On Γ-Semi-prime and Quasi Γ-Semi-prime Ideals in Γ-AG-Groupoids

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บทคัดย่อ

งานวิจัยฉบับนี้ได้ศึกษาแกมมา-ไอดีล, แกมมา-เอจี-เสมือนไอดีลกึ่งเฉพาะและแกมมา-เอจี-ไอดีลกึ่งเฉพาะและได้จำแนกลักษณะเฉพาะบางประการของสิ่งที่ได้กล่าวมาข้างต้น นอกจากนี้ยังได้ศึกษาความสัมพันธ์ระหว่างแกมมา-ไอดีล, แกมมา-เอจี-เสมือนไอดีลกึ่งเฉพาะและแกมมา-เอจี-ไอดีลกึ่งเฉพาะของ $(A:\Gamma:B)$ และ $(A:\gamma:r)$ ในแกมมา-เอจี-กรุปพอยด์ที่มีเอกลักษณ์ทางซ้าย

คำสำคัญ : แกมมา-เอจี-กรุปพอยด์ แกมมา-เอจี-3-แถบ แกมมา-เอจี-เสมือนไอดีลกึ่งเฉพาะ แกมมา-เอจี-ไอดีลกึ่งเฉพาะ แกมมา-สลับที่ทางขวา

Abstract

This study investigated Γ-ideals, Γ-semi-prime, and quasi Γ-semi-prime ideals in Γ-AG-groupoids. Some characterizations of Γ-semi-prime ideals and quasi Γ-semi-prime ideals were obtained. Moreover, the relationships between Γ-semi-prime and quasi Γ-semi-prime ideals of $(A:\Gamma:B)$ and $(A:\gamma:r)$ in Γ-AG-groupoids were investigated. Finally, necessary and sufficient conditions were obtained of Γ-semi-prime ideals to be quasi Γ-semi-prime ideals in Γ-AG-groupoids with left identity.

Keywords: Γ-AG-groupoid, Γ-AG-3-band, quasi Γ-semi-prime ideal, Γ-semi-prime ideal, right Γ-alternative.

Introduction

Abel-Grassmann's groupoid (AG-groupoid) is the generalization of semigroup theory with wide range of usages in theory of flocks [6]. The fundamentals of this non-associative algebraic structure were first discovered by Kazim and Naseeruddin [1]. AG-groupoid is a non-associative algebraic structure mid way between a groupoid and a commutative semigroup. It is interesting to note that an AG-groupoid with right identity becomes a commutative monoid [4]. This structure is closely related with a commutative semigroup because if an AG-groupoid contains a right identity, then it becomes a commutative monoid [5]. A left identity in an AG-groupoid is unique [5]. It is a mid structure between a groupoid and a commutative semigroup with wide range of applications in theory of flocks [6]. Ideals in AG-groupoids have been discussed in [4] and [5]. In 1981 the notion of Γ-semigroups was introduced by M. K. Sen [8] and [12]. Let S and Γ be any nonempty sets. If there exists a mapping

$$S\times\Gamma\times\Gamma\to S$$

written (a,γ,b) by $a\gamma b,$ S is called a Γ-AG-groupoid if S satisfies the identity

$$(a\alpha b)\beta c=(c\alpha b)\beta c)$$

for all $a,b,c\in S$ and $\alpha,\beta\in S$ (See [2]). This structure is also known as left almost semigroup (LA-semigroup). In this paper, we are going to investigate some interesting properties of newly discovered classes of namely; Γ-AG-groupoid S always satisfies the Γ-medial law:

$$(a\gamma b)\beta(c\delta d)=(a\gamma c)\beta(b\delta d)$$

for all $a,b,c,d\in S$ and $\gamma,\beta,\delta\in\Gamma$ (See [2]), while a Γ-AG-groupoid S with left identity e always satisfies Γ-paramedial law:

$$(a\gamma b)\beta(c\delta d)=(d\gamma c)\beta(b\delta a)$$

for all $a,b,c,d\in S$ and $\gamma,\beta,\delta\in\Gamma$ (See[2]).

