THE COMPARISON OF EFFICIENCY OF CONTROL CHART BY WEIHTED VARIANCE METHOD, SCALED WEIGHTED VARIANCE METHOD, EMPIRICAL QUANTILES METHOD AND EXTREME-VALUE THEORY FOR SKEWED POPULATIONS

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ABSTRACT

The objective of this study is to compare the efficiency of control chart using Weighted Variance Method, Scaled Weighted Variance Method, Empirical Quan-tiles Method and Extreme-value Theory for skewed populations. The efficiencies of control chart are determined by average run length. The control charts in the study is $\bar{x}$ chart. Various values of the coefficient of skew ness are 0.1,0.5,1.0,2.0,3. 0,4.0,5.0,6.0,7.0,8.0 and 9.0.Various values of the level of the mean shift equals to 0σ , 0.5σ , 1.0σ , 1.5σ , 2.5σ , 3.0σ The sample size are 3 , 5 and 7 . The data for the experiment are obtained through the Monte Carlo Simulation Technique and the experiment were constructed from 10.000 samples and repeated 1,000 times for each case. The result of the study is that the data have Weibull distribution at coefficient of skew ness 0.1,0.5, 1.0,2.0 and 3.0. The Scaled Weighted Variance Method have the most efficiency sample size of 3 at coefficient of skew- ness 0,4.0.5.0,6.0,7.0,8.0 and 9.0. Extreme – value Theory has the most efficiency sample size of 3,with Lognormal distribution at coefficient of skew ness 0.1,0.5 and 0.1The Weighted Variance Method has the most efficiency sample size of 3 at coefficient of skewness 2.0,3.0,4.0,5.0,6.0,7.0,8.0 and 9.0. The Scaled Weighted Variance Method has the most efficiency sample size of 3, with Burr's distribution. At coefficient of skewness.0.1 and 0.5.TheWeighted Variance Method has the most Efficiency sample size of 3, at coefficient of skew ness 1.0,2.0,3. 0,4,and 0.5. The Scaled Weighted Variance Method has the most efficiency sample size of 3.

KEYWORDS : Average Run Length , Control Chart

1. INTRODUCTION

The control chart originated in the early 1920s, it has become a powerful tool in statistical process control(SPC). The control chart has two types which are parametric control chart and non-parametric control chart. The non-parametric control chart must simulate the value of average and standard deviation for generating chart.

Non-parametric control chart is the unknown parametric distribution and non -normality distribution which suitable for the large data set. Woodall and Mont gomery(1999) consider the non-parametric situation underlying distribution function is assumed to be unimodal that have an increasing or decreasing density. This control chart is depended on the study model Empirical

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Quantiles, it have relative with Bootstrap method [6][7], Kernel estimators and Extreme-value theory . Pipassorn (2003) had studied the efficiency of control chart by weight method (i.e., SWV method, and WV method) which considering the weight only. So, it is not the best way to construct the control chart.

This study present the methods to construct the control chart in case of non-normality distribution which are WV method, SWV method, Empirical Quantiles and Extreme-value theory for skewed populations by weibull distribution, lognormal distribution and burr's distribution.

2. MATERIALS AND METHODS

1.1 Weibull distribution

Density function

$$f(x;\theta,\beta)=\frac{\beta}{\theta^{\beta}}x^{\beta-1}e^{-(x/\theta)^{\beta}} \qquad x>0$$

Mean

$$\mu=E(X)=\frac{\theta}{\beta}\Gamma\left(\frac{1}{\beta}\right)$$

Variance

$$\sigma^2=V(X)=\frac{\theta^2}{\beta}\left\{2\Gamma\left(\frac{2}{\beta}\right)-\frac{1}{\beta}\left[\Gamma\left(\frac{1}{\beta}\right)\right]^2\right\}$$

When θ : scale param ,

β : shape parameter ,

α_3 : coefficient of skew ness

In this study θ = 0.1,0.5,1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0,9.0

And β = 3.2219,2.211,1.563,1.0,0.7686,0.6478,0.5737,0.5237, 0.4873, 0.4596, 0.4376 that relevant with coefficient of skew ness (α_3) at α_3 {0.1,0.5,1.0,2.0,3.0,4.0,5.0, 6.0,7.0,8.0 , 9.0}

