

## Vibration Monitoring and Damage Assessment of a Retrofitted RC Frame Model

Atiwat Vimuttasoongviriya\*, Naveen Kwatra and Maneek Kumar

Department of Civil Engineering, Thapar University, Patiala, Punjab, India

### Abstract

The need for structural damage assessment is continuously growing to monitor the damage level of existing civil structures. The information of damage assessment is utilized to make decisions on maintenance of damaged structure. This paper intends to investigate the effect of lateral load on the damage indexes of RC frame model strengthened using FRP sheets. Failure mechanism of the experimental frame model has been investigated. The effects of FRP wrap have been reported. Subsequently, the damage indexes based on modal parameters method are investigated with the help of impact hammer excitation test. Results of this study show that the use of FRP wrapped for structural retrofitting provides increased significant lateral load capacity and ductile behaviour. The damage indexes of retrofitted frame reduce indicating better performance as compare to the control frame.

**Keywords:** Damage index, non-ductile structure, retrofitted RC frame, FRP sheet, FRF.

### 1. Introduction

The need for structural health assessment is continuously growing to maintain existing civil structures. The damage of structure may be due to natural hazards such as earthquakes, windstorms and due to long duration ageing. Information on the damage is always utilized to make decisions on maintenance of damaged structure. In high seismic region of north India, existing reinforced concrete (RC) buildings have been constructed at time when seismic zones were not recognized. These RC buildings have inadequate reinforcement detailing which results in deficient lateral load resistance. Kanwar *et al.* [1] found that non-ductile RC moment resisting frames result in the need of retrofitting to increase the load resistance capacities. During the past decades, there are various methods of strengthening the damaged RC structures. The use of steel plate jacket and ferro-cement jacket are disruptive to the operation of the facility, labor intensive and time consuming. The Fiber Reinforcement Polymer (FRP) is one popular strengthening technique because FRP with epoxy have received considerable attention due to its high strength, light weight, easy manageability on-site and high resistance against corrosion. This advance material has been successfully used to increase bending and shear capacity of flexural elements. The retrofitted connections using FRP sheets have shown to prevent its brittle shear failure and also significantly improved their displacement ductility and energy dissipation capacity [2]. The use of FRP systems have also been found to be an effective method for upgrading deficient RC columns.

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\*Corresponding author: Tel: +91-175-2393028 Fax: +91-175-2393354

E-mail: atiwat.vimut@gmail.com

For the purpose of structural damage assessment, it is necessary to monitor the structures for location and extent of damage. Damage index is a quantity that is used for estimating the damage. This index depends on different specific damage parameters such as deformation, structural stiffness, energy dissipation and dynamic properties of structure.

Generally speaking, the well known combined damage index method is proposed by Park and Ang [3]. This index is calculated as a linear combination of maximum displacement response and total hysteretic energy dissipation under cyclic load. However, this damage index is not able to monitor the damage of retrofitted structure in proper way because cracks are covered by strengthening material layers. Damage detection using vibration measurement of dynamic properties, such as natural frequency, mode shape and frequency response function (FRF), is non-destructive technique and it has widely used to monitor the damage of structure. Several researchers have relied on the used of vibration measurement for system identification and damage detection. Dipasquale and Cakmak [4] presented damage index based on the change ratio of frequency. This index considers structural fundamental natural frequencies before and after damaged. Rodriguez and Barroso [5] proposed stiffness-mass ratios damage theory. This damage index is based on stiffness-mass ratio in form of modal parameters. The Modal flexibility damage index method is the well known one that it is presented by Ko *et al.* [6]. The principle of this method is on the basis of the comparison of the flexibility matrices obtained from two sets of mode shapes. Maia *et al.* [7] presented FRF based mode shape method that it uses FRF data over modal parameters which can be measured directly on structure without any intermediate steps.

This paper intends to investigate both damage indexes and effects on a retrofitted RC frame model under quasi-static loads. To study the change in dynamic properties, the RC moment resisting frame model has been built in laboratory and tested under different level of controlled damage. Effects of FRP wrap on RC element have been reported. The first part involves determination of load-displacement relationships. Failure mechanism of the experimental frame model has been investigated. Subsequently, some of the damage indexes based on modal parameters method have been selected to indicate the damage of structure.

