LOWER BOUND OF AGV SCHEDULING PROBLEM WITH ALTERNATIVE PICK UP AND DELIVERY NODES BY BENDERS DECOMPOSITION APPROACH

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ABSTRACT

The original single/multi Automated Guided Vehicle (AGV) scheduling problem with a specific pick up and delivery node can be formulated and solved as single/multi traveling salesman problem (TSP/MTSP). When the original AGV problem is modified to capture the special network structure that is the network which has alternative for some node, the problem becomes the AGV scheduling problem with alternative pick up and delivery nodes. TSP/MTSP with alternative nodes will be considered for finding the solution of this kind of AGV scheduling problem with alternative pick up and delivery nodes. The purpose of this paper is to provide the mathematical model of AGV scheduling problem with alternative pick up and delivery nodes which is TSP/MTSP with alternative nodes, the lower bound model which is the assignment problem with alternative nodes, and Benders decomposition approach for solving this model. The Benders decomposition approach is verified and tested by using Excel Solver to determine the lower bound of some simulated example of AGV scheduling problem with alternative pick up and delivery nodes.

KEYWORDS: AGV scheduling problem, Benders decomposition, Traveling salesman problem, Integer linear programming, and Alternative nodes

1. INTRODUTION

Job scheduling and sequencing is an important part of any kind of vehicle routing design problem, include an Automated Guided Vehicle (AGV) system design. Designing an AGV system is a complex task. One of the main purposes of scheduling problem for single/multi AGV concerns about how the scheduling can provide the minimum of total traveling distance of AGV. Normally the scheduling problem have been considered or designed with the routing problem concomitantly for any kind of vehicle system management, Laporte (1997). The ordinary vehicle scheduling and routing problem as single/multi AGV scheduling problem is the problem with single specific pick up and delivery node that can be simulated by some network problem approach such as single/multi Traveling Salesman Problem (TSP/MTSP). Dantzig, Fulkerson and Johnson (1954) proposed that determining the optimal of TSP for large number of nodes requires too much time so that many papers propose the heuristic algorithm for finding the AGV scheduling and traveling path. As NP-hard nature of original TSP, the vehicle management problem with alternative pick up and delivery nodes may be considered as the class of NP-hard problem also when the problem structure fall into the TSP category, Chartrand and Oellermann (1993). According to this point, the potential problem for studying the single/multi AGV scheduling problem is extended to be more realistic problem that the original TSP problem is modified by adding the structure of alternative pick up and delivery nodes. The main purpose is to find the scheduling of AGV problem with alternative nodes. This AGV problem is presented in section 2.1.

The original TSP/MTSP is one of the applications of network problems, it is necessary to choose a sequence of nodes to visit so as to accomplish a specified objective. TSP/MTSP is a network problem that given a network and a cost (or distance) associated with each arc, it is necessary to start from a specified originating or depot node, visit each and every other node exactly one, and return to the starting node in the least cost manner. For example, a bus that leaves the school yard must stop at

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various locations once to pick up students and ultimately return to the school yard in the shortest possible time. As another example, we consider the AGV system that can start from a specified originating or depot node, visit each and every other node, which has some alternative for selection to visit only one, exactly one, and return to the starting node in the least distance. The TSP/MTSP is presented in section 2.2.

The mathematical models of TSP/MTSP are formulated in form of integer linear programming which is 0-1 integer programming. Many applications of linear programming that are applied to formulate the real world problem can be formed as integer, mixed integer, or 0-1 integer programming. There are so many approaches for solving this kind of linear programming. Úsing linear programming duality theory, it is possible to show that any mixed integer program (MIP) can be written as an integer programming. This mean it may be worth solving a MIP by solving its equivalent integer problem. J. F. Benders (1962) proposed a technique in which the equivalent integer program is solved after generating only a subset of its constraints. The Benders decomposition procedure for formulating the MIP into an integer programming is presented in section 2.3.

The TSP/MTSP can be solved for determining the scheduling of normal vehicle routing problem, but we have to modify the original TSP/MTSP to support the AGV problem with alternative pick up and delivery nodes. The concept of TSP/MTSP will be applied with some generated technique of assignment problem to solve the AGV scheduling problem with alternative pick up and delivery nodes for determining the minimum traveling distance of each AGV from depot to some appropriate selected nodes and then come back to depot. This procedure based on the branch and bound with solving assignment sub problem for searching the optimal. The formulated mathematical model is presented in section 2.4.

