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WEIGTHED DESIGN OPTIMALITY CRITERIA FOR SPHERICAL RESPONSE SURFACE DESIGNS

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ABSTRACT

In this paper, D, A, G, and IV optimality criteria are developed by using prior probability assignments to model effects. Hence, one and three center point for response surface designs for three design variables ($K = 3$) in a spherical design region are considered over sets of reduced models for weak and strong heredity. These two specific classes of reduced models are formed by removing terms based on hierarchical structures. The spherical response surface designs are central composite design (CCDs), Box-Behnken design (BBDs), small composite design (SCDs), uniform shell design (UNFSDs), and hybrid 310, 311A, 311B designs. The results of weak and strong heredity models provide reliable interpolation estimates of weighted optimality criterion values for other choices of p_l, p_1, p_2, and p_q.

KEYWORDS: design optimality criteria, spherical response surface design, reduced models.

1. INTRODUCTION

New types of D, A, G, and IV optimality criteria for response surface designs for three design variables ($K = 3$) in a spherical design region are developed by using prior probability assignments to model effects. Thus, one and three center point for central composite design (CCDs), Box-Behnken design (BBDs), small composite design (SCDs), uniform shell design (UNFSDs), and hybrid design 310, 311A, 311B are considered over a set of reduced models.

By studying the optimality criteria, the adequacy of a proposed experimental design can be assessed prior to running it. In addition, if several alternative designs are proposed, their optimality properties can be compared to aid in the choice of design. Also, the experimenter needs to be aware that although a design may be best among several designs by one optimality criterion, it may perform poorly when evaluated by a different criterion. Hence, the choice of design will also depend upon the choice of the evaluation criterion.

Design optimality criteria are primarily concerned with "optimal properties" of the $\mathbf{X'X}$ matrix for the design matrix $\mathbf{X}$. The D, A, G, and IV design optimality measures used in this paper and calculated over a set of reduced model of the second-order model can be written as:

$$D\text{-efficiency} = 100\frac{|\mathbf{X'X}|^{1/p}}{N}, \quad A\text{-efficiency} = 100\frac{p}{trace[N(\mathbf{X'X})^{-1}]}$$

$$G\text{-efficiency} = 100\frac{p}{N\hat{\sigma}^2_{\max}}, \quad IV\text{-criterion} = N\sigma^2_{ave}$$

where $\mathbf{X}$ is the design matrix, p is the number of model parameters, N is the design size, $\hat{\sigma}^2_{\max}$ is the maximum of $\mathbf{f}'(\mathbf{x})(\mathbf{X'X})^{-1}\mathbf{f}(\mathbf{x})$ approximated over the set of candidate points, and σ^2_{ave} is the average of $\mathbf{f}'(\mathbf{x})(\mathbf{X'X})^{-1}\mathbf{f}(\mathbf{x})$ over the design space. For $D, A,$ and G efficiencies, larger values imply a better design, while for IV criterion, a smaller value implies a better design.

The four new $D, A, G,$ and IV criteria that use prior probability assignment to the model effects will be referred to as *weighted design optimality criteria*. The terminology and notation of Chipman [1] and the set of reduced models for weak heredity and strong heredity of Borkowski [2] are adopted.

2. INHERITANCE PRINCIPLES FOR REDUCED MODELS

Because design selection based on an optimality criterion is highly dependent upon the approximating response surface model, we will get different design optimality criterion values from different models. In practice, this means the experimenter selects a design that is based on a model proposed prior to a data collection. When data are collected and the model's parameters are fitted, it is often determined that many parameter estimates are not statistically significant. Thus, a *reduced model* retaining only significant terms is adopted. Therefore, a robust design should be considered over the set of potential reduced models and not over a single model.

Chipman [1] and Chipman and Hamada [3] studied classes of reduced models. Two specific classes of reduced models are formed by removing terms based on hierarchical structures. These models are based on the following two *heredity* concepts.

1. *Weak heredity* (WH) requires that (i) if model contains an x_i^2 term, then it must contain the corresponding x_i term and (ii) if a model contains an $x_i x_j$ term, then it must contain either the x_i or x_j term (or both).

2. *Strong heredity* (SH) requires that (i) if model contains an x_i^2 term, then it must contain the corresponding x_i term and (ii) if a model contains an $x_i x_j$ term, then it must contain both the x_i and x_j terms.

