Some properties of distributive addition of Γ-AG-rings

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Abstract

Abel-Grassmann's ring (AG-rings) introduced by Yusuf. In this paper researcher studies some properties of distributive addition of Γ-AG-ring which is an important basic properties of it.

Keywords: Γ-AG-ring, Γ-invertive law, Γ-medial law, Γ-paramedial law

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Introduction

Kazim and Naseeruddin (1977) was introduces the concept of an AG-groupoid.

Defintion 1.1 (Kazim and Naseeruddin, 1977) A groupoid $(S,\cdot)$ is called an *AG*-groupoid, if it satisfies left invertive law

$(ab)c = (cb)a$ for all $a,b,c \in S$

Lemma 1.2 (Kazim and Naseeruddin, 1977) An AG-groupoid S, is called a *medial law* if it satisfies

$(ab)(cd) = (ac)(bd)$ for all $a,b,c,d \in S$

Definition 1.3 (Shah and Rehman, 2010b). An AG-groupoid S, is called a *paramedical* if it satisfies

$(ab)(cd) = (db)(ca)$ for all $a,b,c,d \in S$

Proposition 1.4 (Shah and Rehman, 2010). If S is an AG-groupoid with left identity, then $a(bc) = b(ac)$ for all $a,b,c,d \in S$

Definition 1.5 (Sarwar, 1993). A groupoid *G* is called a *AG-group*, if

1. there exists $e \in G$ such that $ea = a$ for all $a \in G$,
2. for every $a \in G$ there exists $a^{-1} \in G$ such that, $a^{-1}a = a$
3. $(ab)c = (cb)a$ for all $a, b, c \in G$.

Yusuf (as cited in Shah and Rehman, 2010b) introduces the concept of an AG-ring.

Definition 1.6 (Shah and Rehman, 2010b). An AG-ring *(R, +, •)* is a set *R* together with two binary operation "+" addition, and "•" multiplication, defined on *R* such that the following axioms are satisfied:

1. *(R, +)* is an AG-group,
2. *(R,•)* is an AG-groupoid,
3. For all $a,b,c \in R$, the left distributive law $a(b+c) = ab + ac$ and the right distributive law $(b+c)a = ba + ca$ holds.

Shah and Rehman (2010b). asserted that a commutative ring *(R, +, •)* we can always obtain an AG-ring *(R, ⊕, •)* by defining, for *a, b* $\in R$, $a \oplus b = b - a$ and *ab* is same as in the ring. We can assume the addition to be commutative in an AG-ring.

Definition 1.7 An AG-ring *(R, +, •)* is said to be *AG-integral domain* if $ab = 0$ for all $a, b \in R$ then $a = 0$ or $b = 0$.

Definition 1.8 Let *(R, +,•)* be an AG-ring and *S* be a non-empty subset of *R* and *S* is itself and AG-ring under the binary operation induced by *R*, the *S* is called an *AG-subring* of *R*, then *S* is called an AG-subring of *(R, +, •).*

Definition 1.9 If *S* is an AG-subring of an AG-ring *(R, +, •)* then S is called a *left* (*right*) *ideal* of *R* if $RS \subseteq S$ ($SR \subseteq S$) and is called *ideal* if it is left as well as right ideal.

Shah and Rehman (2010a) asserted that, the notion of Γ-semigroups was introduced by Sen, Let M and Γ be any nonempty sets. If there exists a mapping $M \times \Gamma \times M \rightarrow M$ written (a, α, c) by $a\alpha c$, M is called a Γ-semigroups if M satisfies the identity $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$. A Γ-AG-groupoids analogous to Γ-semigroups.

Definition 1.10 (Shah and Rehman, 2010a) Let G and Γ be two non-empty sets. G is said to be Γ-AG-groupiod if there exists a mapping $G \times \Gamma \times G \rightarrow G$, written (a, α, b) by $a\alpha b$, such that G satisfies the identity $(a\alpha b)\beta c = (c\alpha b)\beta a$ for all $a, b, c \in G$ and $\alpha, \beta \in \Gamma$.