In this paper we characterize the Γ-AG-groupoid. We investigate relationships between Γ-semiprime and quasi Γ-semiprime ideals of $(A:\Gamma:B)$ and $(A:\gamma:r)$ in Γ-AG-groupoids with left identity.

Basic results

In this section we refer to [7, 8, 11] for some elementary aspects and quote few definitions and examples which are essential to step up this study. For more details we refer to the papers in the references.

Example 2.1 [11] (1). Let S be an arbitrary AG-groupoid and Γ any non-empty set. Define a mapping

$$S\times\Gamma\times S\to S;$$

by $a\gamma b=ab$ for all $a,b\in S$ and $\gamma\in\Gamma$. It is easy to see that S is a Γ-AG-groupoid.

(2). Let $\Gamma=\{1,2,3\}$. Define a mapping

$$\mathbb{Z}\times\Gamma\times\mathbb{Z}\to\mathbb{Z} \text{ by } a\gamma b=b-\gamma-a$$

for all $a,b\in\mathbb{Z}$ and $\gamma\in\Gamma$ where "$-$" is a usual subtraction of integers. Then $\mathbb{Z}$ is a Γ-AG-groupoid.

Lemma 2.2 [12] Every Γ-AG-groupoid is Γ-medial.

Lemma 2.3 [10, 12] Let S be a Γ-AG-groupoid with a left identity, then

$$a\gamma(b\alpha c)=b\gamma(a\alpha c)$$

for all $a,b,c\in S$ and $\gamma,\alpha\in\Gamma$.

Definition 2.4 [11] Let S be a Γ-AG-groupoid. A nonempty subset A of S is called a sub Γ-AG-groupoid of S if $A\Gamma A\subseteq A$.

Definition 2.5 [11] A sub Γ-AG-groupoid A of S is called a left (right) Γ-ideal of S if $S\Gamma A\subseteq A$ $(A\Gamma S\subseteq A)$ and is called a Γ-ideal if it is left as well as right Γ-ideal.

Definition 2.6 [11] An element $e\in S$ is called a left identity of Γ-AG-groupoid if $e\gamma a=a$ for all $a\in S$ and $\gamma\in\Gamma$.

Lemma 2.7 [11] If a Γ-AG-groupoid S has a left identity, then every right Γ-ideal is a left Γ-ideal.

Proof. Let I be a right Γ-ideal of S. Then for $i\in I, s\in S$ and $\gamma,\alpha\in\Gamma$, consider

$$\begin{aligned} s\gamma i &= (e\alpha s)\gamma i \\ &= (i\alpha s)\gamma e\in I \end{aligned}$$

where $e\in S$ is a left identity. Hence I is a left Γ-ideal. □

Lemma 2.8 [11] If A is a left Γ-ideal of a Γ-AG-groupoid S with left identity, and if for any $a\in S$, there exists $\gamma\in\Gamma$, then $a\gamma A$ is a left Γ-ideal of S.

Proof. Let A be a left Γ-ideal of S, consider

$$s\gamma(a\gamma i) = (e\gamma s)\gamma(a\gamma i)$$

$$= (e\gamma a)\gamma(s\gamma i)$$
$$= a\gamma(s\gamma i) \in a\gamma A$$

for all $s \in S, i \in A$. Hence $a\gamma A$ is a left Γ-ideal of S. □

Definition 2.9 [11] A Γ-ideal P is called Γ-semiprime if $A^2 = A\Gamma A \subseteq P$ implies that $A \subseteq P$ for any Γ-ideals A of S.

Definition 2.10 [11] A left Γ-ideal P is called quasi Γ-semiprime if $A^2 = A\Gamma A \subseteq P$ implies that $A \subseteq P$ for any left Γ-ideals A of S.

It is easy to see that every quasi Γ-semiprime ideal is Γ-semiprime.