The assignment problem with alternative node which is the lower bound of the AGV problem with alternative nodes is considered as an important part of this paper. We will describe the assignment model and the solving approach by applying the Benders decomposition concept for finding the lower bound of the AGV problem with alternative pick up and delivery nodes in section 2.5. The ordinary assignment problem is 0-1 integer linear programming. By the unimodular property of network problem, we can solve the assignment problem as a regular linear programming without concerning of integer constrain. The result is an integer solution automatically. When the alternative nodes constraint is added to the system, the problem will lose the unimodular property. The new 0-1 integer programming model of assignment problem with alternative and the applied Benders decomposition solving approach are created. We test implementation of the generated model by formulated the Excel Solver. Finally, the result and conclusion are presented in section 3.

2. MATERIAL AND METHOD

2.1 AGV Scheduling Problem

Productivity and flexibility, which are the primary goals of today's automation technology, can only be achieved in fully integrated manufacturing environments. A carefully designed and efficiently managed material handling system is an important part. Tanchoco and Moodie (1987) proposed the study of AGV system. Automated guided vehicles (AGV) are among the fastest growing classes of equipment in the material handling system of industry. Blair, Charnsethikul and Vasques (1987) modeled the optimum routing problem of AGV between the workstations as TSP. An algorithm for the near optimal routing of AGV in such a system is presented which seeks to organize material moves into tours with the objective of minimizing the maximum tour length.

The original AGV problem can be transformed to TSP/MTSP for solving the special situation that the jobs of AGV have the alternative nodes. The TSP/MTSP solution can be generated by many approach, but the branch and bound approach with solving assignment sub problem is considered, Ahuja, Magnanti and Orlin (1993). Suppose some part of the same example list of jobs that one AGV is used to complete all jobs is showed as following.

Job No.	*Pick up Department*	*Delivery Department*
1	B	C
2	A	I
3	B	H or G or I
4	G	C
5	D	E
6	D or H	F

Table 1: The example of a part of job list for one AGV

The table of TSP is consists of the distance of the AGV that move from the starting point of previous job to the starting point of next job. So the TSP distance table for the AGV problem is asymmetric distance table. Suppose the table of the previous AGV job list that is the distance from the considered job to the others in form of TSP is showed as following.

T F	job#	#1	#2	#3			#4	#5	#6	
Job#	Alternative (job i, alt. a)	1.1	2.1	3.1	3.2	3.3	4.1	5.1	6.1	6.2
#1	1.1	-	3	2	2	2	5	4	2	4
#2	2.1	7	-	7	7	7	6	7	7	7
#3	3.1	2	3	-	-	-	3	2	2	2
	3.2	6	5	-	-	-	3	4	6	4
	3.3	6	7	-	-	-	5	6	6	6
#4	4.1	5	6	5	5	5	-	7	5	7
#5	5.1	2	3	2	2	2	3	-	2	2
#6	6.1	4	7	4	4	4	5	4	-	-
	6.2	3	5	7	4	2	6	7	-	-

Table 2: The cost matrix $\mathbf{c_{ij}}$ of the AGV problem with alternative nodes in form of TSP

2.2 TSP Mathematical Model

When we consider mathematical formulation of the routing problem, there are so many kind of mathematical model for many kind of problem, Chartrand and Oellermann (1993). For example, vehicle routing problem (VRP), the Chinese postman problem (CPP), and Traveling salesman problem (TSP) is the focused model. Lawler, Lenstra, Kan and Shmoys (1995) proposed the survey of general TSP. TSP/MTSP can be formulated as integer programming. Orman and Williams (2004) presented many formulations of the TSP as an integer programming such as the conventional formulation that is presented by Dantzig, Fulkerson and Johnson (1954) and the others sequential formulation such as Miller, Tucker and Zemilin (1960), Gavish and Graves (1978), Finke, Claus and Gunn (1983) and etc. We try to analysis the structure of mathematical model of TSP with structure of assignment sub problem. Consider the existing mathematical model as follows:

- <u>VRP</u>

$$\text{Minimize} \sum_{j} c_j \cdot x_j$$

Subject to:

$$(\text{SP}) \sum_{j} a_j \cdot x_j = e$$

X is binary for all j

The Miller-Tucker-Zemlin (1960) ***formulation of classical TSP*** is given as following:

- <u>TSP</u>

$$MinZ = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ji} \qquad (1)$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1, 2, \ldots, n \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, 2, \ldots, n \qquad (3)$$

$$y_i - y_j + n x_{ij} \le n - 1 \qquad \forall i \ne j \qquad (4)$$

$$x_{ij} = 0 or 1 \qquad \forall i, j. \qquad (5)$$

The number of cities is n, the distances are c_{ij} and the arcs in the tour are represented by the variable x_{ij}. The c_{ij} is the distance from city i to j ($c_{ij} = \alpha$ for i = j). x_{ij} is 1 if the salesman travels from city i to j and 0 otherwise. The variables y_i are arbitrary real numbers which satisfy the constrain (4). Bellmore and Hong (1974) proposed the transformation of MTSP for m-salesman to the classical TSP by adding m-1 dummy to the original network as the artificial starting node and solved MTSP from solving TSP of the modified network. The mathematical formulation of MTSP can be formed by applying the transformation idea to The Miller-Tucker-Zemlin formulation.

Svestka and Huckfeldt (1973) gave ***the multi traveling salesman (MTSP) formulation for m salesman as following.***

$$MinZ = \sum_{i=1}^{r}\sum_{j=1}^{r} d_{ij} x_{ji} \qquad ; r = n + m - 1 \qquad (1)$$

Subject to

$$\sum_{i=1}^{r} x_{ij} = 1, \qquad j = 1, 2, \ldots, r \qquad (2)$$

$$\sum_{j=1}^{r} x_{ij} = 1, \qquad i = 1, 2, \ldots, r \qquad (3)$$

$$y_i - y_j + (n+m-1) x_{ij} \le n + m - 2 \qquad \forall i \ne j \qquad (4)$$

$$x_{ij} = 0 or 1 \qquad \forall i, j. \qquad (5)$$

The new distances d_{ij} are defined from the original distance c_{ij} as following: Augment the matrix [c_{ij}] with m-1 new rows and columns, where each new row and column is a duplicate of the first row and column of the matrix [c_{ij}]. It is assumed that the first row and column correspond to the home city. Set all other new element of the augment matrix to infinity. All other terms have the same definition as previous model.

2.3 Benders Decomposition

J. F. Benders (1962) proposed a technique in which the mixed integer program (MIP) can be written as an integer program. The equivalent integer program is solved after generating only a subset of its constraints. That is, the remaining "implicitly enumerated" constrains do not further constrain the integer program. The Benders decomposition procedure partitions the mixed integer program into an integer and a linear program, consisting respectively of the integer and the continuous variable of the original problem. The decomposition algorithm works by successive solving a linear program and an integer program. The linear program produces the extreme point and a single constraint for the integer program. Also, the value of the linear programming optimal solution gives an upper bound for the optimal solution to the mixed integer program. When solved, the integer program, which is the mixed integer program's equivalent when it has all its constraints, yields a nondecreasing lower bound. When the two bounds coincide, the optimal mixed integer solution has been found and the process terminates. Consider the mixed integer program as following.

[MIP] Minimize $cx + dy$

Subject to $\mathbf{A}x + \mathbf{B}y \ge b$,

$x \ge 0$, $y \ge 0$.

and y is integer or binary.

where **A** is an *m* by *n* matrix, **B** is an *m* by *n'* matrix.

A key concept in Benders' algorithm is that of partitioning the variables into two sets (x and y) and projecting the problem onto the y variables. If we let **Y** denote the set of all feasible nonnegative integer vectors y, then MIP may be written as following.