A model can be defined by a vector δ where each element of δ is either "1"or "0". The "1" indicates an active effect and the "0" indicates an inactive effect. Let δ_i, δ_{ii}, and δ_{ij} represent the indicator function values of the i^{th} first-order effect, the i^{th} second-order effect, and the ij^{th} interaction effect, respectively. Then, $\delta = (\delta_1, \delta_2, \delta_3, \delta_{12}, \delta_{13}, \delta_{23}, \delta_{11}, \delta_{22}, \delta_{33})$ for $K = 3$ will represent the corresponding δ-vectors. For example, $\delta = (1, 0, 1, 1, 0, 1, 0, 0, 1)$ corresponds to the model $y = \beta_0 + \beta_1 x_1 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{23} x_2 x_3 + \beta_{33} x_3^2 + \varepsilon$. For WH, there are $\sum_{i=0}^{K} \binom{K}{i} (2^{Ki - i(i-1)/2})$ reduced models of the full second-order model. Thus, for $K = 3$ design variables, there are 185 WH models. For SH, there are $\sum_{i=0}^{K} \binom{K}{i} (2^{i(i+1)/2})$ reduced models of the full second-order model. Thus, there are 95 SH models for $K = 3$ design variables (Borkowski [4]).

3. MODEL PROBABILITIES

When prior specification of probabilities are applied to the weak and strong heredity models, the independence-of-effects principle is assumed as in Chipman [1]. That is, we assume (i) the linear effects (δ_i's) are independent, (ii) the interaction effects (δ_{ij}'s) are independent of each other and each δ_{ij} only depends on its parents (δ_i and δ_j), and (iii) the quadratic effects (δ_{ii}'s) are independent of each other and each δ_{ii} only depends on the parent δ_i. Therefore, if either weak or strong heredity can be reasonably assumed, the joint density of δ can be written as:

$$\Pr(\delta) = \left(\prod_{i=1}^{K} \Pr(\delta_i)\right)\left(\prod_{i<j}^{K} \Pr(\delta_{ij} \mid \delta_i, \delta_j)\right)\left(\prod_{i=1}^{K} \Pr(\delta_{ii} \mid \delta_i\right) \quad (1)$$

where $\Pr(\delta_i)$, $\Pr(\delta_{ij} \mid \delta_i, \delta_j)$, and $\Pr(\delta_{ii} \mid \delta_i)$ are experimenter-assigned prior probabilities that the corresponding $x_i, x_i x_j$, and x_i^2 terms, respectively, are in the model.

If each variable is treated as equally important so that prior probabilities do not depend on particular variables, but only on the type of model term, prior probabilities are equal for linear effects (2), for interaction effects (3), and for quadratic effects (4). That is,

$$\Pr(\delta_i = 1) = p_l \quad \text{for all } i, \quad (2)$$

$$\begin{aligned} \Pr(\delta_{ij} = 1 \mid \delta_i, \delta_j) &= p_0 \quad \text{if } (\delta_i, \delta_j) = (0,0), \\ &= p_1 \quad \text{if } (\delta_i, \delta_j) = (0,1) \text{ or } (1,0), \\ &= p_2 \quad \text{if } (\delta_i, \delta_j) = (1,1), \end{aligned} \quad (3)$$

$$\begin{aligned} \Pr(\delta_{ii} = 1 \mid \delta_i) &= p_q \quad \text{if } \delta_i = 1, \\ &= p_3 \quad \text{if } \delta_i = 0. \end{aligned} \quad (4)$$

Hence, for WH, (p_0, p_1, p_2) = (0, p_1, p_2) for some specified values of p_1 and p_2. For SH, (p_0, p_1, p_2) = (0, 0, p_2) for some specified values of p_2. For both WH and SH, $p_3 = 0$. In this paper. WH and SH models with $p_l \in \{.6, .7, .8, .9\}$, $p_1 \in \{.4, .6, .8\}$, $p_2 \in \{.5, .7, .9\}$, and $p_q \in \{.5, .7, .9\}$ for the response surface designs in a spherical design region are presented.

4. WEIGHTED DESIGN OPTIMALITY CRITERIA

To study how robust a design is to model misspecification, the assumption of either WH or SH and the experimenter-assignment of prior probabilities are used to calculate a weighted average of the criterion values across all WH or SH models.