Nobusawa (1964) was introduced the definition of a Γ-ring. Nobusawa (1964) introduced some conditions in the definition of Γ-ring but these two definitions is paralleled results in ring theory.

Definition 1.11 (Banes, 1996). If $M = \{a, b, c, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ are additive abelian groups, and for all $a, b, c \in M$ and all $\alpha, \beta, \delta, \in \Gamma$ the following conditions are satisfied

1. $a\alpha b$ is an element of M
2. $a\alpha(b + c) = a\alpha b + a\alpha c$
3. $(a + b)\alpha c = a\alpha c + b\alpha a$,
4. $a(\alpha + \beta)b = a\alpha b + a\beta b$
5. $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$, then M is called a Γ-ring.

Γ-Abel-Grassmann's ring

In this paper, we introduce the concept of a Γ-Abel-Grassmann's ring (Γ-Grassmann's ring AG-ring), and we study some properties of distributive addition of Γ-AG-ring.

Definition 2.1. (Wanicharpichat and Gaketem, 2011) Let $(R, +)$ and $(\Gamma, +)$ be two AG-group, R is called gamma Abel-Grassmann's ring (Γ-AG-ring) if there exists a mapping $f: R \times \Gamma \times R \rightarrow R$, $f(a, \alpha, b)$ is denoted by $a\alpha b$, $a, b \in R$ and $\alpha \in \Gamma$, satisfying the following conditions for all $a, b, c \in R$ and for all $\alpha, \beta \in \Gamma$.

1. $a\alpha(b + c) = a\alpha b + a\alpha c$,
2. $(a + b)\alpha c = a\alpha c + b\alpha a$,
3. $a(\alpha + \beta)b = a\alpha b + a\beta b$,
4. $(a\alpha b)\beta c = (c\alpha b)\beta a$ for all $a, b, c \in R$ and $\alpha, \beta \in \Gamma$.

The property (4) is called a Γ-invertive law in Γ-AG-ring R. We have the following lemma. The following theorem with proved is analogous as in (Shah and Shah, 2011).

Theorem 2.2. Let R be a Γ-AG-ring. Then $(a + b)\alpha(c + d) = (b + a)\alpha(d + c)$ for all $a, b, c, d \in R$ and $\alpha \in \Gamma$.

Proof. Let $a, b, c, d \in R$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}(a + b)\alpha(c + d) &= a\alpha(c + d) + b\alpha(c + d)\\ &= (a\alpha c + a\alpha d) + (b\alpha c + b\alpha d)\\ &= (a\alpha c + b\alpha c) + (a\alpha d + b\alpha d)\end{aligned}$$

$= (b\alpha d + a\alpha d) + (b\alpha c + a\alpha c)$

$= (b + a)\alpha d + (b + a)\alpha c$

$= (b + a)\alpha(d + c)$

Thus $(a + b)\alpha(c + d) = (b + a)\alpha(d + c)$. □

Next we give a number of corollaries of Theorem 2.2 which present some distinguished features of Γ-AG-ring.

Corollary 2.3. Let R be a Γ-AG-ring. Then $a\alpha c = (a + 0)\alpha(c + 0)$ for all $a, c \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.2 if $b = d = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(a + 0)\alpha(c + 0) &= a\alpha(c + 0) + 0\alpha(c + 0)\\ &= (a\alpha c + a\alpha 0) + (0\alpha c + 0\alpha 0)\\ &= a\alpha c.\end{aligned}$$

Thus $a\alpha c = (a + 0)\alpha(c + 0)$. □

The following corollaries are application by Theorem 2.2 and Corollary 2.3

Corollary 2.4. Let R be a Γ-AG-ring. Then $b\alpha d = (0 + b)\alpha(0 + d)$ for all $b, d \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.2 if $a = c = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(0 + b)\alpha(0 + d) &= 0\alpha(0 + d) + b\alpha(0 + d) \text{ ; by (2)}\\ &= (0\alpha 0 + 0\alpha d) + (b\alpha 0 + b\alpha d)\\ &= b\alpha d.\end{aligned}$$

Thus $b\alpha d = (0 + b)\alpha(0 + d)$. □

Theorem 2.5. Let R be a Γ-AG-ring. Then $(a + b)^2 = (b + a)^2$ for all $a, b \in R$.