Let $v(y) = dy + \min \{cx \mid \mathbf{A}x \ge b - \mathbf{B}y, x \ge 0\}$

The original problem is clearly seen to be equivalent to:

Minimize $v(y) = dy + \min \{cx \mid \mathbf{A}x \geq b - \mathbf{B}y, x \geq 0\}$

Subject to $y \in \mathbf{Y}$

For a fixed y, the minimization problem is the linear program

[L] Minimize cx

Subject to $\mathbf{A}x \geq b - \mathbf{B}y,$

$x \geq 0$

its dual is

[DL] Maximize $(b - \mathbf{B}y)^T u$

Subject to $\mathbf{A}^T u \leq c,$

$u \geq 0$

In principle it is possible to identify and enumerate all of the extreme points of the dual feasible region and choose the best. That is, we can evaluate the function v (y) by: $v(y) = dy + \text{maximize}_{1 \leq j \leq P} \{(b - \mathbf{B}y)^T u^{\wedge j}\}$, or

$$v(y) = \text{maximize}_{1 \leq j \leq P} \{\alpha^{\wedge j} y + \beta^{\wedge j}\}$$

where $\alpha^{\wedge j} = [u^{\wedge j}]^T \mathbf{B} + d,\ \beta^{\wedge j} = b^T u^{\wedge j}$

However, the v (y) is to be evaluated by solving a linear programming, not by identifying all of the dual extreme points and computing the corresponding linear objective function of y. if k supports are used (where k < P), we get an *underestimate* of v (y) :$v^{\wedge}_k(y) = \text{maximize}_{1 \leq j \leq k} \{\alpha^{\wedge j} y + \beta^{\wedge j}\}$

Benders' Decomposition Algorithm.

Step 0 : Initialization: Select arbitrary $y' \in \mathbf{Y}$, $k \leftarrow 0$

Step 1 : Solve Subproblem: Evaluate v (y') by solving LP with fixed y'

Step 2 : Stopping Criterion: If $v^{\wedge}_k(y') = v(y')$ then stop, otherwise go to step 3

Step 3 : Improve Approximation: by using dual extreme point to generate new support, $k \leftarrow k+1$

Step 4 : Solve Partial Master Problem: that is minimizing $v^{\wedge}_k(y)$ st. $y \in \mathbf{Y}$, go to step 1.

2.4 Problem Formulation

From the previous original TSP/MTSP mathematical math model, we do the analysis and create the model with alternative node as following when n = number of job and m = number of AGV.

$$MinZ = \sum_{iajb} x_{iajb} d_{iajb} \qquad i, j = 1, 2, \ldots, r \quad (r = n + m - 1) \qquad (1)$$

Subject to

$$\sum_{iab} x_{iajb} = 1, \qquad \forall j = 1, 2, \ldots, r \qquad (2)$$

$$\sum_{jab} x_{iajb} = 1, \qquad \forall i = 1, 2, \ldots, r \qquad (3)$$

$$y_i - y_j + r x_{iajb} \leq r - 1 \qquad \forall i \neq j \qquad (4)$$

$x_{iajb} = 0 \text{ or } 1 \quad ; \forall i, a, j, b$

$a, b \in I^+ \ ; a, b > 0$

x_{iajb} = 1 : If one AGV travel from *job **i** with alternative **a** to job **j** with alternative **b**.*

0 : otherwise

d_{iajb} represent the distance from pick up node of *job **I** with alternative **a** (i.a)* (Through the path to delivery node of this job + empty travel distance to the pick up node of the next job, which is job j) to the pick up node of *job **j** with alternative **b** (j.b)*

m = 1 when we consider single AGV

This mathematical model is a TSP/MTSP with alternative node that can simulate the model of single/multi AGV scheduling problem with alternative pick up and delivery nodes. When we relax constrain (4) that is the subtour elimination constrain and consider the single AGV case, this problem looks like the assignment problem, but there are the alternative nodes for each job. This assignment problem with alternative nodes is the 0-1integer programming that is the relaxation of TSP/MTSP with alternative nodes. For solving TSP/MTSP with alternative nodes, we have to apply the branch and bound approach with solving the assignment problem with alternative nodes as a sub problem of each branching. The solution from solving this assignment problem provides the lower bound of AGV scheduling problem with alternative pick up and delivery nodes. Let consider the following assignment problem model.