## 2. Damage Indexes based on Dynamic Characteristics

In structural damage detection, a quantity of damage index is used to monitor the damage state of structure. This value is equal to zero when there is no damage and is equal to one when total collapse occurs. Park and Ang [3] suggested the relation between damage index and various states as examined in Table 1. There are various parameters of physical responses to formulate damage index of structure. These damage parameters can be classified as deformation, change in stiffness, energy dissipation and changes in dynamical parameters. The change in dynamic characteristics of structure such as resonant frequency, mode shape and FRF is always used to calculate the damage of structure. This approach calls modal parameters damage index method. Based on the change in dynamic characteristics of structure, damage in different locations and components actually leads to different frequency changes in various modes. Some of more significant damage indexes were studied and presented in this paper.

**Table 1** Relation between damage index and various states [3].

Damage State	Damage Index, $DI$	State of Building
No damage	0	No damage
Slight damage	0-0.1	No damage
Minor damage	0.1-0.25	Minor damage
Moderate damage	0.25-0.4	Repairable
Severe damage	0.4-1.0	Beyond repair
Collapse	> 1.0	Loss of building

### 2.1 Dipsasquale and Cakmak damage index

Dipsasquale and Cakmak [4] defined the modal plastic softening index for the one-dimensional case, where the fundamental eigen frequency is considered. This damage index is given by

$$DI_{Dip} = 1 - \frac{\omega_n^{*2}}{\omega_n^2} \quad (1)$$

where  $\omega_n$  and  $\omega_n^*$  are the fundamental eigen frequency and damage frequency parameter, respectively. The asterisk (\*) denotes the damage state.

### 2.2 Stiffness-mass ratios method

Rodriguez and Barroso [5] presented the damage index which is based on stiffness-mass ratios method in form of structural modal parameters. Considering multi-degree of freedom (MDOF) system, the mass at each floor is lumped together, given by  $m_l$ . Similarly, the lateral stiffness of the  $l^{th}$  story from 1 to  $n$  is given by  $k_l$ . Based on eigen problem solving, a general expression of the  $l^{th}$  story stiffness can be obtain as

$$k_l = \omega_j^2 \sum_{i=1}^n \frac{m_i \phi_{ij}}{\Delta \phi_{ij}} \quad \text{for } j = 1, 2, \dots, n \quad (2)$$

Defining the damage of a structure as the reduction percentage of story stiffness before and after damage, and assuming that the floor mass does not change due to the damage. This damage index of each story can be expressed as

$$DI_{SMR} = 1 - \frac{k_l^* / m_l}{k_l / m_l} = 1 - \frac{\omega_j^{*2} \sum_{i=1}^n \frac{\phi_{ij}^*}{\Delta \phi_{ij}^*}}{\omega_j^2 \sum_{i=1}^n \frac{\phi_{ij}}{\Delta \phi_{ij}}} \quad (3)$$

where  $\omega_j$  is natural frequency and  $\phi_{ij}$  is mode shape of the  $j^{th}$  modal frequency.

### 2.3 Modal flexibility damage index method

Ko *et al.* [6] suggested modal flexibility damage index method. The principle of modal flexibility damage index method is based on the comparison of flexibility matrices obtained from two sets of experimental fundamental frequency and mode shape. The method is applicable if the mode shapes are mass normalized to unity which implies that the estimation of structural mass is required. The damage index for the  $l^{th}$  story using modal flexibility is defined as

$$DI_{MFDI} = 1 - \frac{F_l}{F_l^*} = 1 - \frac{\sum_{i=1}^n \phi_{li}^2 / \omega_i^2}{\sum_{i=1}^n \phi_{li}^{*2} / \omega_i^{*2}} \quad (4)$$

where  $F_l$  and  $F_l^*$  are the diagonal terms at coordinate  $l$  of the modal flexibility matrix of undamaged and damaged structure.

## 2.4 FRF damage index method

Maia *et al.* [7] presented damage index based on the assumption that the damage is located at the point where the change of an operational mode shape function is the greatest to the whole frequency ranges. This method calls FRF based mode shape method (*FRF\_MS*), which can be defined as

$$\Delta H_{lj}(\omega) = |H_{lj}^*(\omega) - H_{lj}(\omega)| \quad (5)$$

If more than one frequency and force are considered, the value of *FRF\_MS* is the sum from each frequency and force:

$$FRF\_MS_l = \sum_{\omega} \sum_j \Delta H_{lj}(\omega) \quad (6)$$

Using the above equation, the damage index based on *FRF\_MS* method is defined as

$$DI_{FRF\_MS_l} = \frac{\sum_{\omega} \sum_j \Delta H_{lj}(\omega)}{\sum_{\omega} \sum_j H_{lj}(\omega)} \quad (7)$$

where  $H_{lj}(\omega)$  and  $H_{lj}^*(\omega)$  are the value of FRF magnitude of undamaged and damaged structure at  $j^{\text{th}}$  excitation point and  $l^{\text{th}}$  measurement point.

