Particle Sizing and Tracking by using Digital Analysis

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Abstract

A new digital method for sizing particle and tracking its position from particle holograms is proposed. In our proposed method, the hologram is recorded on a CCD sensor. The digitized holograms are analyzed by using a combination of wavelet transform (WT) and a reconstruction of envelope function. The experimental results show that the error of measuring particle size is slightly higher than the error of measuring position. This is caused by speckle noise. The system limitation of the method is presented.

Keywords: holography, image processing

1. INTRODUCTION

Particle sizing and tracking is one of potential applications of an in-line Fraunhofer hologram [1]. In the in-line hologram, opaque or semi-transparent object is illuminated by a coherent light as shown in Fig. 1.

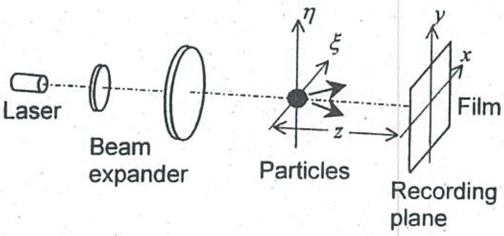

Figure 1. Schematic diagram for recording the in-line hologram of a particle.

An interference pattern produced by light waves diffracted from the particles and directly transmitted light wave is recorded on light-sensitive media such as photographic films. This interference pattern contains information of both three-dimensional (3-D) spatial distribution and diameter of particles. In conventional method, this information is extracted by illuminating the developed hologram by the coherent light. The transmitted light reconstructs back the image of particles at the same distance as the recording distance. Although this method allows us to freeze the moving particles and to analyze it later, in real world applications, we may deal with a huge number of particles. As a consequence, the reconstruction process is very tedious and time consuming. In order to overcome this problem, Murakami employed a microscope to directly observe the transmittance of the hologram [2]. He established a relation between the density and the fringe diameter of the interference pattern which could provide the desired information. However, his method is applicable only to a limited range of measurements of either a very big diameter of particle or a very short recording distance. Recently, automatic analysis of particle hologram has also been proposed by Widjaja by using an optical correlation technique [3]. However the price paid for this technique is that the optical system becomes complex.

In this work, we propose a new method for analyzing particle hologram by using the CCD sensor for capturing the interference pattern. The captured pattern is stored into a frame memory and is then digitally analyzed. In the digital analysis, we employ the WT to extract the position of the particle with respect to the recording plane and reconstructs the envelope function of the interference pattern in order to obtain the size of the particle. In comparison with the previous methods, our proposed method has advantages such as: first, it is free from chemical developing process. Second, it is accurate and has a wider dynamic range of measurement.

2. METHODOLOGY

As for a small particle with a radius of *a*, the amplitude transmittance of the particle hologram can be mathematically expressed as [1]

$$I(r) = 1 - \frac{2\pi a}{\lambda z}\sin\left(\frac{\pi r^2}{\lambda z}\right)\left[\frac{2J_1\left(\frac{2\pi ar}{\lambda z}\right)}{\frac{2\pi ar}{\lambda z}}\right] + \frac{\pi^2 a^4}{\lambda^2 z^2}\left[\frac{2J_1\left(\frac{2\pi ar}{\lambda z}\right)}{\frac{2\pi ar}{\lambda z}}\right]^2, \quad (1)$$

where λ and z are the wavelength of the illuminating light and the distance between the particle and the recording plane, respectively. Here, r represent the polar coordinate in the hologram plane, while J_1 denotes the first-order Bessel function. The first term of Eq. (1) is the directly transmitted light. The second term consists of two functions: first, the chirp signal with frequency which is inversely proportional to the distance z. Second, an Airy function whose position of its sidelobe are determined by the size of particle. Here, the Airy function modulates the chirp signal. The third term is the square of the Airy function which has a much smaller amplitude compared to the second term, so that it can be neglected.

In our proposed method, we employ the WT to extract the instantaneous frequency of the chirp function, because it is useful for analyzing non-stationary signal [4]. The WT analysis of this holographic signal gives information on how the frequency varies as the function of position. From a set of data of the position-varying frequency, the position of the particle z is then estimated. In order to obtain the radius of the particle, the modulating Airy function is reconstructed back by locating the maximum and minimum amplitudes of the modulated chirp signal. The nodal positions of the oscillating signal, which are located at the points where the different between the maximum and minimum amplitudes is small, are determined. The particle radius, the only unknown value in the argument of the Airy function, is finally obtained. Since, we record the interference pattern by using the CCD sensor, the pattern is automatically digitized. Therefore, the WT computation and reconstruction of the modulating function could be easily computed. In this work, all computations are done by using Matlab.