2.5 The lower bound of AGV scheduling problem with alternative nodes

The mathematical model of assignment problem with alternative nodes is presented as follows:

$$MinZ = \sum_{iajb} x_{iajb} d_{iajb} \qquad i, j = 1, 2, \ldots, n \qquad (1)$$

Subject to

$$\sum_{iab} x_{iajb} = 1, \qquad \forall j = 1, 2, \ldots, n \qquad (2)$$

$$\sum_{jab} x_{iajb} = 1, \qquad \forall i = 1, 2, \ldots, n \qquad (3)$$

$$x_{iajb} = 0 \; or \; 1 \quad ; \forall i, a, j, b$$

The above assignment problem with alternative nodes can be modified to be the simpler mathematical model as following.

$$MinZ = \sum_{ij} x_{ij} d_{ij} \qquad i, j = 1, 2, \ldots, n \qquad (1)$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = Z_j, \qquad \forall j = 1, 2, \ldots, n \qquad (2)$$

$$\sum_{j=1}^{n} x_{ij} = Z_i, \qquad \forall i = 1, 2, \ldots, n \qquad (3)$$

$$\sum_{i \in S_K} Z_i = 1 \qquad K = 1, 2, \ldots, m \qquad (4)$$

$$Z_i - Z_j = 0 \qquad \forall i = j \qquad (5)$$

$$x_{ij} = 0 or 1 \qquad \forall i, j$$

Set S_K is the set of any job #i that consist with all alternative of these job. For example, we can define set S_K of table 1 (n = 9, m = 6) that are $S_1 = \{1\}$, $S_2 = \{2\}$, $S_3 = \{3, 4, 5\}$, $S_4 = \{6\}$, $S_5 = \{7\}$, $S_6 = \{8, 9\}$. Now we can solve the assignment problem with alternative nodes by integer linear programming (IP) model which can be formulated in Excel Solver for finding the lower bound of AGV scheduling problem with alternative pick up and delivery nodes.

Suppose the given network can be express in form of matrix as follows:

T F		job#	#1	#2	#3			#4	#5	#6	
	No.		1	2	3	4	5	6	7	8	9
Job#		Alternative (job i, alt. a)	1.1	2.1	3.1	3.2	3.3	4.1	5.1	6.1	6.2
#1	1	1.1	-	3	2	2	2	5	4	2	4
#2	2	2.1	7	-	7	7	7	6	7	7	7
#3	3	3.1	2	3	-	-	-	3	2	2	2
	4	3.2	6	5	-	-	-	3	4	6	4
	5	3.3	6	7	-	-	-	5	6	6	6
#4	6	4.1	5	6	5	5	5	-	7	5	7
#5	7	5.1	2	3	2	2	2	3	-	2	2
#6	8	6.1	4	7	4	4	4	5	4	-	-
	9	6.2	3	5	7	4	2	6	7	-	-

Table 3: The cost matrix $\mathbf{c}_{ij}$ of the assignment problem with alternative nodes

We consider the Benders' decomposition of MIP from section 2.3. We can write our lower bound model in similar form of Benders' decomposition of MIP. The lower bound of AGV scheduling problem with alternative pick up and delivery nodes for this simulated example can be written as following.

