Fuzzy-Set Method For Grade Evaluation

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Abstract*:*

This paper is comparing 3 methods of student grading evaluation i.e. by instructor experiences, T-score and Fuzzy-set. The Instructor experiences method depends on the criteria strictly. The second Method is obtained by transforming the raw score into T-score. The third Method can be obtained by transforming the raw score into the fuzzy number by the expert opinion. The result shows that the Fuzzy Method is the compromised method between the criteria referenced and the T-score Method since it is not only the expert experience dependence as in the first method but also mathematically formulated as in the second method as well. Thus, the Fuzzy method is more reasonable.

Keywords: component, *Grade evaluation, T-score, Fuzzy grading system*

Introduction

As students find it is really difficult to be success ful in each subject and get a good grade from any educational institute, the grading process is also really difficult for each teacher or professor to make it as standard as it should be. The question is how can we evaluate whether a student is good enough to get an A. Since grading does not mean a harsh process or feedback that might decrease the students' motivation to study further but it should encourage them to continue leaning instead.

When we have already got students' aggregate test scores, we might apply the criterion referenced system or norm referenced system (sometimes we call group referenced system) to evaluate students' academic abilities. Some questions are raised because of the fairness issue. For example, is it possible to claim that two students have the same intelligence level because their aggregate scores are equal?. To what degree does each test score explain the aggregate scores?. How can we formulate it and compute it mathematically?. How to translate a set of scores into letter-grades?. Letter-grades are recognized to be *fuzzy* descriptors of students' performance.

From [1] and [6], there are some advantages and disadvantages of both method as follows:

Table 1 Benefits and drawbacks of criterian reference system

Advantages	Disadvantages
1. It is suitable for tracking the progress of the students within the curriculum.	1. Creating tests that are both valid and reliable requires fairly extensive and expensive time and effort.
2. Test item can be designed to match specific program objectives.	2. Results cannot be generalized beyond the specific course or program.
3. It reports how well of the students answer correctly on the lessons being study.	3. Criterion-referenced tests are specific to a program and cannot be used to measure the performance of large groups.
4. It is easy to calculate grade.	4. Fixed scales are arbitrary, so sometimes is meaningless.
5. It reduces competition between students.	5. This method can allow all students to receive the same grade and thus not provide information needed to screen students in competitive circumstances.

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Table 2 Benefits and drawbacks of group referenced system

Advantages	Disadvantages
1. It allows for screening students according to their performance relative to their peers.	1. It does not provide feedback as to actual content mastered by student.
2. It is useful for competitive circumstances where students need feedback as to how they compare to their peers.	2. The curve grade based on single class meaningless unless provided in relation to group student is being scored against.
3. It is very easy for instructors to use.	3. It discourages cooperation, as competition becomes central.
4. It is generally appropriate in large courses.	4. It is not suitable to compare among many classes.
5. It is independent from the difficulty of the test.	5. It uses some complicated statistics.

The aim of this research is to develop a new reasonable method to grade each student in my Linear Algebra class. Actually, the grades have been given already, so the new grade from this method will not affect on their previous evaluation but, fortunately, we can compare the differences between the actual grade and the new grading result from another grading system. This research will focus on grading method using fuzzy logic.

Literature Reviews

Fuzzy concept has distracted people's mind for many decades and become a popular and very interesting subject among computer engineers, mathematicians and statisticians as well as philosophers and psychologists. One of the reasons why fuzzy or vague concept cannot be formulated by ordinary mathematics easily is this concept does not include mathematics definite results. So, we need to know more different mathematical concepts to explain the mathematical modeling of the fuzzy idea.

The idea of fuzzy concept is related to the[3] boundary-line view. A concept is fuzzy if there are some objects which cannot be classified either to the concept or to its complements but are members of the concepts' boundary. The first successful approach to fuzziness was the notion of a fuzzy set proposed by[9]. In this approach, sets are defined by partial membership in contrast to crisp membership used in classical definition of a set.

Fuzziness can occurr in so many areas where human judgment, conclusion, and decision are involved, for example, engineering, medicine, artificial intelligence, pattern recognition, meteorology, computer science education, psychology, sociology etc.

There are some benefits to studying about fuzzy concept. One of them is to give us the interpretation of "non-random uncertainty" since the vague concept provides us a fantastic tool to measure the level of uncertainty or imprecision and this concept also helps us when we lack of information regarding a particular respond of the subject but we have to decide or give a conclusion.

[7]has combined the traditional way of evaluation of students' knowledge and success with the application of fuzzy logic and theory of fuzzy sets. He considered all the students activities and achievements and applied the "Centroid Method" or "Centroid clustering" where centroid is a point in the coordinate system where arithmetic average of scores earned in various activities of the students is used. It presents the average of centroid of each category, measured with the value being graded. That means the final scores or final results are affected by every activity.