Lemma 2.11 [11] If S is a Γ-AG-groupoid with left identity, then a left Γ-ideal P of S is quasi Γ-semiprime if and only if $a\alpha(S\gamma a) \subseteq P$ implies that $a \in P$, for all $a \in S$ and any $\alpha, \gamma \in \Gamma$.

Lemma 2.12 [11] If A is a proper right (left) Γ-ideal of a Γ-AG-groupoid S with left identity e, then $e \notin A$.

Proof. Assume on contrary that and let $e \in A$ and $\gamma \in \Gamma$. Then

$$S = e\gamma S \subseteq A\Gamma S \subseteq A.$$

A contradiction. Hence $e \notin A$. □

Γ-ideals in Γ-AG-Groupoids

The results of the following lemmas seem to be at the heart of the theory of Γ-AG-groupoids; these facts will be used very frequently that normally we shall make no reference to this lemma.

Lemma 3.1 If S is a Γ-AG-groupoid with left identity, then $a\gamma b = a\beta b$, for all $a,b \in S$ and $\gamma, \beta \in \Gamma$

Proof. Let S be a Γ-AG-groupoid and e be the left identity of S, $a,b \in S$ and let $\gamma, \beta \in \Gamma$ therefore we have

$$a\gamma b = a\gamma(e\beta b)$$
$$= e\gamma(a\beta b)$$
$$= a\beta b.$$

Hence $a\gamma b = a\beta b$. □

Lemma 3.2 Let S be a Γ-AG-groupoid with left identity, and let B be a left Γ-ideal of S. Then

$$A\Gamma B = \{a\gamma b : a \in A, b \in B, \gamma \in \Gamma\}$$

is a left Γ-ideal in S, where $\varnothing \neq A \subseteq S$.

Proof. Suppose that S is a Γ-AG-groupoid with left identity . Let B be a left Γ-ideal of S. Then

$$S\Gamma(A\Gamma B) = A\Gamma(S\Gamma B) \subseteq A\Gamma B.$$

By definition of left Γ-ideal, we get $A\Gamma B$ is a left Γ-ideal in S. □

Lemma 3.3 Let S be a Γ-AG-groupoid with left identity and let $a \in S$. Then $a^2\gamma S$ is a Γ-ideal in S, where $\gamma \in \Gamma$.

Proof. By Lemma 2.8, we have $a^2\gamma S$ is a left Γ-ideal of S. Now consider

$$(a^2\gamma r)\alpha s = (s\gamma r)\alpha a^2$$
$$= a^2\alpha(r\gamma s)$$
$$= a^2\gamma(r\gamma s) \in a^2\gamma S$$

for all $r,s \in S$ and $\gamma, \alpha \in \Gamma$. Therefore $a^2\gamma S$ is a Γ-ideal in S. □

Lemma 3.4 Let S be a Γ-AG-groupoid with left identity, and let A be a left Γ-ideal of S. Then $(A{:}\gamma{:}r)$ is a left Γ-ideal in S, where

$$(A{:}\gamma{:}r) = \{a \in S : r\gamma a \in A\}.$$

and $r \in S, \gamma \in \Gamma$.

Proof. Suppose that S is a Γ-AG-groupoid. Let $s \in S$ and let $a \in (A{:}\gamma{:}r)$. Then $r\gamma a \in A$ and so that

$$r\gamma(s\alpha a) = s\gamma(r\alpha a) \in s\gamma A \subseteq S\Gamma A \subseteq A$$

for all $\gamma \in \Gamma$. Therefore $s\alpha a \in (A{:}\gamma{:}r)$ so that

$$S\Gamma(A{:}\gamma{:}r) \subseteq (A{:}\gamma{:}r).$$

Hence $(A{:}\gamma{:}r)$ is a left Γ-ideal in S. □

Remark Let S be a Γ-AG-groupoid and let A be a left Γ-ideal of S. It is easy to verify that $A \subseteq (A{:}\gamma{:}r)$.