Proof. By Theorem 2.2 if $a = c$ and $d = b$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(a + b)^2 &= (a + b)\alpha(a + b)\\ &= a\alpha(a + b) + b\alpha(a + b)\\ &= (a\alpha a + a\alpha b) + (b\alpha a + b\alpha b)\\ &= (a\alpha a + b\alpha a) + (a\alpha b + b\alpha b)\\ &= (b\alpha b + a\alpha b) + (b\alpha a + a\alpha a)\\ &= (b + a)\alpha b + (b + a)\alpha a\\ &= (b + a)\alpha(b + a)\\ &= (b + a)^2.\end{aligned}$$

Thus $(a + b)^2 = (b + a)^2$. □

The following corollaries are application by Theorem 2.5

Corollary 2.6. Let R be a Γ-AG-ring. Then $a^2 = a\alpha a = (a + 0)^2$ for all $a \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.5 if $b = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(a + 0)^2 &= (a + 0)\alpha(a + 0)\\ &= a\alpha(a + 0) + 0\alpha(a + 0)\\ &= a\alpha a + a\alpha 0 + 0\alpha a + 0\alpha 0\\ &= a\alpha a\\ &= a^2.\end{aligned}$$

Thus $a^2 = a\alpha a = (a + 0)^2$. □

Corollary 2.7. Let R be a Γ-AG-ring. Then $b^2 = b\alpha b = (0 + b)^2$ for all $b \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.5 if $a = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(0 + b)^2 &= (0 + b)\alpha(0 + b)\\ &= 0\alpha(0 + b) + b\alpha(0 + b)\\ &= 0\alpha 0 + 0\alpha b + b\alpha 0 + b\alpha b\\ &= b\alpha b\\ &= b^2.\end{aligned}$$

Thus $b^2 = b\alpha b = (0 + b)^2$. □

The following theorem is application by Theorem 2.2 with proved is analogous as in Theorem 2.5

Theorem 2.8. Let R be a Γ-AG-ring. Then $(c + d)^2 = (d + c)^2$ for all $c, d \in R$.

Proof. By Theorem 2.2 if $a = c$ and $d = b$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(c + d)^2 &= (c + d)\alpha(c + d)\\ &= c\alpha(c + d) + d\alpha(c + d)\\ &= (c\alpha c + c\alpha d) + (d\alpha c + d\alpha d)\\ &= (c\alpha c + d\alpha c) + (c\alpha d + d\alpha d)\\ &= (d\alpha d + c\alpha d) + (d\alpha c + c\alpha c)\\ &= (d + c)\alpha d + (d + c)\alpha c\\ &= (d + c)\alpha(d + c)\\ &= (d + c)^2.\end{aligned}$$

Thus $(c + d)^2 = (d + c)^2$. □

The following corollaries are application by Theorem 2.8

Corollary 2.9. Let R be a Γ-AG-ring. Then $c^2 = c\alpha c = (c + 0)^2$ for all $c \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.8 if $d = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(c + 0)^2 &= (c + 0)\alpha(c + 0)\\ &= c\alpha (c + 0) + 0\alpha (c + 0)\\ &= c\alpha c + c\alpha 0 + 0\alpha c + 0\alpha 0\\ &= c\alpha c\\ &= c^2.\end{aligned}$$

Thus $c^2 = c\alpha c = (c + 0)^2$. □

Corollary 2.10. Let R be a Γ -AG-ring. Then $d^2 = d\alpha d = (0 + d)^2$ for all $d \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.8 if $c = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(0 + d)^2 &= (0 + d)\alpha(0 + d)\\ &= 0\alpha(0 + d) + d\alpha(0 + d)\\ &= 0\alpha 0 + 0\alpha d + d\alpha 0 + d\alpha d\\ &= d\alpha d\\ &= d^2.\end{aligned}$$