1.2 Lognormal distribution

Density function

$$f(x;\mu,\sigma)=\frac{1}{x\sigma(2\pi)^{1/2}}\exp\left[\frac{-(\ln x-\mu)^2}{2\sigma^2}\right],\ x>0, \quad (1)$$

Mean

$$\mu=E(X)=e^{\mu+\frac{\sigma^2}{2}}$$

Variance

$$\sigma^2=V(X)=e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$$

When $\exp(\mu)$: scale parameter

σ : shape parameter

α_3 : coefficient of skewnessIn

In this study μ =0.1,0.5,1.0,2.0,3.0,4.0,5.0,6.0,7.0,8.0 , 9.0

and σ =0.0334,0.1641,0.3142,0.5513,0.7156,0.8326,0.9202,0.9889,1.0446,1.0911,1.1307 that relevant with coefficient of skew ness (α_3) at α_3 {0.1,0.5,1.0,2.0,3.0,4.0,5.0, 6.0,7.0,8.0 , 9.0}

1.3 Burr's distribution

Density function

$$f(x) = \begin{cases} \dfrac{kcx^{c-1}}{\left(1+x^{c}\right)^{k+1}} & , \; x \geq 0 \\ 0 & , \; x < 0 \end{cases} \qquad (2)$$

Mean

$$\mu = E(X)$$

Variance

$$\sigma^2 = V(X)$$

2 Control chart

2.1 Weighted Variance : WV Control Charts control charts

The control chart Choobinech and Ballard(1987) [4] proposed the theory of WV method for skew ness distribution data. The theory separate distribution into two parts which are the mean of process and another one for constructing symmetry distribution. These distribution has the same mean but difference in standard deviation. Hence

$\overline{x}$ Control Chart

Upper Control Limit is $\quad UCL_{\overline{X}} = \overline{\overline{X}} + W_U \overline{R}$

Central Limit is $\quad CL_{\overline{X}} = \overline{\overline{X}}$

Lower Control Limit is $\quad LCL_{\overline{X}} = \overline{\overline{X}} - W_L \overline{R}$

When W_U and W_L are the constant based on the sample size and $\hat{P}_x$ estimator

$$\hat{P}_X = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\delta\left(\overline{\overline{X}} - X_{ij}\right)}{n \times k} \qquad (3)$$

$$\delta(X) = \begin{cases} 1 & , X \geq 0 \\ 0 & , X < 0 \end{cases}$$

2.2 Scaled Weighted Variance : SWV $\overline{x}$ - Control Chart

Castagliola(2000) [3] said that the scale weighted variance method is sepa- rate function into two parts $f_L(x)$ and $f_u(x)$ which are $\psi(x, \mu, \sigma_L', 2P_x)$ and $\psi(x, \mu, \sigma_U', 2(1-P_x))$ respectively with bell shape function. The center of bell shape function is μ And the second moments are $\sigma_L'^2$ and $\sigma_U'^2$ and the area under curves are equal to $2P_x$ and $2(1-P_x)$ Bell – shape probability function is

$$\psi(x, \mu, t, \kappa) = \frac{\kappa^{3/2}}{t}\varphi\left(\frac{(x-\mu)\sqrt{\kappa}}{t}\right)$$

So, the control chart is

Upper Control Limit is

$$UCL_{\overline{X}} = \overline{\overline{X}} + \frac{W_U}{3}\sqrt{\frac{1}{2\left(1-P_X\right)}}\,\Phi^{-1}\left(1 - \frac{\alpha}{4\left(1-P_X\right)}\right)\overline{R}$$

Central Limit is

$$CL_{\overline{X}} = \overline{\overline{X}}$$

Lower Control Limit is

$$LCL_{\overline{X}} = \overline{\overline{X}} - \frac{W_L}{3}\sqrt{\frac{1}{2P_X}}\,\Phi^{-1}\left(1 - \frac{\alpha}{4P_X}\right)\overline{R}$$