The Fuzzy-Set Method

Definition 1 [5] (membership function) For a set A, we define a membership function μ_A such as

$$\mu_A(x) = \begin{cases} 1, & \text{if and only if} \quad x \in A \\ 0, & \text{if and only if} \quad x \notin A \end{cases}$$

We can say that the function μ_A maps each element in the universal set X to the set $\{0,1\}$, i.e.

$$\mu_A : X \to \{0,1\}$$

More precisely, the membership function μ_A in crisp set maps whole members in the universal set to $\{0,1\}$

Definition 2 [5] (membership function of fuzzy set) In a fuzzy sets A, each element is mapped to $[0,1]$ by a membership function

$$\mu_A : X \rightarrow [0,1],$$

where $[0,1]$ means the set of real numbers between 0 and 1 (including 0 and 1).

Definition 3 [5] (α-cut set) The α-cut set A_α is made up of members whose membership is not less than α, i.e.

$$A_\alpha = \left\{ x \in X \mid \mu_A(x) \geq \alpha \right\}.$$

Note that α is arbitrary. This α-cut set is a crisp set.

Definition 4 [5] (Fuzzy number) If a Fuzzy set is convex and normalized, and its membership function is defined in R and piecewise continuous, then it is called a "Fuzzy number". So fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy.

Definition 5 [5] (Triangular Fuzzy number) It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$ This representation is interpreted as membership functions and holds the following conditions (see Figure 1).

(i) it is an increasing function from a_1 to a_2

(ii) it is a decreasing function from a_2 to a_3

(iii) $a_1 \leq a_2 \leq a_3$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for} & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for} & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for} & a_2 \leq x \leq a_3 \\ 0 & \text{for} & x > a_3 \end{cases}$$

Definition 6 [5] (Trapezoidal Fuzzy number) The trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for} & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for} & a_1 \leq x \leq a_2 \\ 1 & \text{for} & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{for} & a_3 \leq x \leq a_4 \\ 0 & \text{for} & x > a_4 \end{cases}$$

The membership function of this fuzzy number will be interpreted as follows (Figure 2).

Definition 7 [9] (Centroid Method) This procedure (also called center of area or center of gravity) is the most prevalent and physically appealing of all the defuzzification methods. It is given by the algebraic expression

$$x^* = \frac{\int \mu_{\tilde{A}}(x) \cdot x\, dx}{\int \mu_{\tilde{A}}(x)\, dx},$$

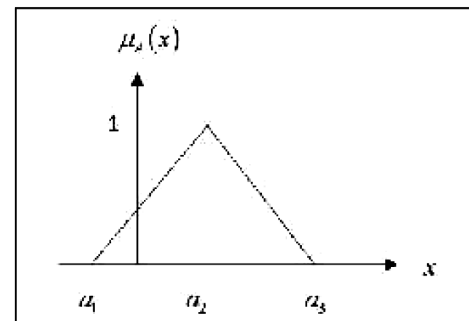where $\int$ denotes an algebraically integration.

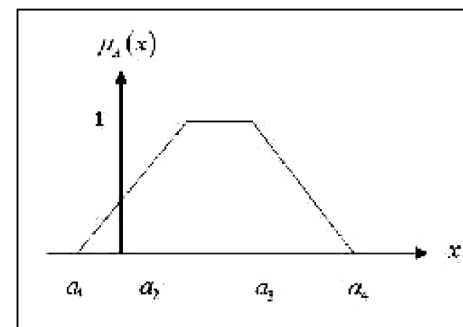**Figure 1** Triangular Fuzzy number

Figure 2 Trapezoidal Fuzzy number

The Main Results

We will compare the result of 2 "traditional" grading systems. Considering from Table 1 and Table 2, we have seen the differences between the numbers of each grade which are given by using those 2 methods. Sometimes, evaluation using the group referenced system seems to work but you might come across with the problems such as "too high" quality for getting an A or "too generous" to give a D to a student (i.e. too low criteria to get an F). Moreover, if we use the criterion referenced system we might be struggle with how to set the criteria and for hardworking student who can just nearly get an A but they cannot since, sometimes, they missed only 0.05 points to achieve our criteria, we might feel so sorry because it looks a bit unfair to those kind of students.

Table 3 The grading using criterian reference system

Interval	Group	Number of students
80-100	A	10
75-79.99	B+	2
70-74.99	B	5
65-69.99	C+	5
60-64.99	C	9
45-59.99	D+	12
35-44.99	D	21
0 - 35	F	12

Table 4 The grading using T-Score (group reference system)

Interval	Group	Number of students
89.5-100	A	5
75.50-89	B+	7
63.75-73	B	12
55-63	C+	14
43.5-52.5	C	14
37-43	D+	12
28-31.5	D	7
17-27.5	F	5