Definition 3.5 A Γ-AG-groupoid S is called Γ-AG-3-band if

$$a\alpha(a\beta a) = (a\alpha a)\beta a = a$$

for all $a \in S$ and $\alpha, \beta \in \Gamma$.

Proposition 3.6 Every left identity in a Γ-AG-3-band is a right identity.

Proof. Let e be a left identity and a be any element in a Γ-AG-3-band S. Then

$$\begin{aligned} a\gamma e &= (a\alpha(a\beta a))\gamma e \\ &= (e\alpha(a\beta a))\gamma a \\ &= (a\beta a)\gamma a \\ &= a \end{aligned}$$

for all $a \in S$ and $\gamma, \alpha, \beta \in \Gamma$. Hence e is right identity. □

Lemma 3.7 If a Γ-AG-3-band S has a left identity, then every left Γ-ideal is a Γ-ideal.

Proof. Let S be a Γ-AG-3-band and let A be a left Γ-ideal. Then

$$\begin{aligned} a\gamma s &= ((a\alpha a)\beta a)\gamma s \\ &= (s\beta a)\gamma(a\alpha a) \\ &\in (S\Gamma A)\Gamma(A\Gamma A) \\ &\subseteq A\Gamma A \\ &\subseteq A \end{aligned}$$

for all $\gamma, \alpha, \beta \in \Gamma$, $a \in A$ and $s \in S$. Hence A is a Γ-ideal. □

Corollary 3.8 Let S be a Γ-AG-3-band with left identity, and let A be a left Γ-ideal of S. Then $(A{:}\gamma{:}r)$ is a Γ-ideal in S.

Proof. This follows from Lemma 3.7. □

Lemma 3.9 Let S be a Γ-AG-groupoid with left identity, and let A, B be left ideals of S. Then $(A{:}\Gamma{:}B)$ is a left Γ-ideal in S, where

$$(A{:}\Gamma{:}B) = \{r \in S : B\Gamma r \subseteq A\}.$$

Proof. Suppose that S is a Γ- AG-groupoid. Let $s \in S$ and let $a \in (A{:}\Gamma{:}B)$. Then $B\Gamma a \subseteq A$ so that

$$B\Gamma(s\gamma a) = s\Gamma(B\gamma a) \subseteq s\Gamma A \subseteq A$$

for all $s \in S$ and $\gamma \in \Gamma$. Therefore $s\gamma a \in (A{:}\Gamma{:}B)$ so that

$$S\Gamma(A{:}\Gamma{:}B) \subseteq (A{:}\Gamma{:}B).$$

Hence $(A{:}\Gamma{:}B)$ is a left Γ-ideal in S. □

Remark Let S be a Γ-AG-groupoid with left identity and let A, B, C be left Γ-ideals of S. It is easy to verify that $(A{:}\Gamma{:}C) \subseteq (A{:}\Gamma{:}B)$, where $B \subseteq C$.

Corollary 3.10 Let S be a Γ-AG-3-band with left identity, and let A, B be left Γ-ideals of S. Then $(A{:}\Gamma{:}B)$ is a Γ-ideal in S.

Proof. The proof is easy. □

Properties of Γ-Semiprime Ideals in Γ-AG-Groupoids.

We start with the following theorem that gives a relation between Γ-semiprime andquasi Γ-semiprime ideal in Γ-AG-groupoid. Our starting points is the following lemma:

Lemma 4.1 If S is a Γ-AG-groupoid with left identity, then a left Γ-ideal P of S is quasi Γ-semiprime if and only if $(S\gamma a)^2 \subseteq P$ implies that $a \in P$, where $a \in S$ and $\gamma \in \Gamma$.