$$MinZ = \sum_{ij} x_{ij} d_{ij} \qquad i, j = 1, 2, \ldots, n$$

Subject to

$$\left(\sum_{i=1}^{n} x_{ij} = Z_i, \quad \forall j = 1, 2, \ldots, n \right)$$

$$\sum_{i=1}^{9} x_{i1} = 1,$$

$$\sum_{i=1}^{9} x_{i2} = 1$$

$$\sum_{i=1}^{9} x_{i3} = Z_1$$

$$\sum_{i=1}^{9} x_{i4} = Z_2$$

$$\sum_{i=1}^{9} x_{i5} = Z_3$$

$$\sum_{i=1}^{9} x_{i6} = 1$$

$$\sum_{i=1}^{9} x_{i7} = 1$$

$$\sum_{i=1}^{9} x_{i8} = Z_4$$

$$\sum_{i=1}^{9} x_{i9} = Z_5$$

$$\left(\sum_{j=1}^{n} x_{i,j} = Z_i \qquad \forall i = 1, 2, \ldots, n\right)$$

$$\sum_{j=1}^{9} x_{1,j} = 1$$

$$\sum_{j=1}^{9} x_{2,j} = 1$$

$$\sum_{j=1}^{9} x_{3,j} = Z_1$$

$$\sum_{j=1}^{9} x_{4,j} = Z_2$$

$$\sum_{j=1}^{9} x_{5,j} = Z_3$$

$$\sum_{j=1}^{9} x_{6,j} = 1$$

$$\sum_{j=1}^{9} x_{7,j} = 1$$

$$\sum_{j=1}^{9} x_{8,j} = Z_4$$

$$\sum_{j=1}^{9} x_{9,j} = Z_5$$

$$Z_1 + Z_2 + Z_3 = 1$$

$$Z_4 + Z_5 = 1$$

$$x_{i,j} \in X = \{0,1\}$$

$$Z_j \in Z = \{0,1\}$$

Benders' algorithm can be applied by partitioning the variables into two sets which are X and Z and projecting the problem onto the Z variables. If we fix Z to be 1 which mean the alternative is selected or 0 otherwise, the problem become the original assignment problem (the constraints that have Z=0 can be ignore). Variables x that are 0-1 integer can be ignore to become $x \geq 0$, because of the unimodularlity of assignment problem. We let **Z** denote the set of all feasible 0-1 integer vectors Z, then MIP may be written as following.

Let $v(z) = cz + \min \{dx \mid \mathbf{A}x \geq b - \mathbf{B}z, x \geq 0\}$

The original problem is clearly seen to be equivalent to:

Minimize $v(z) = cz + \min \{dx \mid \mathbf{A}x \geq b - \mathbf{B}z, x \geq 0\}$
Subject to $z \in \mathbf{Z} = \{0 \text{ or } 1\}$

Now we can apply Benders' decomposition approach to our formulated lower bound model of AGV problem scheduling with alternative nodes. We do not need to solve this problem on IP because Benders' approach can solve this problem on LP which is the original assignment problem after we fix z and apply Benders' algorithm. Let consider the previous example of the given network problem with alternative nodes following.

3. RESULTS AND DISCUSSION

If we need to solve the assignment problem of this example, the key question of the above problem is "How can we select the best alternatives that can provide the minimum assignment cost?" We already known that the assignment problem can be solved efficiently by "Hungarain Method" but this method can not be applied directly to solve this special structure. We implement the mathematical model of assignment problem with alternative nodes by modeling this example network in Excel Solver. The model is a linear integer programming that the Excel Solver can provide the solution which is the lower bound of AGV scheduling problem with alternative pick up and delivery nodes. Now we can solve the assignment problem with alternative nodes as linear programming (LP) or integer programming (IP) by specific $x_{ij} \geq 0$ or x_{ij} is 0 or 1. If the solutions from Solver with constraint of $x_{ij} \geq 0$ are integer, not fractional number, this problem can be solved easier than using IP. If not, we have to add the binary constrain (x_{ij} is 0 or 1) to the model and solve the problem as IP.

After we implement this model by using Solver, we found that some Cost matrix gave the solutions of xij are fractional number. So we can not solve this assignment model by LP ($x_{ij} \geq 0$). Now we can conclude that the sub problem of alternative selection can be solve by this model as 0-1 IP. The solution of this example by Excel Solver is shown as follows:

T F	job#	#1	#2	#3			#4	#5	#6		
Job#	Alternative (job i, alt. a)	1.1	2.1	3.1	3.2	3.3	4.1	5.1	6.1	6.2	
#1	1.1	-	0	1	0	0	0	0	0	0	1
#2	2.1	0	-	0	0	0	1	0	0	0	1
#3	3.1	0	0	-	-	-	0	1	0	0	1
	3.2	0	0	-	-	-	0	0	0	0	
	3.3	0	0	-	-	-	0	0	0	0	
#4	4.1	0	1	0	0	0	-	0	0	0	1
#5	5.1	0	0	0	0	0	0	-	0	1	1
#6	6.1	0	0	0	0	0	0	0	-	-	1
	6.2	1	0	0	0	0	0	0	-	-	
		1	1	1			1	1	1		

Table 4: The matrix $\mathbf{x_{ij}}$ =1of the assignment problem

From the Excel Solver solution, the assignment is:
1 – 3.1, 2 - 4, 3.1 - 5, 4 – 2, 5 – 6.2 and 6.2 - 1
with the solution value of minimum total distances 21 units. Alternative 3.1 and 6.2 are selected.

When we consider the Benders' algorithm for solving the example, let we define: $v(Z_1, Z_2, Z_3, Z_4, Z_5) = \text{maximize}_{1\leq j\leq 18} \{(b - \mathbf{B}z)^T u^{\wedge j} \mid \mathbf{A}^T u \leq c, u \geq 0\}$.