1. For WH and prior p_l, p_1, p_2, and p_q probabilities: the *weighted D – optimality criterion under WH* will be defined as

$$D_w = \sum_{i=1}^{M} D(i)\Pr(\delta = \Delta_i) \tag{5}$$

where M = number of reduced WH models = 185 for $K = 3$ design variables

$\Delta_i = \delta$-vector for model i, $i = 1, 2, \ldots, M$

$D(i) = D$-criterion for model i, $i = 1, 2, \ldots, M$

$\Pr(\delta = \Delta_i)$ = $\Pr(\delta)$ in (1) evaluated for Δ_i

2. For SH and prior p_l, p_2, and p_q probabilities: the *weighted D – optimality criterion under SH* will be defined as

$$D_s = \sum_{i=1}^{N} D(i)\Pr(\delta = \Delta_i) \tag{6}$$

where the summation is now over the N = 95 for $K = 3$ design variables.

The *weighted A, G, and IV -optimality criteria under WH*, denoted A_w, G_w, and IV_w are defined by replacing $D(i)$ with $A(i)$, $G(i)$, and $IV(i)$ in (5), respectively. The *weighted A, G, and IV -optimality criteria under SH*, denoted A_s, G_s, and IV_s are defined by replacing $D(i)$ with $A(i)$, $G(i)$, and $IV(i)$ in (6), respectively.

For symmetric designs: CCDs, BBDs, and 311Bs, an alternate form to exploit the symmetry can be used. That is, the weighted D-optimality criterion under WH can be written as

$$D_w = \sum_{i=1}^{M^*} m(i)D(i)\,\Pr(\delta = \Delta_i^*) \qquad (7)$$

Where M^* = number of reduced nonequivalent WH models = 41 for $K = 3$ design variables

$m(i)$ = the number of models equivalent to model i

Δ_i^* = δ-vector for model i, $i = 1, 2, \ldots, M^*$

$D(i)$ = D-criterion for model i, $i = 1, 2, \ldots, M^*$

$\Pr(\delta = \Delta_i^*) = \Pr(\delta)$ in (1) evaluated at Δ_i^*.

Similarly, the weighted D-optimality criterion under SH can be written as

$$D_s = \sum_{i=1}^{N^*} m(i)D(i)\,\Pr(\delta = \Delta_i^*) \qquad (8)$$

where the summation is over the N^*=25 reduced nonequivalent SH models for $K = 3$ design variables. Alternate forms for A_w, G_w, and IV_w across WH models and A_s, G_s, and IV_s are defined by replacing $D(i)$ with $A(i)$, $G(i)$, and $IV(i)$ in (7) and (8), respectively.

<u>An Example</u>

Suppose a 3-factor 15-point CCD is considered. The $D, A, G,$ and IV-criteria for the subset of 41 nonequivalent WH models are shown in Tables 1. In the Table 1, each row represents one of the 41 nonequivalent WH models. The $m(i)$ column indicates the number of models equivalent to the model i. For the model in row i, the $D, A, G,$ and IV criteria are, respectively, in the $D(i)$, $A(i)$, $G(i)$, and $IV(i)$ columns. This table shows how optimality measures can significantly vary across models and an experimenter should not rely only on the criteria associated with the full second-order model (which is model $i = 41$ in Table 1).

Table 2 shows how the weighted D_w, and D_s optimality criteria for the 3-factor 15-point CCD are calculated. The two set of prior probabilities for 41 nonequivalent WH and 25 nonequivalent SH models are considered. That is, (i) for WH models with (p_l, p_1, p_2, p_q) = (.9, .4, .5, .7) and (ii) for SH models with (p_l, p_2, p_q) = (.9, .5, .7). For the WH and SH models, the $\Pr(\delta = \Delta_i^*)$ columns, respectively, correspond to the WH and SH prior probabilities defined in Equation 1. Then, these probabilities are multiplied by the corresponding $m(i)D(i)$ as it is defined in Equation 7 or 8 and the i^{th} model's contributions to the weighted optimality criteria D_w, and D_s are found. These products are given in their respective columns and rows. To calculate the D_w, and D_s, the summation of these corresponding columns are computed and given at the bottom of the table.