Proof . Let P be a quasi Γ-semiprime ideal of a Γ-AG-groupoid S with left identity and let $\gamma, \beta \in \Gamma$. Now suppose that $(S\gamma a)^2 \subseteq P$. Then by Definition of left Γ-ideal, we obtain

$$\begin{aligned}(S\gamma a)^2 &= (S\gamma a)\beta(S\gamma a)\\ &= (S\gamma S)\beta(a\gamma a)\\ &= S\beta(a\gamma a)\\ &= a\beta(S\gamma a)\end{aligned}$$

that is $a\beta(S\gamma a) = (S\gamma a)^2 \subseteq P$. By Lemma 2.11, we have $a \in P$.

Conversely, assume that if $(S\gamma a)^2 \subseteq P$, then $a \in P$ for all $a \in S$ and $\gamma, \alpha \in \Gamma$. Let $a\alpha(S\gamma a) \subseteq P$. Now consider

$$a\alpha(S\gamma a) = (S\gamma a)^2 \subseteq P.$$

By using given assumption, if $a\alpha(S\gamma a) \subseteq P$, then $a \in P$. Then by Lemma 2.11, we have P is quasi Γ-semiprime ideal in S. □

Theorem 4.2 If S is a Γ-AG-groupoid with left identity, then a left Γ-ideal P of S is quasi Γ-semiprime if and only if $a^2 \in P$ implies that $a \in P$, where $a \in S$.

Proof. Let P be a left Γ-ideal of a Γ-AG-groupoid S with left identity and let $\gamma, \beta \in \Gamma$. Now suppose that $a^2 \in P$. Then by definition of left Γ-ideal, we get

$$\begin{aligned}(S\gamma a)^2 &= (S\gamma a)\beta(S\gamma a)\\ &= (S\gamma S)\beta(a\gamma a)\\ &= S\beta(a\gamma a)\\ &= S\beta a^2\\ &\subseteq S\beta P\\ &\subseteq P\end{aligned}$$

for all $\beta \in \Gamma$. By Lemma 4.1, we have $a \in P$.

Conversely, the proof is easy. □

Definition 4.3 A Γ-AG-groupoid S is called right Γ-alternative if it satisfies the identity,

$$a\gamma(b\alpha b) = (a\gamma b)\alpha b$$

for all $a, b \in S$ and $\alpha, \beta \in \Gamma$.

Theorem 4.4 Let S be aright Γ-alternative, and let A be a quasi Γ-semiprime ideal of S. Then $(A{:}\gamma{:}r^2)$ is a quasi Γ-semiprime ideal in S, where $r \in S, \gamma \in \Gamma.$

Proof. Assume that A is a quasi Γ-semiprime ideal of S and $\gamma, \beta \in \Gamma$. By Lemma 3.4, we have $(A{:}\gamma{:}r^2)$ is a left Γ-ideal in S. Let $a^2 \in (A{:}\gamma{:}r^2)$. Then $r^2\gamma a^2 \in A$ so that

$$\begin{aligned}(r\gamma a)^2 &= (r\gamma a)\beta(r\gamma a)\\ &= (r\gamma r)\beta(a\gamma a)\\ &= r^2\beta a^2\\ &= r^2\gamma a^2 \in A.\end{aligned}$$

By Theorem 4.2, we have $r\gamma a \in A$. Then consider

$$\begin{aligned}r^2\gamma a &= r^2\beta a\\ &= (r\gamma r)\beta a\\ &= r\gamma(r\beta a) \in A.\end{aligned}$$

Therefore $a \in (A{:}\gamma{:}r^2)$ and hence $(A{:}\gamma{:}r^2)$ is a quasi Γ-semiprime ideal in S. □

Theorem 4.5 Let S be aright Γ-alternative, and let A be a quasi Γ-semiprime ideal of S. Then $(A{:}\gamma{:}r)$ is a quasi Γ-semiprime ideal in S ,where $r \in S$ and $\gamma \in \Gamma$.