The dual problem of this example consists of 18 variables of u and 81 constraints. We can write the minimize function $v(Z_1, Z_2, Z_3, Z_4, Z_5)$ and apply Benders' algorithm as following.

$$\text{Min } v(Z_1, Z_2, Z_3, Z_4, Z_5) = \max \{(u_1 + u_2 + Z_1u_3 + Z_2u_4 + Z_3u_5 + u_6 + u_7 + Z_4u_8 + Z_5u_9 + u_{10} + u_{11} + Z_1u_{12} + Z_2u_{13} + Z_3u_{14} + u_{15} + u_{16} + Z_4u_{17} + Z_5u_{18}) \mid \mathbf{A}^T u^{\wedge j} \leq c, u \geq 0\}$$

Subject to $Z \in \mathbf{Z} = \{0 \text{ or } 1\}$

$Z_1 + Z_2 + Z_3 = 1$ and $Z_4 + Z_5 = 1$

Benders' Algorithm

- ***Iteration 1***

<u>*Step 0*</u>: Initialization:

Let select $v(Z_1, Z_2, Z_3, Z_4, Z_5) = v(0, 0, 1, 0, 1)$ and $k \leftarrow 0$.

<u>*Step 1*</u>: Solve Subproblem:

Subproblem maximize $_{1\leq j\leq P} \{(b - \mathbf{B}z)^T u^{\wedge j} \mid \mathbf{A}^T u^{\wedge j} \leq c, u \geq 0\}$ is solved. The maximum occurs at the extreme point $(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18})$ = 0, 5, 0, 2, 4, 3, 0, 1, 0, 2, 3, 2, 2, 2, 1, 2, 2, 2 and maximum value is 24

<u>*Step 2*</u>: Stopping Criterion:

We set If $v^{\wedge}_0(Z) = v(Z) = 24$ then go to step 3

<u>*Step 3*</u>: Improve Approximation and solving Partial Master Problem:

By using dual extreme point to generate new support, $k \leftarrow k+1$. Initial approximation function for v is $v^{\wedge}_1(Z) = u^{\wedge 1} + u^{\wedge 2} + Z_1u^{\wedge 3} + \ldots + Z_5u^{\wedge 18}$.

<u>*Step 4*</u>: Solve Partial Master Problem: that is minimizing $v^{\wedge}_k(Z)$ st. $z \in \mathbf{Z}$, go to step 1.

Partial Master Problem that is:

$$\text{Min } v^{\wedge}_1(Z) = u^{\wedge 1} + u^{\wedge 2} + Z_1u^{\wedge 3} + \ldots + Z_5u^{\wedge 18}$$
$$= 0 + 5 + 0Z_1 + 2Z_2 + 4Z_3 + \ldots + 2Z_5$$

St. $Z \in \mathbf{Z} = \{0 \text{ or } 1\}$

$Z_1 + Z_2 + Z_3 = 1$ and $Z_4 + Z_5 = 1$

The minimum occurs at $Z^*_1 = (1, 0, 0, 0, 1)$ and the minimum value is 20.

- ***Iteration 2***

Step 1: Solve Subproblem:

Let select $v(Z_1, Z_2, Z_3, Z_4, Z_5) = v(1, 0, 0, 0, 1)$ and solve subproblem maximize $_{1 \le j \le P} \{(b - \mathbf{B}z)^T u^{\wedge j} \mid \mathbf{A}^T u^{\wedge j} \le c, u \ge 0\}$ is solved. The maximum occurs at the extreme point $(u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}) = 0, 5, 0, 1, 1, 3, 0, 1, 1, 2, 3, 2, 2, 1, 1, 2, 2, 2$ and maximum value is 21

Step 2: Stopping Criterion:

We setIf $v^\wedge_2(Z) = 21 > v^\wedge_1(Z) = 20$, not terminate, then go to step 3

Step 3: Improve Approximation and solving Partial Master Problem:

By using dual extreme point to generate new support, $k \leftarrow k+1$. Initial approximation function for v is $v^\wedge_3(Z) = u^{\wedge 1} + u^{\wedge 2} + Z_1 u^{\wedge 3} + \ldots + Z_5 u^{\wedge 18}$.