3. RESULTS & DISCUSSION

In our preliminary verification, the in-line hologram of 1-D object such as a wire is simulated. In this case, the envelope function of the interference pattern becomes a sinc function with the same argument [1]. Figure 2 shows the simulated intensity of the interference pattern for a fiber optic of radius 62.48 □m which is located at 80 cm in front of the recording plane under illumination of coherent light with a wavelength of 543.5 nm. The figure shows that the chirp signal is now modulated by the sinc function. This simulated signal is then analyzed by the WT. The resultant wavelet transformation is shown in Fig. 3, where the cross signs correspond to the space-varying frequency, while the solid line corresponds to the theoretical value of this frequency. Note that the frequency is inversely proportional to the scale. It is obvious that by using the WT, the distance z could be determined from this result.

After locating the positions of the maximum and minimum amplitudes of the modulated chirp signal, the magnitude of these points are sequentially compared in order to determine the nodal position of the oscillating signal. Figure 4 shows the resultant reconstruction of the sinc function and its nodal position. The frequency of the sinc function which corresponds to the particle size, is then determined from the distance between the vertical lines.

By using these two computational steps, we calculate the error of measurement for given values.

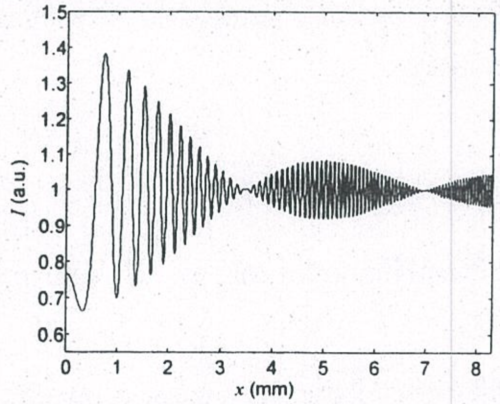

Figure 2. Simulated hologram of a fiber optics.

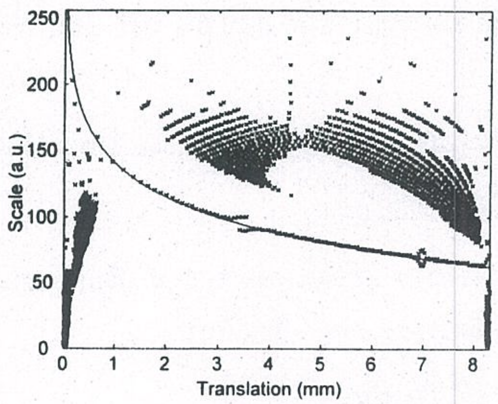

Figure 3. The WT of Fig. (2).

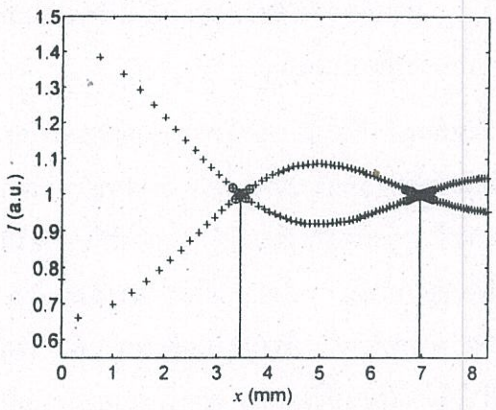

Figure 4. The reconstruction of the sinc function and its nodal position.

of the recording distance z between 15 cm to 100 cm as shown in Fig. 5. The error of measuring distance z is represented by the round sign, while the cross sign corresponds to the error of measuring radius a. It can be obviously observed that the error of measurements is less than 1%.

We further verify our method by generating optically hologram of the same fiber optic. The hologram is recorded by using a CCD camera HAMAMATSU C5948 having 8.3 mm × 6.3 mm photosensitive area with a resolution of 640 × 480 pixels. The fiber optic was illuminated by a collimated He-Ne laser with the wavelength 543.5 nm. Figure 6 shows the errors of measurement of the experiment. The small errors of measurement of the recording distance z are consistent with the simulation results, while the measurement of the particle radius a gives higher error in comparison with the simulation results. This is caused by the speckle noise which smears the hologram. As a consequence, the amplitude of the holographic signal around the nodal positions is comparable to the amplitude of the noise. This causes higher error of measurement.

