$

Second: Copying machine maintenance problem with fuzzy variables

Let $C_1 = C_2 = C_3 = C_4 = 0.25$, $\tilde{SE} = [300, 400, 550]$ persons, $\tilde{B} = [800000, 1000000, 1300000]\$$, $\alpha_1 = 0.8, \alpha_2 = 0.9$ then we obtain $SE_{0.8} = [380, 430]$ persons, $B_{0.9} = [980000, 10300000]\$$.

The copying machine maintenance problem with fuzzy variables is as follows:

$$\max \begin{pmatrix} 38x_1 - 0.50545\dfrac{x_1}{n_1} - 4.81175x_1 n_1 + 35.575x_2 - 1.291675\dfrac{x_2}{n_2} - 14.68625x_2 n_2 \\ +28.925x_3 - 3.048425\dfrac{x_3}{n_3} - 62.04625x_3 n_3 + 27.275x_4 - 9.18215\dfrac{x_4}{n_4} - 213.3x_4 n_4 \end{pmatrix}$$

subject to

$$86.4\frac{x_1}{n_1} - 92.5x_1 n_1 + 36.8\frac{x_2}{n_2} - 49x_2 n_2 + 9.63\frac{x_3}{n_3} + 22.67x_3 n_3$$

$$+4.704\frac{x_4}{n_4} + 13.067x_4 n_4 \le 5760\tilde{SE}$$

$$660\frac{x_1}{n_1} - 1000x_1 n_1 + 253\frac{x_2}{n_2} - 400x_2 n_2 + 49.5\frac{x_3}{n_3} + 133.3x_3 n_3 + 17.6\frac{x_4}{n_4} + 53.3x_4 n_4 \le \tilde{B}$$

$$x_1 \ge 4000$$

$$x_2 \ge 2000$$

$$x_3 \ge 500$$

$$x_4 \ge 10$$

$$n_1 \ge 0.2$$

$$n_2 \ge 0.15$$

$$n_3 \ge 0.1$$

$$n_4 \ge 0.05$$

By applying the mathematical technique which was described in subsection 4.1, we get the following final deterministic form of the concerned problem as follows:

$$\max \begin{pmatrix} 38x_1 - 0.50545\dfrac{x_1}{n_1} - 4.81175x_1 n_1 + 35.575x_2 - 1.291675\dfrac{x_2}{n_2} - 14.68625x_2 n_2 \\ +28.925x_3 - 3.048425\dfrac{x_3}{n_3} - 62.04625x_3 n_3 + 27.275x_4 - 9.18215\dfrac{x_4}{n_4} - 213.3x_4 n_4 \end{pmatrix}$$

subject to

$$86.4\frac{x_1}{n_1}-92.5x_1n_1+36.8\frac{x_2}{n_2}-49x_2n_2+9.63\frac{x_3}{n_3}+22.67x_3n_3$$
$$+4.704\frac{x_4}{n_4}+13.067x_4n_4-5760\,SE\le 0$$
$$660\frac{x_1}{n_1}-1000x_1n_1+253\frac{x_2}{n_2}-400x_2n_2+49.5\frac{x_3}{n_3}+133.3x_3n_3$$
$$+17.6\frac{x_4}{n_4}+53.3x_4n_4-B\le 0$$
$$x_1\ge 4000$$
$$x_2\ge 2000$$
$$x_3\ge 500$$
$$x_4\ge 10$$
$$n_1\ge 0.2$$
$$n_2\ge 0.15$$
$$n_3\ge 0.1$$
$$n_4\ge 0.05$$
$$380\le SE\le 430$$
$$980000\le B\le 10300000$$

By using WinQSB package, we get the following results:

$x_1=4002.5220, x_2=2000.6930, x_3=500, x_4=10, n_1=0.7587,$
$n_2=0.7338, n_3=0.5565, n_4=0.2964$, $Z_1=643187.58$, $Z_2=245092.08$,
$Z_3=13094.6$, $Z_4=287606$

Third: Copying machine maintenance problem with interval number variables

Let $C_1=C_2=C_3=C_4=0.25$, $\theta_1=\theta_2=0.3$, $\left[B^L,B^R\right]=\left[300,550\right]\$$,
$\left[SE^L,SE^R\right]=\left[800000,1300000\right]$ persons.

