Mathematical Modeling of Vibration Sensitivity in Unidirectional Polarimetric Current Sensors

S. Pothiya, P. Tantaswadi, and S. Maheshwari
Sirindhorn International Institute of Technology, Thammasat University
School of Electrical Engineering and Information Technology
PO Box 22, Thammasat Rangsit Post Office, Pathumthani 12121

Abstract

This paper proposed a novel mathematical modeling for vibration sensitivity in unidirectional polarimetric current sensors. We believe that it is the first time to model the effect of vibration to the single-mode sensing fiber on the current measurement error in unidirectional polarimetric current sensors. In this paper, simulation results agree well with experimental results done by Tantaswadi [1].

Keywords:linear birefringence, fiber-optic current sensors, magneto-optic current transformers, simulation and modeling, vibrations.

1. INTRODUCTION

Fiber-optic current sensors, which based on magneto-optic Faraday effect and Ampere's law, It have been received considerable attention for possible application in electric power industry as magneto-optic current transformers (MOCTs) in the past few decades [1]-[5].

However, the optical current sensor exhibits sensitivity to changes in the linear birefringence, δ, in their sensing coils. Changing stresses and temperature may change the total δ or its distribution along the sensing coil, which in turn changes the evolution of the state of polarization of the light through the sensing coil.

The basic theory of birefringence and Faraday rotation in optical fiber is well known, but some interesting conclusions from this theory concerning the application to vibration sensitivity in fiber optic current sensors have not been drawn yet. Although there are a few literatures [1]-[4] related to the vibration sensitivity in polarimetric current sensor, a general mathematical model of vibration sensitivity due to birefringence effect at the sensing part in unidirectional polarimetric current sensor has not been shown. Reference [1] shows experimental study of this system. Simulation results agree well with experimental results.

2. PRINCIPLES

When light propagates in an optical fiber wound around a current carrying wire, the induced magnetic field causes a rotation of the linear polarization plane of lightwave by the magneto-optic Faraday effect. This angle of Faraday rotation, F, through which the plane of polarization rotates, is given by

$$F = V\oint_C \vec{H}\cdot d\vec{l}\,, \qquad (1)$$

where V is the Verdet constant of the optical fiber, $\vec{H}$ is the magnetic field intensity along the direction of light propagation, and l is the optical path along the fiber loop. From the Ampere's law, this closed loop integral of magnetic field around a wire is proportional the current, I, flowing through it, *i.e.* $I = \oint_C \vec{H}\cdot d\vec{l}$. (2)

Therefore, the angle of rotation, F, in the fiber loop configuration is given by

$$F = VNI \qquad (3)$$

where N is the integral number of turns of fiber wrapped around the current carrying wire. The stability of Faraday rotation based current sensors, through the Verdet constant, depends on source wavelength and temperature.

To measure current I, with constants N and V, we can use polarimeter sensor to measure Faraday rotation F. Figure 1 shows unidirectional polarimetric current sensor set-up. Linear polarized light from HeNe laser is coupled to the single-mode fiber. Polarization controller in front of the sensing fiber is to launch the linear polarized into the fast axis of fiber loop forming a sensing part. Light passes through a polarizer (we usually call an "analyzer") at the fiber output. θ is the angle between the analyzer and fast birefringence axis at the end of sensing fiber loop. A simplified configuration consists of input linear polarized electric field (E_{in}) entering the sensing fiber loop, an analyzer aligned θ degrees with respect to the birefringence fast axis of the fiber loop, and a detector. The analyzer angle θ of 45^o provides maximum sensitivity and linear dynamic range.

