The Coupling Soliton Equations For 3x3 Optical Coupler Characterization

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Abstract

In this research we report the numerical studies of the propagation of soliton pulses in three-core nonlinear fiber with a triangular coupler device. It described in terms of three linearly coupled nonlinear Schodiger equations. The numerical method used the finite element method base on the Galerkin method. First, we discrete the time domain using quadratic line elements, then we use the iterative techniques to clarify the element position. It is dependent from the half-beat length (Lc). This case $Lc=\pi/3\kappa$, where κ is linear coupling coefficient. The interative techniques use θ scheme method, when $\theta=1/2$ known as the Crank-Nicolson scheme. In the part of calculation, input fundamental soliton into core 1part core 2 and 3 are zeros. A core 1 has shown that the transfer amplitude into core 3 and 2 is observed.

Keywords: finite element methods, soliton switching

1. INTRODUCTION

It has been recognized that temporal optical solitons present a unique opportunity for performing a wide range of all-optical processing functions in nonlinear-optical fibers. Doran and Wood first suggested that solitons are natural bits for ultra-fast all-optical signal proccessing[1], based on the observation that, even in nonlinear fiber systems that have no exact solitons, an injected soliton-like pulse displays a remarkable degree of phase coherence over the whole pulse, making it possible to process individual soliton bits. The potential of all-optical switching devices, on the other hand, to operate at speeds and bandwidths much greater than those possible with electronic switches has led to a growing interest in the study of the former for use in signal-processing systems. Among those devices is the nonlinear fiber coupler whose two core version has been investigated extensively, and many interesting continuous wave (CW) and pulsed operational regimes have been proposed [2]. Limitations on its operation, arising from linear loss, which can be eliminated by introducing gain in the cores while the use of solitons as input pulses prevents pulse stripping. The three-core nonlinear fiber coupler has also attracted considerable attention recently because it possesses significant advantages, particularly sharper fiber coupler [3]. Buah *et al*. find a numerical based on the use of a finite-element-based beam propagation algorithm, of all-optical switching of solitons in three-core nonlinear fiber couplers array. Linear gain is found to lead to sharper transmitted characteristics, a considerable lowering of the switching powers, and also to an increase in the power exchange between the input and center core for a nearest-neighbor core arrangement [4].Coupled nonlinear Schrodiger (**CNLS**) equation of the type that describes soliton dynamics in three-core fiber coupler are formally systems of second-order nonlinear partial differential equations and can be treated as a problem of the evolution of a dynamical system with an infinite number of degree of freedom which are usually Hamiltonian, i.e., they can be derived from Hamiltonian functions. Their analysis is based on the theory of infinite dimensional Hamiltonian systems with the energy of the solutions usually being conserved. Particular solutions of the systems can be found by making a substitution of the required form of solution, usually with some variables separated, and then finding relations concerning various parameter. In such methods, one tries to reduce given CNLS systems of PDE's to coupled ordinary differential equations and to proceed more systematical with the resulting systems. If all the effects of pulse dispersion and damping are neglected and only CW interactions are considered, the system of equations can be solved analytically. It is, however, extremely difficult, even through coupled soliton solutions [5] have been usually studied numerically both because of the non-integratability and because the equations is much more complicated than the single-axis nonlinear Schrodinger equations [4]. Here, we are only interested in the amplitude characteristics of a coupler of a half-beat length for switching purposes, the solution to which can only be obtained numerically.

The numerical study reported is organized as follows: The CNLS equations are described briefly for a tri-core nonlinear fiber coupler, in the next section, while the finite element algorithm is described in detail in section 3. Section 4 presents the results of the simulation with some explanation when it use in optical communication for highly speed communication and network.

2. THROEY

When the ultrashort soliton pulse propagates in a three-core of nonlinear fiber coupler with a triangular basis, then the output behavior can be predicted by solving the dimensionless three-coupled system of NLS equations as the following form [3], [5], [6]:

$$j\frac{\partial u_1}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_1}{\partial \tau^2} + |u_1|^2 u_1 + \kappa(u_2 + u_3) = 0 \qquad (1)$$

$$j\frac{\partial u_2}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_2}{\partial \tau^2} + |u_2|^2 u_2 + \kappa(u_1 + u_3) = 0 \qquad (2)$$

$$j\frac{\partial u_3}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u_3}{\partial \tau^2} + |u_3|^2 u_3 + \kappa(u_1 + u_2) = 0 \qquad (3)$$

$$0 < \kappa \le 1$$

where the boundary condition (BC) is

$$\frac{\partial u_1}{\partial \tau} = \frac{\partial u_2}{\partial \tau} = \frac{\partial u_3}{\partial \tau} = 0 \qquad (4)$$

where the amplitudes u_i (i=1,2,3) are the normalized components of the slowly varying pulse envelopes, ξ and τ are the axial coordinate and the time in a reference frame moving with a common group velocity, while κ is the normalized linear coupling coefficient. In practice, the pulse width of the input soliton is the very intense pulse with the pulse with in the time interval of ps.