The copying machine maintenance problem with interval number variables is as follows:

$$\max\begin{pmatrix}38x_1-0.50545\frac{x_1}{n_1}-4.81175x_1n_1+35.575x_2-1.291675\frac{x_2}{n_2}-14.68625x_2n_2\\+28.925x_3-3.048425\frac{x_3}{n_3}-62.04625x_3n_3+27.275x_4-9.18215\frac{x_4}{n_4}-213.3x_4n_4\end{pmatrix}$$

subject to

$$86.4\frac{x_1}{n_1}-92.5x_1n_1+36.8\frac{x_2}{n_2}-49x_2n_2+9.63\frac{x_3}{n_3}+22.67x_3n_3+4.704\frac{x_4}{n_4}$$
$$+13.067x_4n_4\le 5760\left[SE^L,SE^R\right]$$

$$660\frac{x_1}{n_1}-1000x_1n_1+253\frac{x_2}{n_2}-400x_2n_2+49.5\frac{x_3}{n_3}+133.3x_3n_3+17.6\frac{x_4}{n_4}+53.3x_4n_4\le\left[B^L,B^R\right]$$

$x_1\ge 4000$
$x_2\ge 2000$
$x_3\ge 500$
$x_4\ge 10$
$n_1\ge 0.2$
$n_2\ge 0.15$
$n_3\ge 0.1$
$n_4\ge 0.05$

By applying the mathematical technique which was described in subsection 5.1, we get the following final deterministic form of the concerned problem as follows:

$$\max\begin{pmatrix}38x_1-0.50545\frac{x_1}{n_1}-4.81175x_1n_1+35.575x_2-1.291675\frac{x_2}{n_2}-14.68625x_2n_2\\ +28.925x_3-3.048425\frac{x_3}{n_3}-62.04625x_3n_3+27.275x_4-9.18215\frac{x_4}{n_4}-213.3x_4n_4\end{pmatrix}$$

subject to

$$86.4\frac{x_1}{n_1}-92.5x_1n_1+36.8\frac{x_2}{n_2}-49x_2n_2+9.63\frac{x_3}{n_3}+22.67x_3n_3+4.704\frac{x_4}{n_4}$$
$$+13.067x_4n_4\le 3024000$$

$$660\frac{x_1}{n_1}-1000x_1n_1+253\frac{x_2}{n_2}-400x_2n_2+49.5\frac{x_3}{n_3}+133.3x_3n_3+17.6\frac{x_4}{n_4}+53.3x_4n_4\le 8400000$$

$x_1\ge 4000$
$x_2\ge 2000$
$x_3\ge 500$
$x_4\ge 10$
$n_1\ge 0.2$
$n_2\ge 0.15$
$n_3\ge 0.1$
$n_4\ge 0.05$

By using WinQSB package, we get the following results:
$x_1 = 4006.9230, x_2 = 2002.0780, x_3 = 501.3311, x_4 = 10, n_1 = 0.7341,$
$n_2 = 0.3488, n_3 = 0.2299, n_4 = 0.1993$, $Z_1 = 640348.85$, $Z_2 = 185378.707$, $Z_3 = 13111$,
$Z_4 = 287898.04$.

For the concerned problem, only one objective function, named Z_3, must be maximized for the company. From the obtained results, we found that the interval number approach gives us the best solution for Z_3. Three objective functions, named Z_1, Z_2 and Z_4 must be achieved for the customer. From the obtained results, we found that the fuzzy approach gives us the maximum customer profit (Z_1) and the interval number approach gives us the minimum customer downtime (Z_2) and the maximum maintenance capacity from customer's preference Z_4.

7. Conclusions

In this paper we treated the copying machine maintenance problem. We suggested three approaches of uncertainty for the concerned problem. These approaches are stochastic programming approach, fuzzy programming approach and interval number programming approach. We found that the uncertainty approaches are more realistic for the practical problems such as copying machine maintenance problem. A case study for our problem is considered to clarify the proposed approach. From the obtained results, we determined which of stochastic programming approach, fuzzy programming approach and interval number programming approach is more suitable for problem of concern.

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