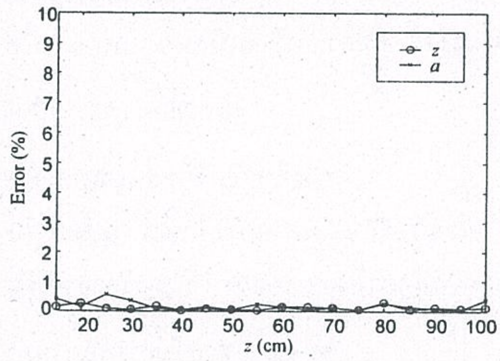

Figure 5. Error of finding a and z from simulated holograms.

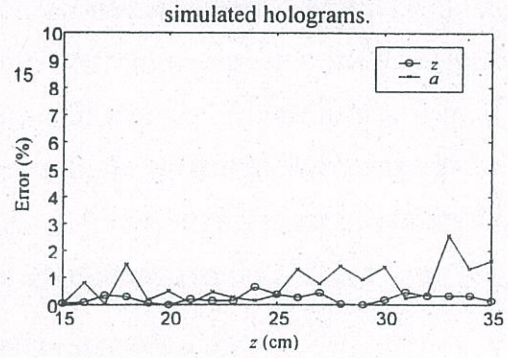

Figure 6. Error of finding a and z from holograms of fiber optic.

4. SYSTEM PERFORMANCE

The system performance of our proposed method is determined by the limited size and resolution of the employed CCD sensor. This can be explained by using 1-D notation as follows. The spatial resolution of the sensor must be high enough in order to assure that the interference pattern is not under sampled. The adequate size of the CCD is also required in order to reconstruct its nodal position correctly. Assume that the CCD sensor has the size of X and the resolution of N_x pixels. The spatial sampling frequency of the CCD can be mathematically expressed as

$$f_{CCD} = \frac{N_x - 1}{X}. \quad (2)$$

According to the Nyquist sampling theorem [5], the relationship between the frequency of the chirp signal and the sampling frequency f_{CCD} is given by

$$f_{CCD} > 2f_{chirp}. \quad (3)$$

Substitutions of the chirp frequency f_{chirp} given by $x/\Box z$ and Eq. (2) into Eq. (3) gives

$$x < \frac{\lambda z(N_x - 1)}{2X}. \quad (4)$$

Equation (4) describes the relationship between the analyzable area x in the hologram plane with the recording distance z. When the right term of the inequality of Eq. (4) is bigger than the CCD size, then the whole recorded hologram $x \le X$ can be analyzed. This condition is achieved if the recording distance $z > 2X^2/[\Box(N_x-1)]$. Then the analyzable area is determined as follows

$$x < \frac{\lambda z(N_x - 1)}{2X} \quad \text{if} \quad 0 < z \le \frac{2X^2}{\lambda(N_x - 1)} \quad (5a)$$

and

$$x \le X \quad \text{if} \quad z > \frac{2X^2}{\lambda(N_x - 1)}. \quad (5b)$$

On the other hand in order to extract the particle size, a minimum number of nodes n_{min} is required to be recorded within the CCD area. Since the position of the node is determined by the argument of the sinc function $2ax/\Box z$, the following relationship

$$\frac{2ax}{\lambda z} \ge n_{min}, \quad (6)$$

is obtained. The smallest diameter that can be measured is found by substituting Eqs. (5a) and (5b) into Eq. (6)

$$a \ge \begin{cases} \dfrac{Xn_{min}}{N_x - 1} & \text{if } 0 < z \le \dfrac{2X^2}{\lambda(N_x - 1)} \\[2ex] \dfrac{\lambda z n_{min}}{2X} & \text{if } z > \dfrac{2X^2}{\lambda(N_x - 1)} \end{cases}. \quad (7)$$

However, since the modulating function can be considered as being sampled by the chirp signal, the maximum frequency of the chirp must be bigger than the sinc function such as

$$\frac{x}{\lambda z} > c\frac{a}{\lambda z}, \quad (8)$$

where c is a constant of greater than 2. The biggest diameter of particle is also determined by the range of the recording distance. By substituting the corresponding analyzable size of the hologram Eqs. (5a) and (5b) into Eq. (8) we obtain

$$a < \begin{cases} \dfrac{\lambda z(N_x - 1)}{2cX}, & \text{if } 0 < z \le \dfrac{2X^2}{\lambda(N_x - 1)} \\[2ex] \dfrac{X}{c}, & \text{if } z > \dfrac{2X^2}{\lambda(N_x - 1)} \end{cases}. \quad (9)$$

The measurable size of particle and its recording distance are shown in a logarithmic scale in Fig. 7. The values of constant n_{min} and c are chosen based on the allowable error of the measurement.