## 3. Experimental Program

### 3.1 Material System

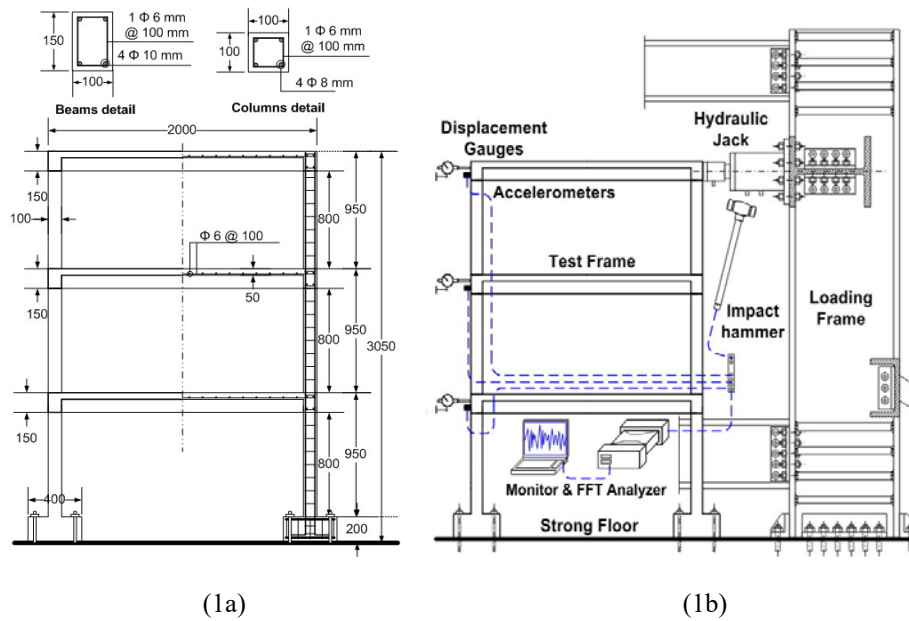
The concrete used in the RC frame model was grade M20 (20 MPa). The longitudinal steel reinforcements were deformed bars and steel quantity was provided as per provisions of Indian Standard of IS:456-2000. The steel bars exhibited yield strength of 415 MPa and a Young's modulus of 200 GPa. The glass fiber reinforced polymer (GFRP) was used. It had a tensile strength of 3.4 GPa, elastic modulus of 63 GPa and density of 2.6 g/cm<sup>3</sup>. The thickness of GFRP sheet used is 0.34 mm. Table 2 presents the details of the materials.

**Table 2** The details of the materials.

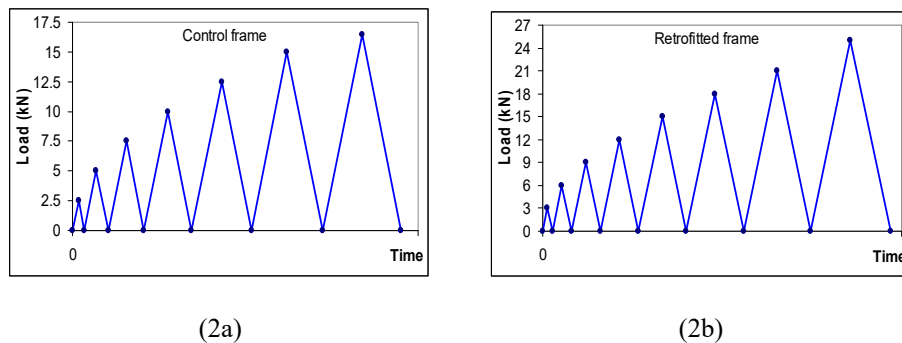
Compressive strength of concrete		20 MPa	
Tensile strength of steel bars (MPa)	Diameter	Yield	Ultimate
	10 mm	475.68 MPa	586.60 MPa
	8 mm	516.65 MPa	628.91 MPa
Mechanical properties of FRP laminate			
Fibre	Thickness	Density	Tensile
GFRP	0.34 mm	2.6 g/cm <sup>3</sup>	3.4 GPa
			$E$
			63 GPa