When

W_U and W_L are the constant of WV method

α is type I error

2.3 Empirical Quantile $\overline{x}$ Control Chart

The bootstrap method, introduced by Efron(1979), is a powerful tool for estimating the sampling distribution of statistic. Let $X_1, X_2, \dots, X_N$ be an inde-pendent and identically distributed sample with mean and variance. The standard bootstrap procedure is to draw with replacement a random sample of size N from $X_1, X_2, \dots, X_N$ .Let $x_1^*, x_2^*, \dots, x_n^*$ are Bootstrap Sample

$\overline{x}$ is mean of subgroup

$\overline{\overline{x}}$ is mean of Bootstrap Sample

S_N^* is standard deviation of Bootstrap Sample

F_N is distribution Empirical Quantiles of $x_1^*, x_2^*, \dots, x_n^*$

Let N = kn; with n the subgroup size and k the number of subgroups. Because the $\overline{x}$ chart plots the subgroup sample means, the control limits should be obtained from $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ quantiles of the sampling distribution of $\sqrt{n}(\overline{X}_N^* - \overline{\overline{X}}_N)$ Hence, this sampling distribution can be approximated by bootstrap from any observation

$$P(\sqrt{n}(\overline{X}_n^* - \overline{\overline{X}}_N) \le x \mid F_N) \approx P(\sqrt{n}(\overline{X}_n - \mu) \le x \mid F) \qquad (5)$$

From epuation (5)leads to an alternative approach to constructing and $\overline{x}$ chart for iid observation by repeating the bootstrap procedure k times and form a histogram of the resulting k terms of $\sqrt{n}(\overline{X}_N^* - \overline{\overline{X}}_N)$, and then locate the $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ quan- tiles.These are then used as the estimated $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ quantiles to obtain $\tau_{\alpha/2}$ So,

$$\frac{\alpha}{2} = P(\sqrt{n}(\overline{X}_n^* - \overline{\overline{X}}_N) \le \tau_{\alpha/2} \mid F_N)$$

$$\frac{\alpha}{2} \approx P(\overline{X} \le \mu + \tau_{\alpha/2}/\sqrt{n} \mid F)$$

In summary, we conclude that the Empirical Quantiles control chart obtained by $\tau_{\alpha/2}$ which are constructed from repeating $\sqrt{n}(\overline{X}_N^* - \overline{\overline{X}}_N)$ of random sample of the distribution (As shown in appendix(16),(18),(20))

The control limits of weibull distribution
Upper Control Limit is

$$UCL = \overline{\theta} + \tau_{(1-\alpha/2)}/\sqrt{n}$$

Central Limit is

$$CL = \overline{\theta}.$$

Lower Control Limit is

$$LCL = \overline{\theta} + \tau_{\alpha/2}/\sqrt{n} \qquad (6)$$

when

θ is sample mean of each subgroup

$\overline{\theta}$ is sample mean

$$\overline{\theta} = \frac{\sum_{i=1}^{k}\theta_i}{k}$$

k is the number of sample class

The control limits of Lognormal distribution
Upper Control Limit is

$$UCL = \overline{\mu} + \tau_{(1-\alpha/2)}/\sqrt{n}$$

Central Limit is

$$CL = \overline{\mu}$$

Lower Control Limit is

$$LCL = \overline{\mu} + \tau_{\alpha/2} / \sqrt{n} \qquad (7)$$

when

μ is sample mean of each subgroup

$\overline{\mu}$ is sample mean

$$\overline{\mu} = \frac{\sum_{i=1}^{k} \mu_i}{k} \qquad (8)$$

(2-5)

k is the number of sample class

The control limits of Burr's distribution
Upper Control Limit is

$$UCL = \overline{k} + \tau_{(1-\alpha/2)} / \sqrt{n}$$

Central Limit is

$$CL = \overline{k}$$

Lower Control Limit is

$$LCL = \overline{k} + \tau_{\alpha/2} / \sqrt{n} \qquad (9)$$

when

k is sample mean of each subgroup

$\overline{k}$ is sample mean

$$\overline{k} = \frac{\sum_{i=1}^{k} k_i}{m} \qquad (10)$$

m is the number of sample class

2.4 Extreme-value Theory : $\overline{x}$ Control Chart

To obtain Extreme value of $\overline{x}$ Chart must beestimated $M_k^{(r)}$ by moment method (As shown in appendix in equation(24),(26),(28)) $\tau\left(\frac{k+\beta-i}{\beta}\right)$ is the gamma function which

$$\tau(z) = \int_0^{\infty} t^{z-1} e^{-t} dt \quad , z > 0$$

and z is the real number in gamma function table. Simulating data to estimate $\hat{\gamma}_k$ from (2-11), we obtain Extreme value theory control chart. The control limits of weibull distribution:

Upper Control Limit is

$$UCL = X_{(k-m)} + \frac{(m/(kq))^{\gamma_k} - 1}{\hat{\gamma}_k}(1 - (\hat{\gamma}_k \wedge 0)) X_{(k-m)} M_k^{(1)}$$

Lower Control Limit is

$$LCL = X_{(m+1)} + \frac{(m/(k\alpha/2))\overline{\gamma}_k - 1}{\overline{\gamma}_k}(1 - (\overline{\gamma}_k \wedge 0)) X_{(m+1)} \overline{M}_k^{(1)} \qquad (11)$$

when

$$\overline{M}_k^{(r)} = \frac{\sum_{k=1}^{n} M_k^{(r)}}{n}$$

n is the number of class

The control limits of Lognormal distribution
Upper Control Limit is

$$UCL = X_{(k-m)} + \frac{(m/(kq))^{\gamma_k} - 1}{\hat{\gamma}_k}(1 - (\hat{\gamma}_k \wedge 0))X_{(k-m)}M_k^{(1)}$$

Lower Control Limit is

$$LCL = X_{(m+1)} + \frac{(m/(k\alpha/2))\bar{\gamma}_k - 1}{\bar{\gamma}_k}(1 - (\bar{\gamma}_k \wedge 0))X_{(m+1)}\overline{M}_k^{(1)} \qquad (12)$$

when

$$\overline{M}_k^{(r)} = \frac{\sum_{k=1}^{n} M_k^{(r)}}{n}$$

n is the number of class

The control limits of Burr's distribution
Upper Control Limit is

$$UCL = X_{(k-m)} + \frac{(m/(kq))^{\gamma_k} - 1}{\hat{\gamma}_k}(1 - (\hat{\gamma}_k \wedge 0))X_{(k-m)}M_k^{(1)}$$

Lower Control Limit is

$$LCL = X_{(m+1)} + \frac{(m/(k\alpha/2))\bar{\gamma}_k - 1}{\bar{\gamma}_k}(1 - (\bar{\gamma}_k \wedge 0))X_{(m+1)}\overline{M}_k^{(1)} \qquad (13)$$

when

$$\overline{M}_k^{(r)} = \frac{\sum_{k=1}^{n} M_k^{(r)}}{n}$$

$$\hat{\gamma}_k = M_k^1 + 1 - \frac{1}{2}\left\{1 - \frac{(M_k^{(1)})^2}{M_k^{(2)}}\right\}^{-1}$$

$$\bar{\gamma}_k = \overline{M}_k^1 + 1 - \frac{1}{2}\left\{1 - \frac{(\overline{M}_k^{(1)})^2}{\overline{M}_k^{(2)}}\right\}^{-1}$$

n is the number of class
The sample sizes of this study are 3, 5 and 7. The value of the coefficient of skew ness α_3 ε 0,1,0.5,1,2,3,4,5,6,7,8,9 The values of the level of the mean shift equals to 0σ to 3σ The results of this study are simulated under :

1. For WV method, Let α = 0.0027 to comparison the efficiency of control chart with Weibull distribution, lognormal distribution and burr's distribution.
2. For SWV method, Let α = 0.0027 to comparison the efficiency of control chart with Weibull distribution, lognormal distribution and Burr's distribution.
3. For Empirical..,using Weibull distribution, lognormal distribution and Burr's distribution to comparison the efficiency of control chart..
4. For Extreme method, Let α = 0.0027 to comparison the efficiency of control chart with Weibull distribution, lognormal distribution and Burr's distribution.

