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Optimal control for the traffic congestion in Bangkok

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Abstract

We investigate the optimal control for the traffic congestion in Bangkok. In the queueing model we refer the traffic in a crossing as the flow of jobs, the traffic time in the crossing as those of the service time. The traffic consists of two kinds of flows, main and secondary ones. Furthermore we define a traffic controller as a server in the queueing system. At the instant of service completion (cars passed the crossing) in the main flow, the controller is continuously busy as long as there is any job in the main flow. As soon as the controller finds the main flow empty, however, he takes another job in the secondary flow (control of against flows). The service time of job is assumed to be a random variable with exponential distribution. As regards taking services in the secondary flows, the server returns to the main flow immediately after a single job whether there is a job or not in the main flow. By taking a job the server utilizes a part or all of his idle time for additional job in the secondary flow.

For the model above, we obtained the stationary distribution of the system size and that of the waiting time ([1]). In this paper we have the optimal secondary service rate for the model.

1 Introduction

To investigate the traffic congestion in Bangkok we proposed the mathematical model ([1]): the GI/M/1 queue with the secondary system's service whose length is assumed to be an exponential distributed random variable.

It is the queueing model where the controller takes aonther kind of job of secondary flow immediately after he becomes idle at the instant of service completion for the main flow. Upon termination the secondary flow's service he returns to serve the main flow. If any job is present in the main flow upon termination of main flow's service, the controller keeps for giving his service to each job, that is, the system operates as an ordinary queue.

In this paper we concern with Model 1([1]): Immediately after a single job of secondary flow, the controller returns to the main flow and becomes ready for his service. If there is any job kept waiting during the service of secondary flow, the controller begins to serve at once in the main flow. Otherwise he waits for the first job to arrive at the main flow.

While the controller is away from the main flow by taking a secondary flow's service, he provides the secondary flow with the service as an additional work. Thus the server utilizes the idle time. Let the idle time of the controller be the elapsed time during which he sojourns with no main work for him in the main flow waiting for the first job to arrive.

In Bangkok we consider a crossing which have a main flow and secondary one. Inter-arrival times to this crossing are assumed to be the generally distributed random variables. The length of service times for the main flow or secondary one can be assumed to be exponentially distributed random variables. A controller (a signal or a policeman) is controlling the traffic congestion for the crossing.

The aim of this paper is to obtain the optimal secondary service rate which minimize the total system energy.

In the next section we describe the result on the probability that the main flow or the secondary one is served and the mean waiting time.

2 Mean waiting time

Inter-arrival times to the main flow are assumed to be independent and identically distributed (i.i.d.) random variables with distribution function $A(x)$ with finite mean:

$$\frac{1}{\lambda} = \int_0^\infty x dA(x)$$

The service times are i.i.d. random variables with common exponential distribution $1 - e^{-\mu x}$. If at the instant of service completion the server finds the system empty he leaves for a secondary flow's service whose duration is a random variable with exponential distribution $1 - e^{-\nu x}$. On the other hand as long as any job is present in the system upon termination of aservice, the server continues to give his service to each job as an ordinary queue.

The following parameters are introduced;

$$\rho = \frac{\lambda}{\mu} < 1,$$
$$\delta = \frac{\nu}{\mu}.$$

Furthermore we define

$$a[s] = \int_0^\infty e^{-sx} dA(x) \qquad \text{for} \quad Re \ \ s > 0,$$
$$f(z) = \{a[\nu] - a[\mu(1-z)]\}/(1-z-\delta)$$

Let ζ denote the solution of the equation

$$\zeta = a[\mu(1-\zeta)], \quad 0 < \zeta < 1.$$

In Bangkok we consider the special case of $M/M/1$ for the traffic model.

Let us denote π_1 , π_2 as the probability that the main flow is served and that of the secondary one is served, respectively. Since $\zeta = \rho$ we have

$$\pi_1 = \frac{1-\rho}{1-\rho+\kappa}\left(\frac{\delta}{1-a[\nu]} + \kappa\frac{1}{1-\rho}\right), \tag{1}$$
$$\pi_2 = \frac{(1-\rho)(1-a[\nu]-\delta)}{(1-\rho+\kappa)(1-a[\nu])}, \tag{2}$$
$$\text{where } \kappa = \frac{(1-\rho)\delta^2 - 2\rho^2\delta - \rho^3}{\rho(\rho+\delta)} \tag{3}$$

Finallay we have the following Theorem 1 (see [1]).

THEOREM 1 *The mean waiting time W is obtained as :*

$$W = \frac{\rho}{\mu(1-\rho)} + \frac{1-\rho-\delta}{\mu\delta(1-\rho+\kappa)} \tag{4}$$

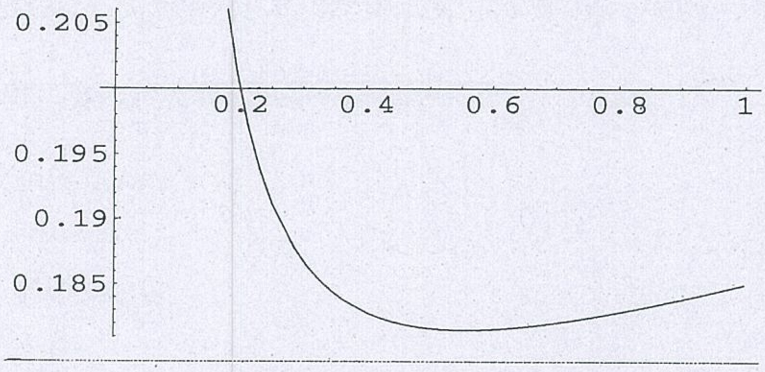
We hereafter define the total system energy for the queueing system and obtain the optimal secondary service rate to minimize the energy.

3 Total system energy and optimal secondary service rate

We assume the energy of secondary flow 1.5 times as large as that of main flow. Hence we define the total system energy by

$$S_E(\delta) = \frac{(\pi_1 \cdot 1 + \pi_2 \cdot 1.5)}{W + \frac{1}{\mu}}. \tag{5}$$

In the case of $\rho = 0.8, \mu = 1.0,$, we have the numerical result: we can find $\delta = 0.557$ and $\min S_E = 0.181$ in the figure where the horizontal axis means δ and the vertical one is S_E.



Hence we find the optimal secondary service rate is 0.557 if the traffic intensity is 0.8.

References

[1] M. Doi, H. Ōsawa, *A queueing model for the traffic congestion in Bangkok*, KMITL Science Journal, Vol.3-1, pp.47-54, 2003.