Step 4: Solve Partial Master Problem: that is minimizing $v^\wedge_k(Z)$ st. $z \in \mathbf{Z}$, go to step 1.

Partial Master Problem that is:

$$\text{Min } v^\wedge_3(Z) = u^{\wedge 1} + u^{\wedge 2} + Z_1 u^{\wedge 3} + \ldots + Z_5 u^{\wedge 18}$$
$$= 0 + 5 + 0Z_1 + 1Z_2 + 1Z_3 + \ldots + 2Z_5$$
$$\text{St.} \quad Z \in \mathbf{Z} = \{0 \text{ or } 1\}$$
$$Z_1 + Z_2 + Z_3 = 1 \text{ and } Z_4 + Z_5 = 1$$

The minimum occurs at $Z'_2 = (1, 0, 0, 0, 1)$ and the minimum value is $v^\wedge_3(Z) = 21 = v^\wedge_2(Z)$ that we can terminate the algorithm. The solution from the Benders' algorithm is the same as the solution from IP which is the assignment is 1 – 3.1, 2 - 4, 3.1 - 5, 4 – 2, 5 – 6.2 and 6.2 – 1 with the solution value of minimum total distances 21 units. Alternative 3.1 and 6.2 are selected.

We can see that this solution provide two subtour that can assign for 2 AGVs or any kind of vehicle, which stat at node 1 and node 2, because there are 2 subtours. This solution can be the lower bound of single/multiple AGV scheduling problem with alternative pick up and delivery nodes. This lower bound can be used in branch and bound approach to find the TSP that is the optimal scheduling of AGV problem.

We simulate some amount of different network problems with six nodes of problem size to test the mathematical model. The results show that the model can provide the solution and satisfy all constraints of all problems. There are some solutions provide the assignment as a single that is the TSP solution also but most of them generate subtour. According to this point, we can apply the model to any size of problem, but the running time is not examined further than six nodes. We found that this mathematical model can provide the solution of assignment problem with alternative nodes.

4. CONCLUSIONS

In this study, the mathematical model of the AGV scheduling problem with alternative pick up and delivery nodes and the relaxation model which is assignment problem with alternative nodes are purposed with the solving approach by integer programming and the Benders decomposition approach. The assignment problem with alternative nodes is solved to find the lower bound of the AGV scheduling problem with alternative pick up and delivery nodes. This study is conducted because the assignment problem is a sub problem that we have to solve in all iteration of solving TSP/MTSP by branch and bound framework. This paper creates the base knowledge for studying the TSP/MTSP with alternative pick up and delivery nodes that will be applied for solving the AGV scheduling problem with alternative pick up and delivery nodes in the future.

The assignment problem with alternative node has some special structure the different from the original assignment problem and can not be solved by the traditional solving approach for original assignment problem. This special structure creates effect on the unimodular property of the assignment problem. The 0-1 integer linear programming of original assignment can be solve as regular linear programming with out concerning of integer constraint by unimodular property of network problem, but when the alternative constraints are added, the model will lose the unimodular property. We create the new mathematical model structure for formulating the assignment problem with alternative node by modifying the original assignment problem structure. The created model is still the 0-1 integer linear programming. Then we create another solving approach by Benders decomposition algorithm which can be solved the problem with linear programming.

The simulated problem is generated to verify the model by using Excel Solve to model each network problem and solve the problem. The model can provide the assignment solution correctly that can be used as the lower bound tour or subtour for TSP/MTSP with alternative pick up and delivery nodes. These assignment solutions from all simulated problems are satisfy all model constraints can provide the lower bound of the AGV scheduling problem with alternative pick up and delivery nodes and the solving procedure by using branch and bound approach in the future.

5. ACKNOWLEDGEMENTS

The completion of this research would not have been possible with out the kind assistance and cooperation of a number of people. My most profound gratitude goes to Mr. Chompon and Mrs. Sommai Khamyat, my parent, who have been gave me all academic support.

I also wish to express my sincere appreciation to Mrs. Waraporn Parditporn, my beloved person, who have been always gave me my inspiration of life for researching and beside me for all of life time with endless love and everlasting support.

My deep thanks also go to Associate Professor Peerayuth Charnsethikul, my research advisor, who has paved the way for me while I am on my quest for academic excellence.

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