To study the behavior of a design with respect to choices of p_l, p_1, p_2, and p_q for WH models and p_l, p_2, and p_q for SH models, the weighted design optimality criteria with respect to various choices prior probabilities are evaluated. In this paper, WH and SH models with $p_l \in \{.6,.7,.8,.9\}$, $p_1 \in \{.4,.6,.8\}$, $p_2 \in \{.5,.7,.9\}$, and $p_q \in \{.5,.7,.9\}$ for the response surface designs in a spherical design region are studied. Then, a full second-order response surface model is fitted. That is, for WH, D_w, A_w, G_w, and IV_w are evaluated at the 108 points of the 4×3^3 factorial design in p_l, p_1, p_2, and p_q. Table 3 shows a subset of the results from the 108 points for the 3-factor 15-point CCD. For WH, the 15-parameter full second-order model to be fit is

$$\text{Eff}_w = \beta_0 + \beta_l p_l + \beta_q p_q + \beta_1 p_1 + \beta_2 p_2 + \beta_{ll} p_l^2 + \beta_{qq} p_q^2 + \beta_{11} p_1^2 + \beta_{22} p_2^2 + \beta_{lq} p_l p_q + \beta_{l1} p_l p_1 + \beta_{l2} p_l p_2 + \beta_{q1} p_q p_1 + \beta_{q2} p_q p_2 + \beta_{12} p_1 p_2 + \varepsilon \quad (9)$$

Where Eff_w is D_w, A_w, G_w, or IV_w. Table 7 contains the estimated β-coefficients for D_w, A_w, G_w, or IV_w of the 3 factor response surface designs.

For SH, D_s, A_s, G_s, and IV_s are evaluated at the 36 points of the 4×3^2 factorial design and the 10-parameter full second-order model to be fit is

$$\mathrm{Eff}_s = \beta_0 + \beta_l p_l + \beta_q p_q + \beta_2 p_2 + \beta_{ll} p_l^2 + \beta_{qq} p_q^2 + \beta_{22} p_2^2$$
$$+ \beta_{lq} p_l p_q + \beta_{l2} p_l p_2 + \beta_{q2} p_q p_2 + \varepsilon \qquad (10)$$

Where Eff_s is D_s, A_s, G_s, or IV_s. For WH and SH for 19 3-factor designs across all 4 optimality criteria, a total of 152 models were fit based on Equation 9 and 10. The minimum R^2 is .9974. These models will provide reliable interpolation estimates of weighted optimality criterion values for other choices of p_l, p_1, p_2, and p_q.

5. WEIGHTED DESIGN OPTIMALITY CRITERIA COMPARISONS

Examples of comparisons of weighted design optimality criteria for 3-factor small spherical response surface designs by ranking within a design optimality criterion and a design size (N = 11,13) are given in Table 4, 5, and 6. Recall that for D, A, and G efficiencies, larger values imply a better design, while for IV criterion, a smaller value implies a better design. That is, Table 4, 5, and 6 show the results of ranking 3-factor small spherical response surface designs when N = 11,13 for each criteria (D, A, G, and IV) for full model optimality criteria, weighted optimality criteria across WH models with $p_l = .9$, $p_1 = .4$, $p_2 = .5$, and $p_q = .7$, and weighted optimality criteria across SH Models with $p_l = .8$, $p_2 = .5$, and $p_q = .7$, respectively. These tables indicate the following results:

1. *D, A, and IV criterion values are conservative while the G criterion value is not*: Every weighted D, A, and IV optimality criterion value in Table 5 and 6 is better than its associated D, A, or IV value in Table 4. However, for G, most of weighted optimality criterion values in Table 5 and 6 are smaller than the associated G criterion values in Table 4 with the exception being the SCDs. Thus, if only the full model optimality values are considered, the experimenter is being conservative for the D, A, and IV criteria because they are underestimates relative to the weighted design optimality criteria values. Conversely, the experimenter is often using a liberal criterion value when the G-criterion is considered.

2. *Large differences can occur between full-model and weighted criterion values*: The 13-

point UNFSD is tied for being the worst design based on $IV = 16.36$. However, in Table 6, its $IV_s = 5.58$ is the third best.

3. *Weighted criteria rankings can dramatically change*: The 13-point UNFSD has A -rank of 5 in Table 4 but A_s -rank of 1 in Table 6.