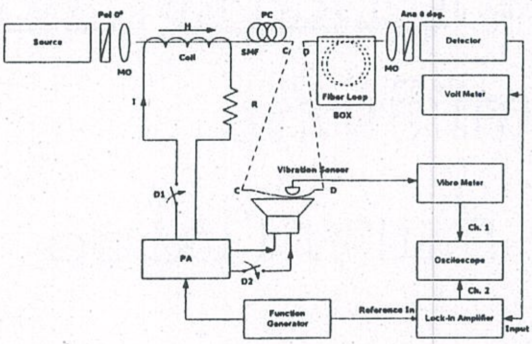

Figure 1 Unidirectional polarimetric current sensor and its experimental set-up, MO: microscope objective lens, SMF: single-mode fiber, PC: polarization controller, Ana: analyzer, PA: power amplifier

The output electric field (E_{out}) of the sensor impinging on the detector can be described by Jones Calculus [2]

$$E_{out} = P \cdot R(\theta) \cdot \vec{L} \cdot E_{in} \tag{4}$$

where $E_{in}, \vec{L}, R(\theta)$, and P represent input linearly polarized light in the x-axis, sensing fiber matrix, coordinate rotation matrix with the angle difference between the output analyzer and the birefringence fast axis at the fiber end (θ), and P is the analyzer matrix, respectively.

$$\vec{L} = \begin{bmatrix} A & -B \\ B & A^* \end{bmatrix} \tag{5}$$

where

$$A = \cos\left(\frac{\alpha}{2}\right) + j\sin\left(\frac{\alpha}{2}\right)\cos(\chi) \tag{6}$$

$$B = \sin\left(\frac{\alpha}{2}\right)\sin(\chi) \tag{7}$$

$$\frac{\alpha}{2} = \sqrt{(F+T)^2 + \left(\frac{\delta}{2}\right)^2} \tag{8}$$

$$\tan\chi = \frac{2(F+T)}{\delta} \tag{9}$$

where δ, F, T, and α are the linear birefringence, Faraday rotation, twisted (circular) birefringence, and total birefringence in radians. Coordination rotation matrix $R(\theta)$ is given by

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{10}$$

In our experiment, we align the analyzer with θ of 60^o, where the output signal-to-noise ratio (SNR) is better than that at θ of 45^o, where the sensitivity of the sensor is maximum. Using equation (4) to (10), normalized output intensity (Intensity $\propto |E_{out}|^2$) on the detector is given by

$$\frac{I_{60^o}}{I_0} = \frac{1}{4} + \frac{\sqrt{3}}{2}\frac{\sin(\alpha)}{\alpha} \cdot (F+T) + 2\left[\frac{\sin(\alpha/2)}{\alpha} \cdot (F+T)\right]^2 \tag{11}$$

where I_0 is the maximum intensity at the detector. Substituting equation (8) into equation (11), we get

$$\frac{I_{60^o}}{I_0} = \frac{1}{4} + \frac{\sqrt{3}}{2}\frac{\sin\left(\sqrt{4(F+T)^2+\delta^2}\right)}{\sqrt{4(F+T)^2+\delta^2}} \cdot (F+T) + \left[1 - \cos\left(\sqrt{4(F+T)^2+\delta^2}\right)\right] \cdot \frac{(F+T)^2}{4(F+T)^2+\delta^2} \tag{12}$$

2.1 Current measurement

Obviously, equation (12) shows that the normalized output intensity is a function of F, δ, and T. With the input light aligned to birefringence fast axis of the sensing fiber, in an ideal fiber ($\delta, T \approx 0$), we would expect only Faraday rotation ($F=VNI$) to occur. Equation (12) becomes

$$\frac{I_{60^o}}{I_0} = \frac{1}{2}\left[1+\sin\left(2F-30^o\right)\right] \quad (13)$$