### 3.2 Specimen detail and testing procedure

A three story non-ductile RC frame was constructed in the laboratory. This frame built without beam-column joint transverse reinforcement bars. The schematic drawing of the frame and experimental test set-up are presented in Figure 1. The frame consisted of three slabs. Each column was equally sized with a square cross section of 100 mm x 100 mm (reinforced with four 8 mm diameter bars) with a floor-to-floor height of 950 mm. All the beams were equally sized rectangular with a cross section of 100 mm x 150 mm (reinforced with two 10 mm diameter bars on the tension and compression faces). All columns and beams were provided with 6 mm diameter stirrups at 100 mm spacing. Each column was cast integrally with stub foundation, which was in turn bolted firmly on the strong floor. Each floor was equipped with one displacement dial gauge in the horizontal direction. A hydraulic jack was horizontally installed along the desired direction at top floor. Quasi-static loads were applied at uniform pace rate to simulate structural damage as shown in Figure 2.



**Figure 1** Schematic drawing of control frame model (1a) and (1b) experimental test set-up.



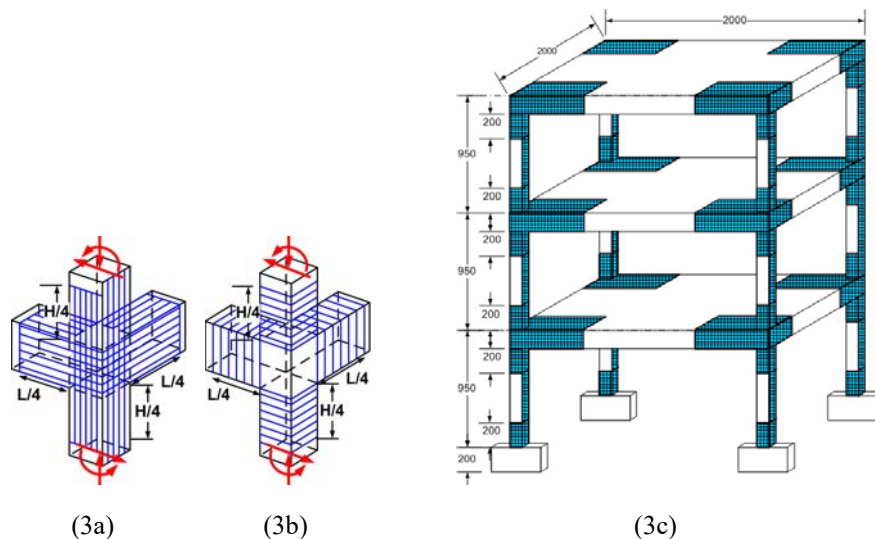
**Figure 2** Applied load history of control frame (2a) and (2b) retrofitted frame.

### 3.3 Impact hammer excitation test

Impact hammer excitation test is a useful tool because it gives information of the health of structure without destructing or affecting the facility when damage is hidden within the structure behind strengthening material layer. Each floor of experimental frame model was equipped with one accelerometer to measure vibration in horizontal direction. The accelerometer was mounted with a thin coating of epoxy on clean flat surface. Impact hammer with a hard rubber tip was used to excite the structure. Dynamic characteristic data of structure was automatically calculated and recorded by OROS software program with the help of fast Fourier transforms spectrum analyzer. The usual aim of vibration measurement is to predict response given force in different damage status. The specific frequencies at which resonance amplitudes occur are called the natural frequencies of the structure. These frequencies and the corresponding distribution of amplitude are global properties. Structural mode shapes are generalized from eigenvalue problem corresponding to natural frequencies.

## 4. Strengthening Scheme

As earlier explained, the horizontal load was applied to the top floor of the control frame model till the desired damage state was reached. The damaged frame was then moved back to its initial state. Loose concrete was removed and the surfaces were cleaned of dirt. All the corners of damaged elements were beveled and rounded to a radius of 10 mm. The cracks were filled with epoxy layer of MBrace primer and surface was smoothed by MBrace concessive. Application of FRP wrap on to the damaged structure and the schematic drawing of retrofitted frame are shown in Figure 3. FRP sheet has been applied in two steps: the first layer was provided with fiber oriented along the beam or column axes on the top and bottom surfaces to increase their flexural strength capacity, and on front and back surfaces to improve their shear strength capacity. The columns and beams were confined at each edge zones by wrapping the other layer in the transverse direction as well.