4. *Designs that were obviously strongest by* D, A, G, *or* IV *, now have competitors*: For the 11-point designs and the IV criterion, the 310 design is 3.75 units better that the 311B design. However, the IV_w values are nearly identical in Table 5. Similarly, the A -efficiencies for the BBD and 311A designs are 15% apart but the A_s values are less than 1% apart in Table 6.

6. CONCLUSIONS

For experimenters considering larger designs and designs with more factors, the results on small designs in this paper serve as a reminder that the chosen design may not be as efficient as you believe it to be. Therefore, if we assume that the full second-order response surface model is reduced after an experiment has been performed, then the experimenter should exercise caution when choosing a design. When a researcher must decide which response surface design is 'best' based on one or more design optimality criteria, it is important that the optimality criteria are determined over a subset of possible reduced models.

Table 1. Optimality Criteria of a 15-Point CCD for WH Models, $K = 3$.

model	δ_1	δ_2	δ_3	δ_{12}	δ_{13}	δ_{23}	δ_{11}	δ_{22}	δ_{33}	$m(i)$	$D(i)$	$A(i)$	$G(i)$	$IV(i)$
1	0	0	0	0	0	0	0	0	0	1	100.0000	100.0000	100.0000	1.0000
2	1	0	0	0	0	0	0	0	0	3	55.7766	47.4567	47.4567	1.0357
3	1	0	0	0	0	0	1	0	0	3	31.0047	19.1536	32.7226	1.5022
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
39	1	1	1	1	1	1	1	0	0	3	75.5678	66.6141	82.3331	5.9801
40	1	1	1	1	1	1	1	1	0	3	75.2769	56.5984	91.0192	8.3327
41	1	1	1	1	1	1	1	1	1	1	71.1296	32.4011	66.6667	17.9453

Table 2. The WH and SH Model Probabilities for a 3 Factor 15-Point CCD with p_l =.9, p_1 =.4, p_2 =.5, and p_q =.7.

Model (i)	Weak Heredity		Strong Heredity	
	$Pr(\delta = \Delta_i^*)$	$\frac{m(i)D(i)}{Pr(\delta = \Delta_i^*)}$	$Pr(\delta = \Delta_i^*)$	$\frac{m(i)D(i)}{Pr(\delta = \Delta_i^*)}$
1	0.001000000	0.1000	0.001000000	0.1000
2	0.000972000	0.1626	0.002700000	0.4518
3	0.002268000	0.2110	0.006300000	0.5860
4	0.000648000	0.2054	-	-
⋮	⋮	⋮	⋮	⋮
40	0.013395375	3.0251	0.013395375	3.0251
41	0.031255875	2.2232	0.031255875	2.2232
		D_w =76.3221		D_s =72.3000

Table 3. Weighted Optimality Criteria for the 3-Factot 15-Point CCD Across WH Models.

p_l	p_q	p_1	p_2	D_w	A_w	G_w	IV_w
.60	.50	.40	.50	60.0636	52.0843	48.2801	4.4597
.60	.50	.40	.70	58.9372	51.1790	49.4657	4.5687
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
.60	.90	.80	.90	63.3258	53.9764	55.5364	5.9185
.70	.50	.40	.50	61.2073	52.8283	48.5234	5.0038
.70	.50	.40	.70	59.7481	51.6875	50.1989	5.1496
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
.90	.90	.80	.70	65.4661	52.7685	59.3899	7.7510
.90	.90	.80	.90	63.9231	52.1464	62.8065	7.9795

Table 4. Full Model Optimality Criteria for Small Response Surface Designs.

Designs	r_s	n_0	N	D		A		G		IV	
SCD	1	1	11	59.0785	*4*	28.1641	*4*	32.7923	*4*	17.0840	*4*
310	-	1	11	60.6397	*3*	45.7457	*1*	45.0198	*3*	10.6710	*1*
311A	-	1	11	67.6003	*2*	37.4090	*3*	78.6243	*2*	14.4549	*3*
311B	-	1	11	70.9973	*1*	37.8798	*2*	90.9091	*1*	14.4290	*2*
BBD	-	1	13	69.5854	*2*	35.5007	*4*	76.9140	*3*	16.3622	*5.5*
SCD	1	3	13	55.7945	*5*	32.8879	*6*	27.7473	*6*	12.1843	*4*
UNFSD	-	1	13	69.5913	*1*	34.0475	*5*	76.9231	*2*	16.3622	*5.5*
310	-	3	13	55.0194	*6*	47.1490	*3*	38.9577	*5*	9.6415	*3*
311A	-	3	13	63.8425	*4*	50.6899	*2*	69.0153	*4*	9.2126	*1.5*
311B	-	3	13	67.0507	*3*	50.9072	*1*	77.4084	*1*	9.2126	*1.5*

Note: Italicized values indicate rank within column (design optimality criterion) and design size (N = 11,13),

r_s = number of star point, n_0 = number of center point.