3. RESULTS AND DISCUSSION

3.1 The propose of this study is to compare the efficiency of control chart by WV,SWV for skewed populations i.e., weibull distribution, lognormal distribution and burr's distribution. From the study, We found that the width of control limits are based on the variance of distribution. Detect has the most efficiency in the narrow control limit.

More than one of Weibull distribution data of skewness has the shape of curve like normal distribution and right skew, Shewhart chart gives the same result. Extreme-value is good detect data which agree with M.B.Vermaat ET A1(2003)[8], is an extreme-value appropriate non-normal distribution. At coefficient less than or equal to 1 the shape of curve like exponential distribution ,studied weight variance method which agree with Adisak (2003).Examine control chart is appear at coefficient 0.1,0.5,1.0,2.0 and 3.0 by SWV method is efficient with most width of control limits and ARL is maximum, but at coefficients 4.0,5.0,6.0,7.0,8.0 and 9.0 by EV method is efficiency with most width of control limits and ARL is maximum. See Figure 1

Lognormal distribution data of skew ness more than one has the shape of the curve like normal distribution and right skew, Shewhart chart give the same re- sult, but detect data not good as non-normal distribution theory which agree with Adisak (2002)[1] .Examine control chart is appear at coefficients 0.1,0.5 and 1.0 by WV method is efficient with most width of control limits and ARL is maximum, but at coefficients 2.0,3.0,4.0,5.0,6.0,7.0,8.0 and 9.0 by SWV method is efficient with most width of control limits and ARL is maximum. See Figure 2

Burr's distribution data of skew ness more than one has the shape of curve very skewed ,when k increase shape of curve like weibull distribution lead to weight variance method which agree with Adisak (2004)[2].Examine control chart is appear at coefficients 0.1 and 0.5 by WV method is efficient with most width of control limits and ARL is maximum, but at coefficients 1.0,2.0,3.0,4.0 and 5.0 by SWV method is efficient with most width of control limits and ARL .Figure 3

3.2 Data are shifted right skewed increasing and average run length decrease. Then control chart increase in efficiency . In this study Weibull distribution has

coefficients of skew ness 0.1,0.5,1.0,2.0 and 3.0, by Scaled Weighted Variance Method is the most efficient. At coefficients of skew ness 4.0,5,6.0,7.0,8.0 and 9.0 ,Extreme-value Theory is the most efficient. Data have Lognormal distribution at coefficients of skew ness 0.1,0.5 and 1.0, Weighted Variance Method has the most efficient. At coefficients of skew ness 2.0, 3.0, 4.0,5.0,6.0,7.0,8.0and 9.0, Scaled Weighted Variance Method is the most efficient.Data has Burr's distribution at coefficient of skew ness 0.1 and 0.5, Weighted Variance Method has the most efficient sample size of 3.At coefficients of skew ness 1.0, 2.0, 3.0, 4.0and 5.0. Scaled Weighted Variance Method is the most efficient.

4. CONCLUSION

Studied($\bar{x}$)control chart by Weighted Variance Method, Scaled Weighted Vari- ance Method, Empirical Quantiles Method and Extreme-value Theory for skewed populations. The result of the study is data have Weibull distribution at coefficients of skew ness 0.1,0.5,1.0,2.0 and 3.0. Scaled Weighted Variance Method is the most efficiency sample size is 3.At coefficients of skew ness 4.0,5.0,6.0,7.0,8.0 and 9.0.Extreme-value Theory has the most efficient sample size of 3.Data have Lognormal distribution at coefficients of skew ness 0.1,0.5 and 1.0, Weighted Vari- ance Method has the most efficient sample size of 3.At coefficients of skew ness 2.0, 3.0, 4.0,5.0,6.0,7.0,8.0and 9.0, Scaled Weighted Variance Method has the most efficient sample size of 3.Data have Burr's distribution at coefficients of skew ness 0.1 and 0.5 by Weighted Variance Method has the most efficient sample size is 3.At coefficients of skew ness1.0,2.0,3.0,4.0and 5.0 by Scaled Weighted Variance Method has the most efficient sample size of 3.