In our next step, we are developing a new grading concept using fuzzy concept to classify the group students' performance. We define the membership function of the input and output as shown in Figure 3. We are considering the aggregate score as input, and see whether the grading result is reasonable for the students.
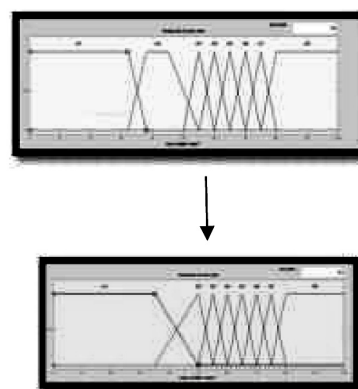
Figure 3   The Membership Function of the Input and Output for the Grading Method
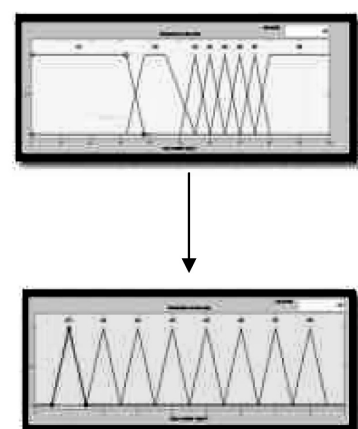
Figure 4   The Membership Function of the Input and a new Membership Function Output for the Grading Method

Table 5 The Grading using Fuzzy system

Interval	Group	Number of students
77-100	A	11
75.5	B+	1
70.5-73	B	5
65-69	C+	5
60-64	C	9
55-58	D+	7
44-52.5	D	13
0-43.5	F	25

Table 6 The Grading using Fuzzy Method with the new membership function

Interval	Group	Number of students
77-100	A	11
72.5-76.5	B+	1
67.5-72	B	5
62.5-67	C+	5
57.5-62	C	9
52-57	D+	7
35-51.5	D	26
0-34.99	F	12

Our first experiment is not quite suitable if we want to use it in the real situation because there is "too high expectation" from the students to let them pass the exams. The reason why this thing happened is the centroid of the membership function of the output was changed to be a bit more, so the number of students who reserve the grade D will be less than it should be. As a result, we have tried the second attempt to improve the result to be more applicable and more reasonable for both sides, i.e. students and teachers. Instead of writing the membership function in terms of the score criteria, we set the condition by using the number from 1, 2, 3 to 9 to representing the different grades and defining the triangular membership functions without overlapping.

Conclusion

From the Table 5, we used the Fuzzy Method with the membership function in Figure 3 for the grade evaluation but it is not satisfortoru since there are too many students who got an F. Thus we tried to improve the method using a new membership function as in Figure 4 and, finally, we got the very reasonable grade evaluation model for the only one input i.e. aggregate scores.

This new grading system can be useful for the teachers who have to mark students' essays because sometimes the students' marks do not only depend on their writing abilities but also the emotion of the marker as well. That means there is some fuzziness on marking because of the marker which is unfair to evaluate the students who have the same level of ability with a different grade.

We do not expect that this paper alone will change the negative attitudes toward fuzzy set theory and fuzzy logic by most researchers in the grading system. However, we consider it a necessary first step. To change the attitudes will require that the utility of fuzzy set theory and fuzzy logic for representing and dealing with concepts be properly demonstrated. This will not be possible without extensive research involving experts from both areas. We intend to make efforts to facilitate such cooperation in the future.

Future Work

To get a "more reasonable" Fuzzy grading system, we need to be fair with the students who have tried more attempts for their achievements or what we call development. Therefore, we will consider Midterm Exams and Final Exams and see whether there is any student who get a good progress. If the Final result is getting improved, we should reward them somehow. For example, If the Midterm and the Final scores of two students, says, Alice and Bob are 30, 20 and 20, 30 respectively, we should reward Bob a better grade since there is some development. Thus, our future work is finding an improved version of this fuzzy method and also explain the model understandably.

Suggestions

This method has been developed to solve some problem on evaluating students using vague questions. We might have seen some of the fuzzy questions in the Ordinary National Educational Test (O-NET), for example,

1. If you get sexually aroused, what should you do?

(O-NET 54)
a) Go play football.
b) Ask parents for some advice.
c) Try to sleep.
d) Go out with the opposite gender.
e) Go to the cinema with close friends.

There are some unclear thought about how to answer this question. It depends on students' experiences

and gender, i.e. boy students might choose the item a. as the correct answer but most of the girl students will not choose that choice because it is not girl preference. So the best answer of each student are different from the other but, in fact, every question has to have just only one correct answer.

　　2.　Which item is the best leisure activities for family members' happiness and health? (O-NET 53)

　　a)　Parents play golf and the children play games at home.

　　b)　Parents send their children to stay with grandparents then go for working.

　　c)　Parents send their children to the tutorial school and then go for shopping.

　　d)　Everyone goes for doing some exercise at a park and come back together to cook some meal for dinner.

　　The best answer of each student may vary because of their personal experience. Therefore, it is difficult to tell whether the item d. is the really best answer for this question.

　　It would be a better way to evaluate the students using fuzzy set method, since the Entrance examination questions are sometimes unclear. Although the newly developing grading system is difficult to understand for most of all school teachers since it needs a lot of understanding on Statistics and Fuzzy set concepts, it is still possible to use if they are trained to use it.

　　Actually, it is really difficult to find a good grade evaluation method which is reliable, mathematical formulated and fair, but the more difficult thing is how to train the teacher or instructor to teach well and also be a good evaluator at the same time.

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