Table 5. Weighted Optimality Criteria for Small Response Surface Designs Across WH Models with $p_l = .9$, $p_1 = .4$, $p_2 = .5$, and $p_q = .7$.

Designs	r_s	n_0	N	D_w		A_w		G_w		IV_w	
SCD	1	1	11	69.9093	*3*	43.0961	*4*	43.0227	*3*	8.9139	*4*
310	-	1	11	62.0937	*4*	48.4534	*3*	38.5379	*4*	7.7151	*1*
311A	-	1	11	71.3371	*2*	50.6298	*2*	60.6570	*2*	8.3490	*3*
311B	-	1	11	76.3152	*1*	53.9327	*1*	69.5248	*1*	7.9384	*2*
BBD	-	1	13	71.9468	*2*	48.8510	*5*	57.3411	*3*	9.4461	*6*
SCD	1	3	13	64.0253	*5*	43.39539	*6*	37.1165	*5*	7.6786	*4*
UNFSD	-	1	13	75.7160	*1*	51.7501	*3*	62.3268	*1*	8.6074	*5*
310	-	3	13	56.4586	*6*	48.1721	*4*	33.9802	*6*	7.2912	*3*
311A	-	3	13	65.7684	*4*	54.3365	*2*	53.0084	*4*	6.6877	*2*
311B	-	3	13	70.0299	*3*	56.6074	*1*	60.3640	*2*	6.4029	*1*

Note: Italicized values indicate rank within column (design optimality criterion) and design size ($N = 11, 13$), r_s = number of star point, n_0 = number of center point.

Table 6. Weighted Optimality Criteria for Small Response Surface Designs Across SH Models with $p_l = .8$, $p_2 = .5$, and $p_q = .7$.

Designs	r_s	n_0	N	D_s		A_s		G_s		IV_s	
SCD	1	1	11	64.1062	*2*	46.5929	*3*	45.7266	*3*	5.9081	*4*
310	-	1	11	56.3093	*4*	45.6463	*4*	38.9048	*4*	5.8809	*3*
311A	-	1	11	64.0995	*3*	50.8736	*2*	54.7577	*2*	5.6721	*2*
311B	-	1	11	67.9600	*1*	54.1921	*1*	61.0947	*1*	5.3346	*1*
BBD	-	1	13	65.1038	*2*	49.4432	*4*	52.3499	*3*	6.2114	*6*
SCD	1	3	13	57.9465	*5*	43.5300	*5*	39.2916	*5*	5.7934	*4*
UNFSD	-	1	13	67.8953	*1*	53.2593	*1*	57.8345	*1*	5.5789	*3*
310	-	3	13	51.0743	*6*	43.4713	*6*	34.5094	*6*	5.8632	*5*
311A	-	3	13	58.3290	*4*	49.5156	*3*	47.8368	*4*	5.2659	*2*
311B	-	3	13	61.5502	*3*	51.8310	*2*	52.6207	*2*	5.0286	*1*

Note: Italicized values indicate rank within column (design optimality criterion) and design size ($N = 11, 13$), r_s = number of star point, n_0 = number of center point.