Proof. Assume that A is a quasi Γ-semiprime ideal of S and let $r \in S, \gamma, \beta \in \Gamma$. By Lemma 3.4, we have

$(A{:}\gamma{:}r)$ is a left Γ-ideal in S. Let $a^2 \in (A{:}\gamma{:}r)$. Then $r\gamma a^2 \in A$ so that

$$\begin{aligned}(r\gamma a)^2 &= (r\gamma a)\beta(r\gamma a)\\ &= r\beta((r\gamma a)\gamma a)\\ &= r\beta(r\gamma(a\gamma a))\\ &= r\beta(r\gamma a^2) \in r\beta A \subseteq A.\end{aligned}$$

By Theorem 4.2, we have $r\gamma a \in A$. Therefore

$$a \in (A{:}\gamma{:}r)$$

and hence $(A{:}\gamma{:}r)$ is a quasi Γ-semiprime ideal in S. □

Theorem 4.6 Let S be a Γ-AG-3-band with left identity. Then P is a quasi Γ-semiprime ideal in S if and only if P is a Γ-semiprime ideal in S.

Proof. The proof is easy. □

Theorem 4.7 Let S be a Γ-AG-groupoid with left identity, and let P be a Γ-semiprime ideal of S. If $(S\gamma a^2)^2 \subseteq P$, then $a^2 \in P$, where $a \in S$ and $\gamma \in \Gamma$.

Proof. Let P be a Γ-semiprime ideal of a Γ-AG-groupoid S with left identity and let $\gamma, \beta \in \Gamma$. Now suppose that

$$(S\gamma a^2)^2 \subseteq P.$$

Then by Definition of left Γ-ideal, we get

$$\begin{aligned}(S\gamma a^2)^2 &= (S\gamma a^2)\beta(S\gamma a^2)\\ &= ((S\gamma a^2)\gamma a^2)\beta S\\ &= ((a^2\gamma a^2)\gamma S)\beta S\\ &= (S\gamma S)\beta(a^2\gamma a^2)\\ &= a^2\beta((S\gamma S)\gamma a^2)\\ &= a^2\beta((a^2\gamma S)\gamma S)\\ &= (a^2\gamma S)\beta(a^2\gamma S)\\ &= (a^2\gamma S)^2\end{aligned}$$

that is $(a^2\gamma S)^2 \subseteq P$. By Lemma 3.3, we have $a^2\gamma S$ is a Γ-ideal in S. Therefore

$$\begin{aligned}a^2 &= a\gamma a\\ &= (e\beta a)\gamma a\\ &= (a\beta a)\gamma e\\ &= a^2\gamma e \in a^2\gamma S \subseteq P.\end{aligned}$$ □

Corollary 4.8 Let S be a Γ-AG-groupoid with left identity, and let P be a Γ-semiprime ideal of S. If $(a^2)^2 \in P$, then $a^2 \in P$, where $a \in S$.

Proof. Let P be a Γ-semiprime ideal of a Γ-AG-groupoid S with left identity and let $\gamma, \beta \in \Gamma$. Now suppose that

$$(a^2)^2 \in P.$$

Then by Definition of left ideal, we get

$$\begin{aligned}(a^2\gamma S)^2 &= (a^2\gamma S)\beta(a^2\gamma S)\\ &= a^2\beta((a^2\gamma S)\gamma S)\\ &= a^2\beta((S\gamma S)\gamma a^2)\\ &= (S\gamma S)\beta(a^2\gamma a^2)\\ &= S\beta(a^2)^2\\ &\subseteq S\Gamma P\\ &\subseteq P\end{aligned}$$

that is $(a^2\gamma S)^2 \subseteq P$. It is easy to see that $a^2 \in P$. □

Conclusions

Many new classes of Γ-AG-groupoids have been discovered recently. All this has attracted researchers of the field to investigate these newly discovered classes in detail.

This current article investigates the Γ-ideals, Γ-semiprime and quasi Γ-semiprime ideals in Γ-AG-groupoids. Some characterizations of Γ-semiprimeand quasi Γ-semiprime ideals are obtained. Moreover, we investigate relationships between Γ-semiprime and quasi Γ-semiprime ideals of $(A:\Gamma:B)$ and $(A:\gamma:r)$ in Γ-AG-groupoids.

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