Equation (13) indicates that the normalized output intensity is a constant when there is no applied cur-rent ($F=0$). When AC current ($I(t)=A\sin\omega t$) is applied (where A is zero-to-peak *(0-p)* amplitude in Ampere), F can be described by

$$F(t)=VNI(t)=(VNA)\sin\omega t = F_0\sin\omega t \quad (14)$$

where $\omega = 2\pi f$ and f is frequency of the applied current. V is the Verdet constant and a function of wavelength (4.68 μ *rad*/A_{rms} for wavelength of 633 nm). Then, $F_0 = VNA$ is a constant and depends on the current amplitude, the number of turns, and wavelength of operation. Substituting equation (14) into (13), we obtain the phase modulation signal. Detector's output voltage (v_D) is proportional to the impinged output intensity ($v_D \propto I_{60^o}$) and is given by

$$v_D = \frac{K}{2}\left[1+\sin\left(2F_0\sin\omega t-30^o\right)\right] \quad (15)$$

where K is the maximum output voltage at the detector. When F_0 is small, the output voltage at the detector of equation (15) can be approximated by

$$v_D(t)=v_{DC}+v_{AC}(t)\approx v_{DC}+v_{(0-p)}\sin\omega t$$
$$\approx \frac{K}{2}\left[1+2\sqrt{3}\sin\omega t\right] ,$$
(16)

when F_0 is small and less than 0.04 radian, the output zero-to-peak *(0-p)* detector voltage $v_{(0-p)}$ is linear proportional to the applied current amplitude (A) as given by $v_{(0-p)} \propto 2\sqrt{3}F_0$.

2.2 Signal Modulation

Current measurement can be found by a ratio of $v_{(0-p)}$ to v_{DC}, which is known as "signal modulation." Thus, signal modulation using equation (16) is given by

$$v_{(0-p)}/v_{DC} = 2\sqrt{3}F_0 = 2\sqrt{3}VNA \quad (17)$$

The sensor output is linear up to 0.04 *rad* ($\sin(x)\approx x$, when $x<0.04\,rad$) or about 8,500 A_{rms} (0.04 *rad*/ V ; $V = 4.68\times10^{-6}$ for wavelength of 633 nm). If the applied current is 1,000 A_{rms}, the signal modulation using equation (17) will be $1.62\text{x}10^{-2}$. In practice, δ is not negligible but T is *e.g.* $\delta = 0.660$ and $T = 0.05$ radians. When the applied current is 1,000 A_{rms}, the signal modulation using equation (12) is $1.378\text{x}10^{-2}$. Scale factor of the sensor is defined by

$$SF = \frac{\text{signal modulation (in experiment)}}{\text{signal modulation of the ideal case}}.$$

In this case, the scale factor SF is 0.85.

3. MATHEMATICAL MODELING OF LINEAR BIREFRINGENCE CHANGE DUE TO VIBRATIONS

We model the linear birefringence in the sensing fiber being composed of bending-induced linear birefringence (δ_{DC}) and vibration-induced linear birefringence caused by transverse strain or vibration (δ_v), which can be described by [3]

$$\delta_v = \frac{2\pi}{\lambda}\Delta nl , \quad (18)$$

where Δn is the refractive index change induced by stress in the medium (silica in this case), l is the effective length (under perturbation) of fiber, and λ is the center wavelength of the source. The refractive index change is given by

$$\Delta n = -\frac{n^3}{2}p\sigma = 0.311\sigma , \quad (19)$$

where n is the (unperturbed) refractive index of medium, σ is the strain and p is the photoelastic constant of fiber ($p=0.2$ in silica). Value of the Faraday rotation (VNI) depends on the Verdet constant (V). Typical value of V is 4.68 μ*rad*/*A* or 0.268°/(*kA*). Because the vibration affects linear birefringence, to induce a π-radian birefringence change $\Delta n = \lambda/2 = 3.165\times10^{-7}$ for a fiber length of 1 m. We can find the strain of $1.019\text{x}10^{-6}$ using Equation (19).

The (static) bending-induced birefringence (δ_{DC}) per turn of a fiber loop with radius R under no tension can be described by [3]

$$\delta_{DC} \approx \frac{0.6\pi}{R} \ ^{o}/\text{turn}. \quad (20)$$

In our case, R of 0.3 m results in δ_{DC} of 6.3 ^{o}/turn. Six turns of fiber loop result in δ_{DC} of 37.8^{o}.