3. THE FINITE ELEMENT

The time domain of equations (1), (2), and (3) are divided by using quadratic line elements, where the amplitudes $u_1(\xi,\tau)$, $u_2(\xi,\tau)$, and $u_3(\xi,\tau)$ are expanded in terms of $u_i(\xi)$, $v_i(\xi)$, and $w_i(\xi)$, i=1,2,3 is the node point, then the equation (4) is expressed in the following forms

$$u_1(\xi,\tau) = \{N\}^T\{u(\xi)\}_e \qquad (5)$$

$$u_2(\xi,\tau) = \{N\}^T\{v(\xi)\}_e \qquad (6)$$

$$u_3(\xi,\tau) = \{N\}^T\{w(\xi)\}_e \qquad (7)$$

where N is the shape function.

The time discretization by using the finite element method based on the Galerkin method is formed the three-coupled first-order matrix equations [8] which are expressed as

$$j[M]\frac{d\{u\}}{d\xi} + [K]\{u\} + N(u)\{u\} + \kappa(\{v\} + \{w\}) = \{0\} \qquad (8)$$

$$j[M]\frac{d\{v\}}{d\xi} + [K]\{v\} + N(v)\{v\} + \kappa(\{u\} + \{w\}) = \{0\} \quad (9)$$

$$j[M]\frac{d\{w\}}{d\xi} + [K]\{w\} + N(w)\{w\} + \kappa(\{u\} + \{w\}) = \{0\} \quad (10)$$

where the matrix parameters are given by

$$[M] = \sum_e \int_e \{N\}\{N\}^T d\tau,$$

$$[K] = \sum_e \int_e \frac{1}{2}\{N_{,\tau}\}\{N_{,\tau}\}^T d\tau,$$

$$N(u) = \sum_e \int_e [\{u\}]^2 \{N\}\{N\}^T d\tau,$$

and

$$N(v) = \sum_e \int_e |\{v\}|^2 \{N\}\{N\}^T d\tau, \quad N(w) = \sum_e \int_e |\{w\}|^2 \{N\}\{N\}^T d\tau.$$

Concerning the differentiation with respect to ξ of (8), (9), and (10), using the θ scheme method, thus gives

$$[L(\theta)]_i\{u\}_{i+1} + \theta\Delta\xi\kappa[M]\{v\}_{i+1} + \theta\Delta\xi\kappa[M]\{w\}_{i+1} = [L(\theta-1)]_i\{u\}_i - (1-\theta)\Delta\xi\kappa[M]\{v\}_i - (1-\theta)\Delta\xi\kappa[M]\{w\}_i \quad (11)$$

$$[Q(\theta)]_i\{v\}_{i+1} + \theta\Delta\xi\kappa[M]\{u\}_{i+1} + \theta\Delta\xi\kappa[M]\{w\}_{i+1} = [Q(\theta-1)]_i\{v\}_i - (1-\theta)\Delta\xi\kappa[M]\{u\}_i - (1-\theta)\Delta\xi\kappa[M]\{w\}_i \quad (12)$$

$$[S(\theta)]_i\{w\}_{i+1} + \theta\Delta\xi\kappa[M]\{u\}_{i+1} + \theta\Delta\xi\kappa[M]\{v\}_{i+1} = [S(\theta-1)]_i\{w\}_i - (1-\theta)\Delta\xi\kappa[M]\{u\}_i - (1-\theta)\Delta\xi\kappa[M]\{v\}_i \quad (13)$$

where the parameters L, S, Q are given by

$$[L(\theta)]_i = j[M] + \theta\Delta\xi([K] + N(u)_i),$$

$$[Q(\theta)]_i = j[M] + \theta\Delta\xi([K] + N(v)_i),$$

$$[S(\theta)]_i = j[M] + \vartheta\Delta\xi([K] + N(w)_i),$$

where $0 \le \theta \le 1$. When θ=1/2 the Crank-Nicholson scheme is employed. We write equations (11), (12), and (13) in the form as

$$\mathbf{Ax = y} \quad (14)$$

Where the matrix A is expressed as

$$A = \begin{bmatrix} [L(\theta)] & \theta\Delta\xi\kappa[M] & \theta\Delta\xi\kappa[M] \\ \theta\Delta\xi\kappa[M] & [Q(\theta)] & \theta\Delta\xi\kappa[M] \\ \theta\Delta\xi\kappa[M] & \theta\Delta\xi\kappa[M] & [S(\theta)] \end{bmatrix},$$

$$x = \begin{Bmatrix} \{u\}_{i+1} \\ \{v\}_{i+1} \\ \{w\}_{i+1} \end{Bmatrix},$$

$$y = \begin{Bmatrix} [L(\theta-1)]\{u\}_i - (1-\theta)\Delta\xi\kappa[M]\{v\}_i - (1-\theta)\Delta\xi\kappa[M]\{w\}_i \\ [Q(\theta-1)]\{v\}_i - (1-\theta)\Delta\xi\kappa[M]\{u\}_i - (1-\theta)\Delta\xi\kappa[M]\{w\}_i \\ [S(\theta-1)]\{w\}_i - (1-\theta)\Delta\xi\kappa[M]\{u\}_i - (1-\theta)\Delta\xi\kappa[M]\{v\}_i \end{Bmatrix}.$$

Thus equation (14) is now reduced into the simplest form i.e. linear equation. The numerical results using FEM is performed in the next section.