5. SUGGESTIONS

5.1 Data has Weibull distribution at coefficients of skew ness 0.1,0.5,1.0,2.0and 3.0 by SWV Method has the most efficient sample size of 3.At coefficients of skewness 4.0,5.0,6.0,7.0,8.0 and 9.0 by Extreme-value Theory has the most efficient sample size of 3.

5.2 Data has Lognormal distribution at coefficients of skew ness 0.1,0.5 and 1.0 by WV Method has the most efficient sample size of 3.At coefficients of skew ness 2.0,3.0, 4.0,5.0,6.0,7.0,8.0 and 9.0 by SWV Method has the most efficient sample size of 3.

5.3 Data has Burr's distribution at coefficients of skew ness 0.1 and 0.5 by WV Method have the most fficiency sample size is 3.At coefficients of skew ness 1.0, 2.0,3.0, 4.0and 5.0 by SWV Method has the most efficient sample size is 3.

5.4 Can is study in Student's t distribution etc.

5.5 Control chart have many methods , Example by Cowden[5], Kernel etc.

5.5 Control chart have many methods , Example by Cowden, Kernel etc.

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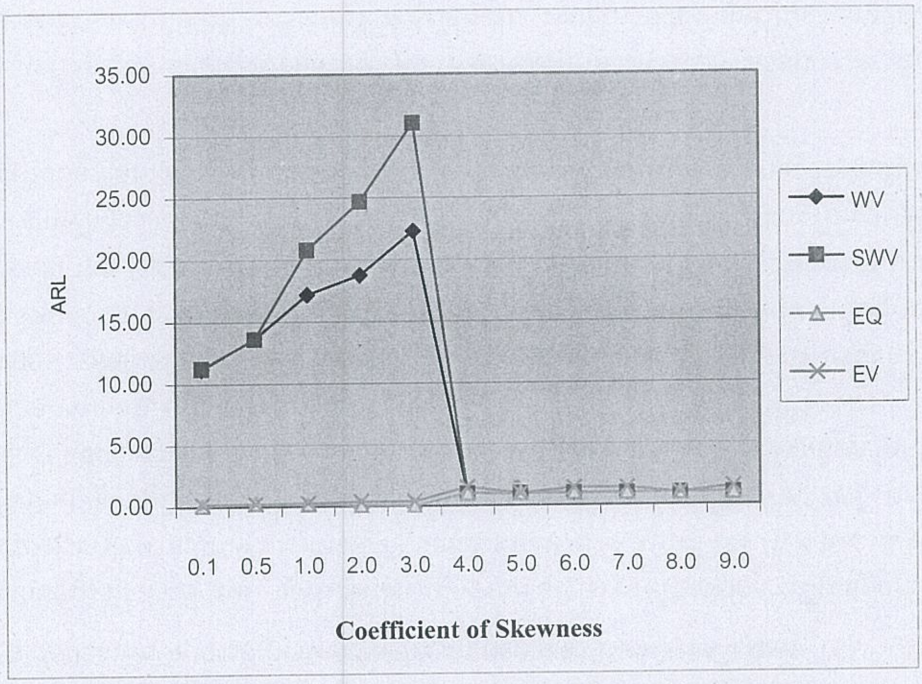

Figure 1: Comparison ARL of Average Control Chart (n=3:0σ)Between WV,SWV,EV and EV for Data from Weibull distribution.