Assume that (total) linear birefringence (δ) in single-mode sensing part is the algebraic sum of the (static) bending-induced linear birefringence (δ_{DC} due to loop radius) and vibration-induced linear birefringence (δ_V). Then, the (total) linear birefringence can be expressed by

$$\delta = \delta_{DC} + \delta_v \sin(\omega_v t) \quad (21)$$

where $\omega_v = 2\pi f_v$ and f_v is the frequency of the vibration δ_v and T is much smaller than δ_{DC} in our configuration.

4. CURRENT MEASUREMENT ERROR DUE TO VIBRATIONS

In this section we formulate a mathematical model of the vibration sensitivity or current measurement error due to vibration. In equation (12), even when no current and no vibration are applied ($F = 0$ and further assume that the δ_{DC} and T to be constant), the output will be constant. However, the presence of vibration perturbs the linear birefringence and modulates the angular rotation, which cannot be distinguishable from that of the Faraday rotation (F). This results in signal modulation even there is no applied current. It is named apparent current or false current. Substituting equation (21) into (12), we obtain a phase modulation signal. Assume the change of δ_{DC} and T (from vibrations) to be small and negligible. In this case, we formulate a mathematical model for vibration sensitivity when only vibration on the sensing fiber ($F = 0$) to be given by

$$\frac{I_{60^{\circ}}}{I_0} = \frac{1}{4} + \frac{\sqrt{3}}{2}\frac{\sin\left(\sqrt{4T^2 + (\delta_{DC} + \delta_v \sin \omega_v t)^2}\right)}{\sqrt{4T^2 + (\delta_{DC} + \delta_v \sin \omega_v t)^2}} \cdot T + \left[1 - \cos\left(\sqrt{4T^2 + (\delta_{DC} + \delta_v \sin \omega_v t)^2}\right)\right] \cdot \frac{T^2}{4T^2 + (\delta_{DC} + \delta_v \sin \omega_v t)^2} \quad (22)$$

Using Lock-in amplifier or the spectrum analyzer, we can examine the frequency components of the DC, first-harmonic, second-harmonic, and higher harmonic components. Assume the values of the δ_{DC} and T to be 0.660 (37.8^{o}) and 0.05 radians, respectively. The value of δ_V is assumed to vary linearly with the mechanical vibration to the sensing fiber and, thus, signal modulation varies with the same frequency. When δ_V is small and the applied vibration is of single-frequency, the signal modulation due to vibration can be approximated by (when $2T, \delta_v << \delta_{DC}$)

$$I_{60^{\circ}}/I_0 = \frac{1}{4} + \frac{\sqrt{3}}{2}\frac{\sin(\delta_{DC} + \delta_v \sin \omega_v t)}{\delta_{DC} + \delta_v \sin \omega_v t} \cdot T + [1 - \cos(\delta_{DC} + \delta_v \sin \omega_v t)] \cdot \frac{T^2}{(\delta_{DC} + \delta_v \sin \omega_v t)^2} \quad (23)$$

Expanding equation (23) using Bessel function(see Appendix), we obtain

$$\left.\frac{I_{60^{\circ}}}{I_0}\right|_{Approx} = \frac{1}{4} + \frac{\sqrt{3}}{2} B_1 \begin{bmatrix} J_0(\delta_v)\sin(\delta_{DC}) + \\ 2J_1(\delta_v)\cos(\delta_{DC})\sin(\omega_v t) \end{bmatrix} + B_2 \begin{bmatrix} 1 - J_0(\delta_v)\cos(\delta_{DC}) + \\ 2J_1(\delta_v)\sin(\delta_{DC})\sin(\omega_v t) \end{bmatrix} \quad (24)$$

where J_k is the Bessel function of the first kind of order k.