Thus $d^2 = d\alpha d = (0 + d)^2$. □

Corollary 2.11. Let R be a Γ-AG-ring. Then $b\alpha(c + d) = (0 + b)\alpha(c + d)$ for all $b, c, d \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.2 if $a = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(0 + b)\alpha(c + d) &= (0 + b)\alpha c + (0 + b)\alpha d\\ &= 0\alpha c + b\alpha c + 0\alpha d + b\alpha d\\ &= b\alpha c + b\alpha d\\ &= b\alpha(c + d)\end{aligned}$$

Thus $b\alpha(c + d) = (0 + b)\alpha(c + d)$. □

Corollary 2.12. Let R be a Γ -AG-ring. Then $a\alpha(c + d) = (a + 0)\alpha(c + d)$ for all $a, c, d \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.2 if $b = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(a + 0)\,\alpha\,(c + d) &= (a + 0)\alpha c + (a + 0)\,\alpha d\\ &= a\alpha c + 0\alpha c + a\alpha d + 0\alpha d\\ &= a\alpha c + a\alpha d\\ &= a\alpha(c + d)\end{aligned}$$

Thus $a\alpha(c + d) = (a + 0)\alpha(c + d)$. □

Corollary 2.13. Let R be a Γ -AG-ring. Then $(a + b)\alpha d = (a + b)\alpha(0 + d)$ for all $a, b, d \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.2 if $c = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(a + b)\alpha(0 + d) &= (a + b)\alpha 0 + (a + b)\alpha d \\ &= a\alpha 0 + b\alpha 0 + a\alpha d + b\alpha d \\ &= a\alpha d + b\alpha d \\ &= (a + b)\alpha d\end{aligned}$$

Thus $(a + b)\alpha d = (a + b)\alpha(0 + d)$. □

Corollary 2.14. Let R be a Γ-AG-ring. Then $(a + b)\alpha c = (a + b)\alpha(c + 0)$ for all $a, b, c \in R$ and $\alpha \in \Gamma$.

Proof. By Theorem 2.2 if $d = 0$ and $\alpha \in \Gamma$ then

$$\begin{aligned}(a + b)\alpha(c + 0) &= (a + b)\alpha c + (a + b)\alpha 0 \\ &= a\alpha b + b\alpha c + a\alpha 0 + b\alpha 0 \\ &= a\alpha b + b\alpha c \\ &= (a + b)\alpha c\end{aligned}$$

Thus $(a + b)\alpha c = (a + b)\alpha(c + 0)$. □

Results and discussion

In this paper, we study properties of distributive addition of Γ-AG-ring which is an important basic properties in prove in paper.

Conclusions

In this paper, we present the theory that, let R be a Γ-AG-ring. Then for all $a, b, c, d \in R$ and $\alpha \in \Gamma$, $(a + b)\alpha(c + d) = (b + a)\alpha(d + c)$, $(a + b)^2 = (b + a)^2$ and $(c + d)^2 = (d + c)^2$.

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References

Banes, W.E. 1996. On the Γ-ring of Nobusawa. *Pacific Journal of Mathematics 18*: 411-422.

Kazim, M.A. and M.D. Naseeruddin. 1977. On almost semigroup. *Portugaliae Mathematica 36*: 41-47.

Nobusawa, N. 1964. On generalization of the ring theory. *Osaka J. Math. 1*: 81-89.

Sarwar (Kamran), M. 1993. Conditions for LA-semigroup to resemble associative structures. Ph.D. Thesis. Quaid-i-Azam University.

Shah, M. and T. Shah. 2011. Some Basic Properties of LA-ring. *International Mathematical Forum 6*: 2195-2199.

Shah, T. and I. Rehman. 2010a. On Γ-ideal and Γ-bi-ideals in Γ-AG-groupoids. *International Journal of Algebra 4*: 267-276.

Shah, T. and I. Rehman. 2010b. On LA-rings of finitely nonzero function. *Int. J. Contemp. Math. Sciences. 5:* 209-222.

Wanicharpichat, W. and T. Gaketem. 2011. Ideal in regular in Γ-AG-ring. M.Sc. Thesis. Naresuan University.