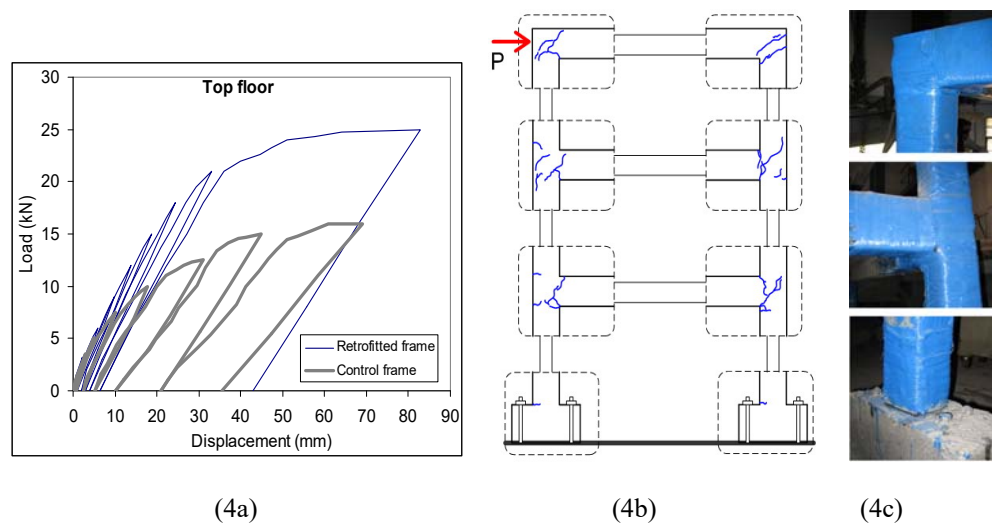


**Figure 3** Application of FRP wrap of flexural layers (3a) and confinement layers (3b) and (3c) schematic drawing of the retrofitted frame.

## 5. Results and Discussion

### 5.1 Load-displacement behaviour

The control frame model was constructed and tested under lateral quasi-static load. Loads were applied at the middle of top floor to simulate different level of controlled damage. The changes of lateral displacements with the increase of the number of loads are shown in Figure 4. The initial diagonal cracks occurred on the beam-column joints of the top floor at a load of 10 kN, indicating that structural elements of top floor were the most stressed. At a load of 12.5 kN, large cracks started to open and small cracks occurred on connection joints of second floor. It indicated that the yield point was visible at a load 12.5 kN, displacement of 31 mm from initial state. Ultimate damage state was at the load of 16.5 kN and a displacement of 69 mm. The ductility index was 2.22 and total energy dissipation was nearly 567.57 kN-mm.



**Figure 4** Lateral load-displacement plots at top floor (4a), failure mode of control frame (4b) and (4c) failure mode of retrofitted frame.

Test for the retrofitted frame was performed in the similar manner as that for the control frame. At the final stage of the damaged control frame was grouting cracks by adhesive epoxy and wrapped it with FRP sheets. The load versus displacement behaviour of retrofitted frame is shown in Figure 4 along with the behaviour for the control frame. The moving sounds of FRP sheets started from the load of 18 kN, displacement of 24.8 mm. The load displacement relation can be roughly considered to be linear when the load is smaller than or equal to the load of 18 kN. It might say that the yield damage state of the retrofitted frame was visible at a load of 18 kN. Ultimate damage state was at the load of 25 kN with displacement of 83 mm from initial state, the displacement ductility index was 3.34 and total energy dissipation was nearly 1039.68 kN-mm. There were breaking sounds of fiber and epoxy layers from connection joints at ultimate state. After remove FRP layers, it was observed that in addition to old cracks which opened up, new flexural cracks also appeared at the connection joints and columns.

Results from experimental test shows that the use of FRP wrapped for structural strengthening provides significant lateral load capacity increases approximately 151.5% as compared to control frame. The FRP wrapped around the structural elements in this manner are intended to provide external confinement and crushing of the concrete cover at larger lateral

displacements. The ductile behaviour of the strengthened frame is largely restored after the FRP composite sheets are engaged.