Table 7. Coefficients of WH D_w Models for 3-Factor Response Surface Designs

					Linear Terms				Quadratic Terms			
Dsgn	r_s	n_0	N	$\hat{\beta}_0$	$\hat{\beta}_l$	$\hat{\beta}_q$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_{ll}$	$\hat{\beta}_{qq}$	$\hat{\beta}_{11}$	$\hat{\beta}_{22}$
310	-	0	10	55.16	0.03	-20.34	52.57	-4.48	26.83	2.57	-7.88	0.33
SCD	1	1	11	51.48	6.27	4.44	22.39	4.16	31.08	-6.63	-7.87	6.20
310	-	1	11	55.71	-8.91	-19.08	48.50	-3.43	30.04	2.38	-7.59	0.50
311A	-	1	11	48.76	12.86	-6.91	46.88	-2.71	25.00	-2.40	-8.78	1.46
311B	-	1	11	45.20	21.20	1.77	41.30	-1.24	24.32	-5.43	-9.31	2.62
BBD	-	1	13	49.60	18.79	-15.64	59.21	-4.52	20.73	0.70	-8.88	0.36
SCD	1	3	13	56.45	-16.86	-2.55	17.93	6.26	38.80	-2.78	-7.01	6.19
UNFSD	-	1	13	45.20	23.47	0.42	43.74	-0.76	23.18	-5.08	-9.52	1.94
310	-	3	13	56.25	-22.26	-15.27	41.68	-2.58	34.57	1.32	-6.92	0.66
311A	-	3	13	52.87	-8.50	-10.36	39.97	-0.69	32.28	-0.36	-8.00	1.70
311B	-	3	13	51.06	-3.68	-5.09	34.78	1.08	32.66	-1.95	-8.38	2.82
CCD	1	1	15	43.66	27.80	0.74	48.31	-2.83	20.64	-5.40	-9.45	1.79
BBD	-	3	15	52.11	1.60	-16.38	52.38	-2.64	26.82	1.50	-8.36	0.64
UNFSD	-	3	15	50.59	0.97	-6.39	37.97	1.61	30.87	-1.67	-8.76	2.20
CCD	1	3	17	49.49	5.97	-7.12	42.85	-0.38	28.29	-1.67	-8.83	2.05
SCD	2	1	17	49.78	10.05	16.40	9.19	1.23	32.88	-10.77	-7.44	7.90
SCD	2	1	19	55.85	-11.23	5.19	7.10	4.57	40.32	-5.33	-6.87	8.05
CCD	2	1	21	40.75	31.80	15.62	33.41	2.32	22.29	-10.97	-9.29	3.71
CCD	2	3	23	48.27	9.56	2.80	30.20	0.82	30.28	-5.22	-8.79	3.98

				Interaction Terms						
Dsgn	r_s	n_0	N	$\hat{\beta}_{lq}$	$\hat{\beta}_{l1}$	$\hat{\beta}_{l2}$	$\hat{\beta}_{q1}$	$\hat{\beta}_{q2}$	$\hat{\beta}_{12}$	R^2
310	-	0	10	-4.62	-46.34	-2.45	5.29	4.98	1.25	.9989
SCD	1	1	11	1.30	-23.25	-46.42	8.19	4.75	3.75	.9975
310	-	1	11	-0.29	-42.80	-4.37	5.33	4.15	1.26	.9986
311A	-	1	11	-4.43	-41.49	-13.23	6.77	4.98	1.82	.9983
311B	-	1	11	-3.63	-36.67	-22.66	7.54	4.68	2.42	.9983
BBD	-	1	13	-7.22	-51.99	-2.79	5.70	4.99	1.36	.9989
SCD	1	3	13	12.24	-18.96	-45.51	7.28	1.45	3.57	.9982
UNFSD	-	1	13	-6.58	-38.41	-21.51	7.26	5.17	2.03	.9983
310	-	3	13	1.47	-36.93	-6.06	5.18	3.49	1.24	.9981
311A	-	3	13	5.57	-35.43	-15.39	6.35	2.93	1.80	.9982
311B	-	3	13	8.47	-30.94	-24.14	6.89	2.09	2.35	.9986
CCD	1	1	15	-9.12	-42.69	-16.01	7.27	5.52	2.06	.9982
BBD	-	3	15	1.92	-46.04	-5.97	5.69	3.46	1.37	.9986
UNFSD	-	3	15	6.16	-33.35	-23.39	6.79	2.54	2.02	.9986
CCD	1	3	17	4.71	-37.37	-18.47	6.89	3.05	2.05	.9985
SCD	2	1	17	-5.77	-11.21	-53.17	7.96	5.84	4.90	.9977
SCD	2	1	19	9.10	-9.15	-53.80	7.28	1.71	4.72	.9986
CCD	2	1	21	-14.23	-30.07	-29.76	8.04	6.70	3.07	.9974
CCD	2	3	23	3.03	-27.19	-32.02	7.59	3.34	3.02	.9986

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