5. SIMULATED FALSE CURRENT DUE TO VIBRATIONS

In Figure2, I and I1f represent exact and approximated signal modulation calculation using equation (23) and (24), respectively. It shows that when the vibration is small and is of single-frequency, the signal modulation varies linearly with δ_v and has the same frequency as that of the vibration. The other higher harmonics are negligible. The relation between vibration-induced linear birefringence δ_v and signal modulation is

$$\text{Signal modulation} = 3.1715 \times 10^{-2} \delta_v \quad (25)$$

For example, δ_v of 10^{-2} *rad* will result in signal modulation of 3.1715x10^{-2}% or false current of 23.02 *Arms. turn* using signal modulation of 1.378x10^{-2} for 1,000 *Arms. turn* (see section 2.2). Similarly, we find the relationship between false current and δ_v *(rad) i.e.*

$$\text{False current } [A_{rms}\cdot turn] = 2.302\times10^{+3}\delta_v \quad (26)$$

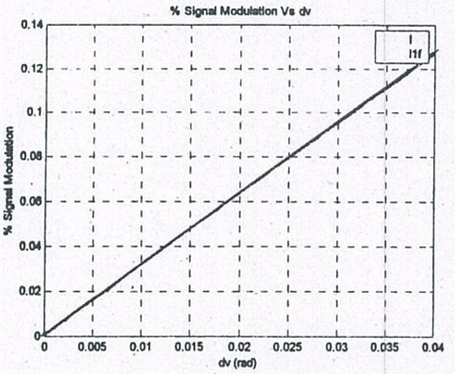

Figure 2 Signal modulation Vs vibration-induced linear birefringence δ_v

6. CONCLUSION

We have demonstrated a novel mathematical modeling for vibration sensitivity in the unidirectional polarimetric current sensor (UDPS). The experimental study shows that the linear birefringence δ_V due to mechanical vibration changes linearly with vibration amplitude. This sensor's residual sensitivities to environmental disturbances can be further reduced by a damped packing such as putting sensing fiber part in a gel-filled tube together with employing two polarization scheme, where the two sensor's output at the +45° and –45° to the output fiber axes. Also, 'spun' low birefringence or annealed fiber can be used to reduce the value of (static) linear birefringence δ_{DC}. Further investigation of the magnitude of linear birefringence change under the influence of vibration should be done experimentally with the help of optical modulator.

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8. APPENDIX

We can expand equation (23) using Bessel function. In equation (23), first we define

$$B_1 = \frac{T}{\sqrt{4T^2+\delta^2}} = \frac{T}{\sqrt{4T^2+\left(\delta_{DC}+\delta_v\sin(\omega_v t)\right)^2}}$$

$$B_2 = \frac{T^2}{4T^2+\delta^2} = \frac{T^2}{4T^2+\left(\delta_{DC}+\delta_v\sin(\omega_v t)\right)^2},$$

where

$$\sin(\delta_v\sin(\omega_v t)) = 2\sum_{k=0}^{\infty} J_{2k+1}(\delta_v)\sin((2k+1)\omega_v t)$$

$$\cos(\delta_v\sin(\omega_v t)) = J_0(\delta_v) + 2\sum_{k=1}^{\infty} J_{2k}(\delta_v)\cos(2k\omega_v t)$$

$$\left.\frac{I_{60^\circ}}{I_0}\right|_{Approx} = \frac{1}{4} + \frac{\sqrt{3}}{2}B_1\left[J_0(\delta_v)\sin(\delta_{DC}) + 2J_1(\delta_v)\cos(\delta_{DC})\sin(\omega_v t)\right]$$
$$+ B_2\left[1 - J_0(\delta_v)\cos(\delta_{DC}) + 2J_1(\delta_v)\sin(\delta_{DC})\sin(\omega_v t)\right]$$
$$+ \text{higher harmonic terms}$$

When $\delta_v \ll 1$, the contribution of higher harmonic terms is negligible and the approximation of $J_0(\delta_v)\approx 1$ and $J_1(\delta_v)\approx 0.5\delta_v$.

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