### 5.2 Failure mechanism

Failure mode at final state of tested frame is also shown in Figure 4. First diagonal cracks began at connection joints of top floor. These opening cracks increased when load was increasing. The second and third diagonal cracks occurred at beam-column connections of second and first floor respectively. It indicated that plastic hinges were started at top floor and the other hinges were formed at second and first floor respectively. The structure suddenly lost their load carrying capacity when cracks crossed from tension surface through compression surface of structural cross section elements. At final state, both diagonal and flexural cracks were occurred on connection joints and structural elements, indicating that the failure mode of control frame was combined between shear mode and flexural mode. Only flexural failure mode was observed at column near stub foundation. The lateral load carrying capacity of control frame was insufficient due to non-ductile reinforcement detailing, which included no beam-column joints transverse reinforcement. Therefore, the top and bottom bars moved in the opposite direction under combined forces. These forces were balanced by bond stress developed between concrete and steel bars. In such circumstances, the plastic hinges were formed by debonded bars and connection joints lost their capacity to carry load.

Debonding of FRP sheets were a failure mechanism for the retrofitted frame. After yield load, the flexural FRP layers delaminated along the interface between the concrete and FRP sheet. The delamination started at the corner of beam-column connection in tension zone. The first cracks with their moving sound were initiated at connection of top floor and the other cracks were formed at second floor and first floor respectively. After initial debonded FRP sheet formed, the retrofitted structure was stating to lose their load carrying capacity. The FRP sheets of confinement layer suddenly delaminated at ultimate load with their breaking sounds. Small cracks were occurred on epoxy resin of FRP layers at final state and huge cracks occurred inside existing RC elements behind strengthening layers. Again, the primary failure mechanism was debonding of the FRP sheets. It occurred at the base of huge cracks in tension zone of the existing damaged RC elements.

### 5.3 Dynamic characteristic results and damage indexes

After applied each load steps, the impact hammer was used to excite the testing frame model. The dynamic characteristics gave the records in OROS software program based on linear analysis setup. These records include trigger hammer plot, time history plot which give damping, frequency response function plot which give the amplitude of vibration along with frequency. The program was set up to make a free zoom measurement with a frequency range of 0 to 50 Hz. In this frequency range there covered all three majority modes of this frame model. The time history plots and comparison of FRF plots are shown in Figures 5 and 6. At undamaged state the natural frequency of first, second and third mode were 6.5 Hz, 19 Hz and 31.5 Hz. At initial cracks of 10 kN,  $DI_{Dip}$  was 0.12,  $DI_{SMR}$  was 0.16,  $DI_{MFDI}$  was 0.16 and  $DI_{FRF\_MS}$  was 0.43. At yield point, the natural frequency of first, second and third mode were 5.5 Hz, 17 Hz and 27.2 Hz.  $DI_{Dip}$  was 0.16,  $DI_{FRF\_MS}$  was 0.47,  $DI_{SMR}$  and  $DI_{MFDI}$  were 0.21, severe damage state. At ultimate damage state,  $DI_{Dip}$  was 0.34,  $DI_{SMR}$  was 0.36,  $DI_{MFDI}$  was 0.34 and  $DI_{FRF\_MS}$  was 0.57, respectively. The frequencies at ultimate state of first, second and third mode were 5.2 Hz, 15.5 Hz and 25.9 Hz. The average changed of corresponding resonant frequency decreased approximately 18.73%. The summary of damage indexes and appearances of control frame are presented in Table 3.

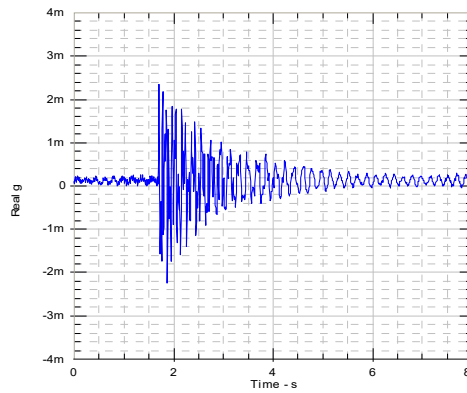
The relation between applied loads and damage indexes based on modal parameters of control and retrofitted frame are also shown in Figure 6. In order to estimate the dynamic properties of the frame under the damaged state, the measurements data after applied load are extracted to represent the behaviour of structure. It is clearly shown that the value of  $DI_{FRF\_MS}$  is