4. RESULTS

The initial conditions as shown in equation (15) are considered. The short input pulse with period of ps(10^{-12} s) is formed by the hyperbolic function. The coupling power between the fiber core of the active coupler is performed associating with the coupling value(□).

$$u_1(\xi = 0, \tau) = \text{sech}(\tau),$$
$$u_2(\xi = 0, \tau) = u_2(\xi = 0, \tau) = 0 \quad (15)$$

κ=0.2,0.5,0.8,1.0

The coupling power between fiber core is calculated and performed by using a half-beat coupling length i.e. Lc=π/3κ. Let $-10 \le \tau \le 10$, $0 \le \xi \le Lc$, Δξ=π/150, and element number (Ne) is 150. The results is shown and discussed as following. Figure 1 the perfect switching is formed by a core 1, while the noisy signal output is obtained by core 2 and core 3.

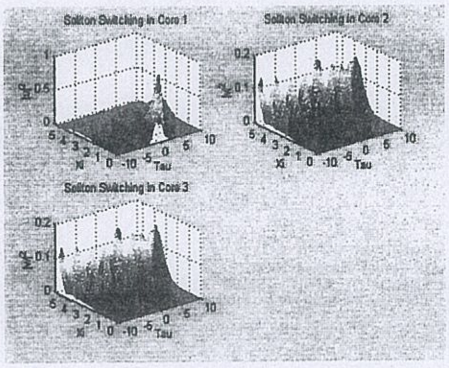

Figure 1.Graph between $|u|^2$,$|v|^2$, and $|w|^2$ by κ=0.2(Lc=π/0.6,Ne=150,Δξ=π/150).

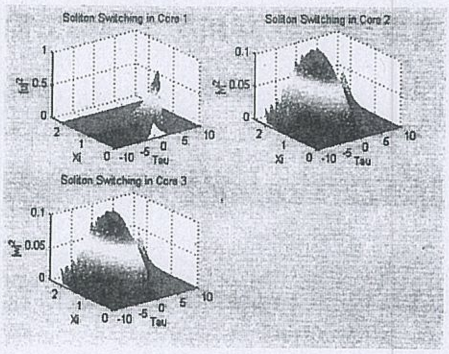

Figure 2. Graph between $|u|^2$,$|v|^2$, and $|w|^2$ by κ=0.5 (Lc=π/1.5,Ne=150,$\Delta\xi$=π/150).

The switching soliton pulse is obtained by the fiber core 1, while the pulse broadening I.e. dispersion, is occurred in the fiber core 2 and core 3. Similarly, fore core 1, but the output pulses broadening with noisy signals are occurred in core 2 and 3 in Figure 3 and 4, respectively.

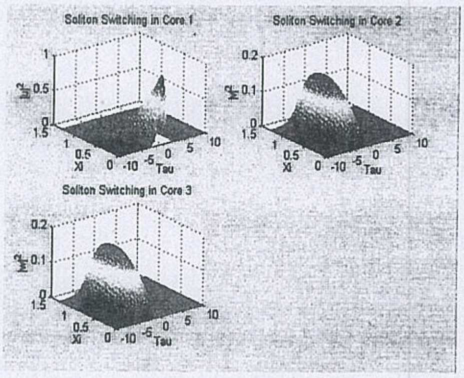

Figure 3. Graph between $|u|^2$,$|v|^2$, and $|w|^2$ by κ=0.8 (Lc=π/0.6,Ne=150,$\Delta\xi$=π/150).

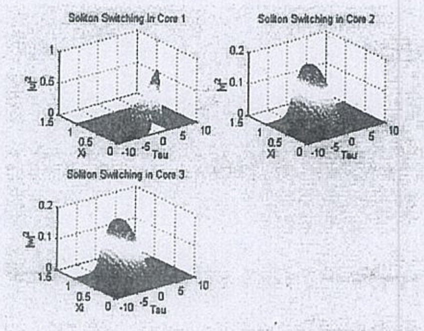

Figure 4. Graph between $|u|^2$,$|v|^2$, and $|w|^2$ by κ=1.0 (Lc=π/3.0,Ne=150,$\Delta\xi$=π/150).

5. CONCLUSION

The CNL soliton equations are studied using FEM to solve and chracterized the optical soliton equation. Result obtained has shown the optical switching may be formed using the appropriated coupling coefficient. The application communication network, when the high capacity network may be implemented using soltiton technique where the in-line connection between optical device may be required to complete the highly speed communication through out the optical network.

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