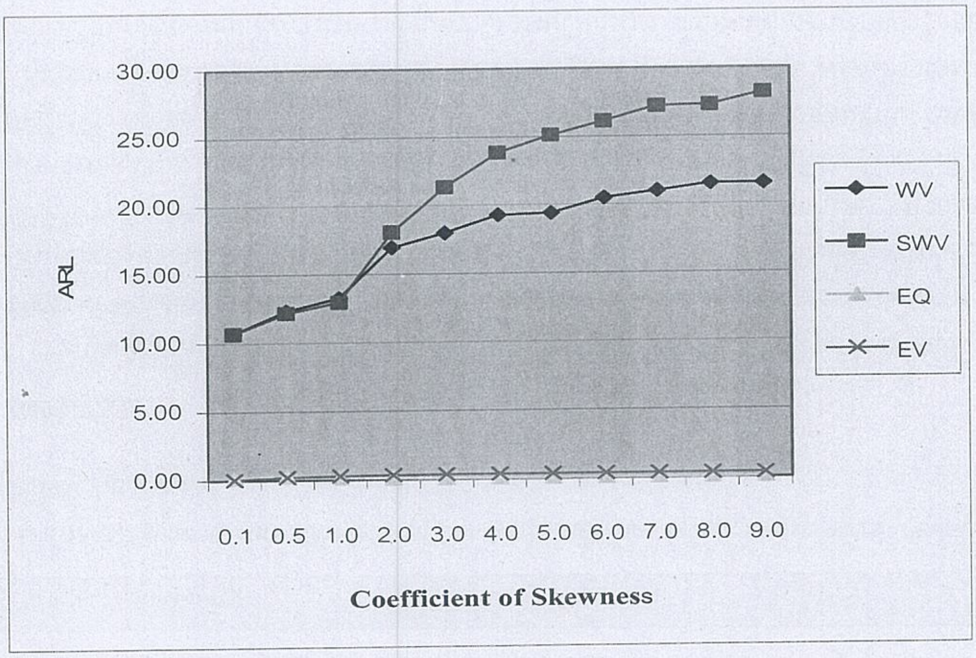

Figure 2: Comparison ARL of Average Control Chart (n=3:0σ)Between WV,SWV,EV and EV for Data from Lognormal distribution.

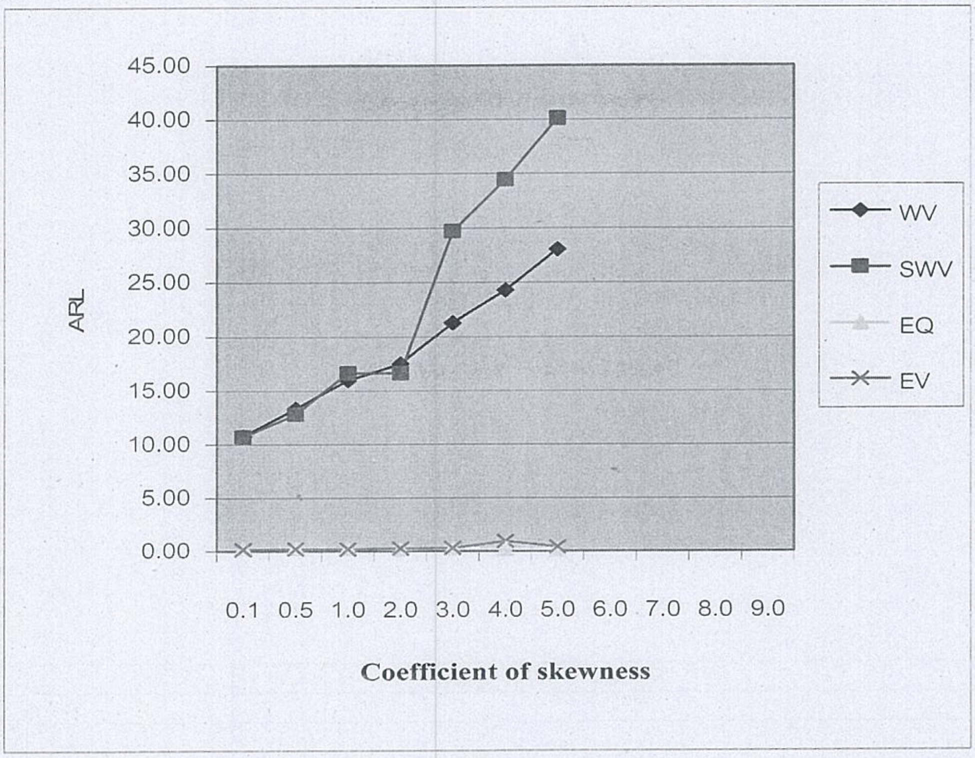

Figure 3: Comparison ARL of Average Control Chart (n=3:0σ)Between WV,SWV,EV and EV for Data from Burr's distribution.