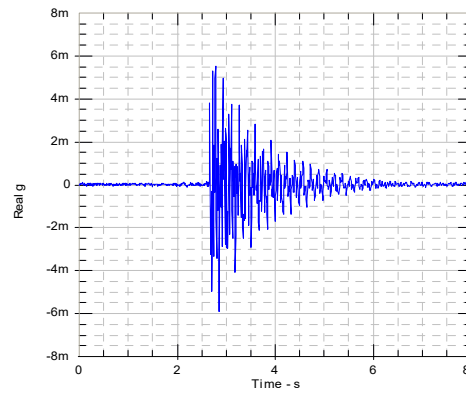
larger than  $DI_{Dip}$ ,  $DI_{SMR}$  and  $DI_{MFDI}$ , indicating that damage indexes based on the change in FRFs show a much acceptable accuracy correlation with damage indexes based on the change in natural frequency and mode shape in general. However, it can be seen that  $DI_{Dip}$ ,  $DI_{SMR}$  and  $DI_{MFDI}$  are small which agree with the fact that no visible damage was reported at load less than yield point. Meanwhile, some index values around 0.25 are obtained. The health of control frame is said to be of great concern when  $DI_{FRF\_MS}$  increases to larger than 0.5. On the other hand, the damaged structure seems to be unsafe when  $DI_{Dip}$ ,  $DI_{SMR}$  and  $DI_{MFDI}$  increase larger than 0.25 or the structure experienced severe damage.

**Table 3** Damage indexes and appearances of control frame.

$P, \text{kN}$	$P/P_{max}$	$DI_{Dip}$	$DI_{SMR}$	$DI_{MFDI}$	$DI_{FRF\_MS}$	Appearance
0	0	0	0	0	0	Un-deformed
5.0	0.30	0.03	0.04	0.04	0.23	Un-cracked
7.5	0.45	0.06	0.09	0.09	0.36	Un-cracked
10.0	0.61	0.12	0.16	0.16	0.43	Minor cracking
12.5	0.76	0.16	0.21	0.21	0.47	Severe cracking
15.0	0.91	0.25	0.27	0.26	0.51	Spalling of concrete cover
16.5	1	0.34	0.36	0.34	0.57	Loss of shear capacity

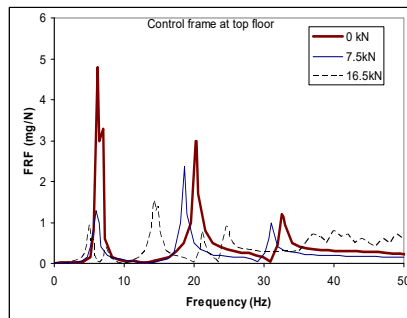


(5a)

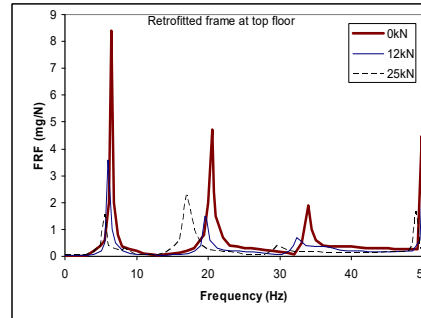


(5b)

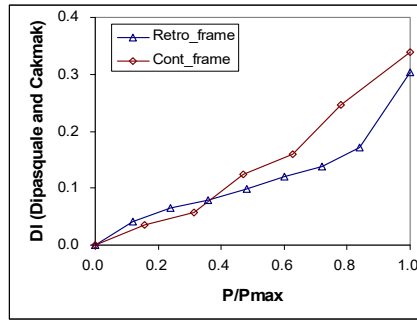
**Figure 5** Time history plot at initial state of control frame (5a) and (5b) retrofitted frame.



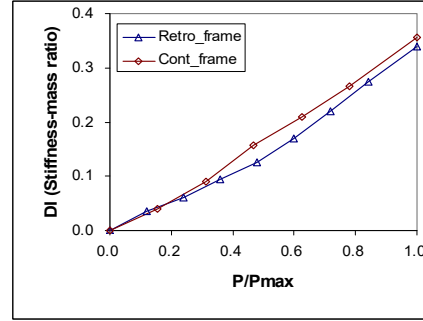
(6a)



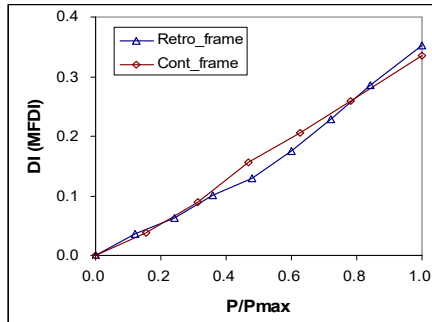
(6b)



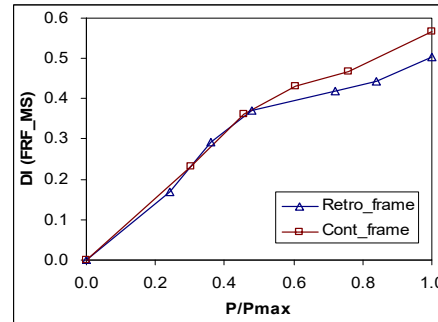
(6c)



(6d)



(6e)



(6f)

**Figure 6** FRF plots at top floor of control frame (6a) and (6b) retrofitted frame. Modal plastic softening index plot (6c) and (6d) Stiffness-mass ratio index plot. Modal flexibility damage index plot (6e) and (6f) FRF based mode shape index plot.

At initial state the natural frequency of first, second and third mode were 6.7 Hz, 20.7 Hz and 34.0 Hz, respectively. At the load of 18 kN, the frequencies of first, second and third mode were 6.1 Hz, 19.3 Hz and 32 Hz respectively. The yield damage state of the strengthened frame was visible at a load 18 kN,  $DI_{Dip}$  was 0.14,  $DI_{SMR}$  was 0.22,  $DI_{MFDI}$  was 0.23 and  $DI_{FRF\_MS}$  was 0.42, severe damage state. At ultimate damage state, the frequencies of first, second and third mode were 5.5 Hz, 17.2 Hz and 29 Hz respectively. The damage index of  $DI_{Dip}$  was 0.30,  $DI_{SMR}$  was 0.34,  $DI_{MFDI}$  was 0.35 and  $DI_{FRF\_MS}$  was 0.50. The average changed of corresponding resonance frequency decreased approximately 16.51%. The time history plot of retrofitted frame indicates that the stiffness of the damaged control frame is regained significantly by wrapping FRP but even then it is not able to bridge the cracks fully.

The summary of damage indexes and appearances of retrofitted frame are presented in Table 4. From Tables 3 and 4, it is shown that the value of  $DI_{SMR}$  and  $DI_{MFDI}$  are larger than  $DI_{Dip}$  for both control and retrofitted frame, indicating that the stiffness-mass ratio method and modal flexibility damage method are more reliable methods than the modal plastic softening damage index. However, damage indexes based on the change in FRF is more accurate method to estimate damage of structure as compare to the other methods in this study. Moreover, the results of damage indexes of retrofitted frame below the curves of control frame indicating better performance. In the present work we found that detecting structural damage using information contained in vibration signatures is a useful technique to monitor health of structure without destructing when damage is hidden within the structure behind strengthening material layers.

**Table 4** Damage indexes and appearances of retrofitted frame.

$P, \text{kN}$	$P/P_{max}$	$DI_{Dip}$	$DI_{SMR}$	$DI_{MFDI}$	$DI_{FRF\_MS}$	Appearance
0	0	0	0	0	0	Un-deformed
3.0	0.12	0.04	0.04	0.04	0.17	Un-cracked
9.0	0.36	0.08	0.10	0.10	0.29	Un-cracked
15.0	0.60	0.12	0.17	0.18	0.37	Un-cracked
18.0	0.72	0.14	0.22	0.23	0.42	Noise of fiber moving
21.0	0.84	0.17	0.28	0.29	0.44	Breaking noise of fiber
25.0	1	0.30	0.34	0.35	0.50	Loss of shear capacity

## 6. Conclusions

Based on the performed research investigation, the following main conclusions can be drawn: 1) Experimental results approve that the use of FRP wrapped for structural retrofitting provides increased significant lateral load capacity and ductile behaviour. 2) Damage index based on the change in FRF shows a much acceptable accuracy correlation with damage based on the change in frequency and mode shape. 3) The damage indexes based on the change in dynamic characteristics of retrofitted frame reduce indicating better performance as compare to the control frame. 4) Although the structural stiffness of the damaged frame is regained significantly by wrapping FRP jacket but it is not able to bridge the cracks fully.

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