

The Noise Model Prediction by Allan Variance

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Abstract

This article deals with the ability of Allan variance to predict noise models in any frequency system. Five noise types were modeled and simulated by computer. The Allan variance is able to identify these noise models. Any type of five noises can be identified by the Allan variance via “sigma-tau” plot. In this study, RMSE was used to measure the potential of Allan Variance.

Keywords: allan variance, noise model

Introduction

The frequency oscillator plays a very important role in telecommunication, global positioning system and scientific instruments, but noise decrease the frequency stability of these systems. The Ability to predict noise models will make the system work more efficient and solve any problems from noise by getting rid of noise source.

A frequency oscillator normally generates a sine wave signal as shown in (1), which ignores amplitude fluctuation and unity amplitude.

$$u(t) = \sin(2\pi f_0 t + \phi(t)) \quad (1)$$

Where $\phi(t)$ is the time dependent phase fluctuations and f_0 is the oscillator nominal frequency. In accuracy or stability measurements another reference frequency source with a higher order of stabilize than oscillator under test. The reference source is ideal with zero term $\phi(t)$. The fractional frequency $y(t)$ is yielded by the comparison of the frequency in the oscillator under test and the reference which $y(t)$ is defined by (2)^{1,2}.

$$y(t) = \frac{f(t) - f_0}{f_0} = \frac{1}{2\pi f_0} \frac{d\phi}{dt} = \frac{dx}{dt} \quad (2)$$

where $f(t)$ is the time variant frequency of oscillator under measurement and $x(t)$ is time fluctuation. The relation between time fluctuations and phase fluctuations is determine with $\phi(t) = 2\pi f_0 x(t)$.

In order to measure the frequency stability, the statistical variances were used. Allan variance his normal time domain frequency stability³. The definition of Allan variance satisfies (3)⁴, in which y_i is order i of fractional frequency averaged over sampling or interval time, τ and M is the number of fractional frequency averaged.

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)} \sum_{i=1}^{M-1} [y_{i+1} - y_i]^2 \quad (3)$$

The fractional frequency data average over time τ with the nonoverlap sample were used to calculate Allan variance. With the plot of Allan variance and sampling time, sigma-tau, the noise models can be determined.

The mentioned Sigma-tau plot in Figure 1 shows some measure of frequency stability versus the time over which the frequency is averaged. The plot was shown in $\log \sigma_y$ or square root of Allan variance versus $\log \tau$ and slope of the plot equal $\mu / 2$. The μ was used to determine the noise models, which the white and flicker phase modulation, μ is equal -2 and white frequency modulation, flicker frequency modulation and random walk the m are -1, 0 and 1 respectively.

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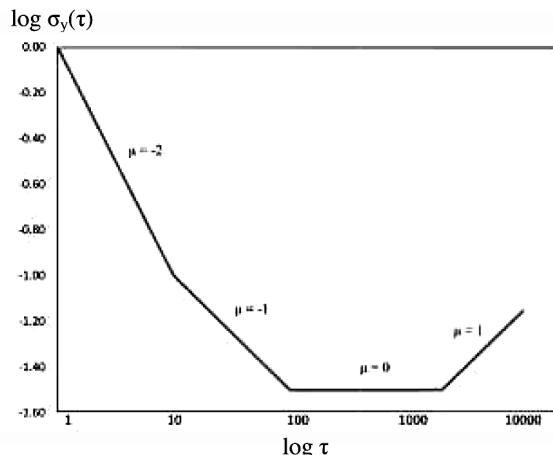


Figure 1 Sigma tau plot.

Methodology

We use program AlaNoise 3.0 to simulate data which have noise models white and flicker phase modulation, white frequency modulating. The size of time series 100, 200, 500 and 1,000 were selected with 100 repeat times. Each time series was used to calculate the Allan variance and the results were plotted. The slope in the sigma-tau plot shows the noise models of plotted data. The corrected predictions of noise models use statistical parameter RMSE.

Results, Discussion and Conclusion

All of m (100 values in each noise model) from the sigma-tau plot were compared with theory by the RMSE as shown in Table 1 and Figure 2. The trend of RMSE decreased slightly in the Flicker phase modulation, Random Walk frequency modulation, White phase and frequency modulation with the increasing of the time series size and the Flicker frequency modulation. The values of RMSE increase while the size of time series increases. The lowest RMSE was found in the Flicker phase modulation noise model.

The RMSE of five noise models with a time series 100, 200, 500 and 1000

Size of time series	RMSE				
	White Phase Modulation	Flicker Phase Modulation	White Frequency Modulation	Flicker Frequency Modulation	Random Walk Frequency Modulation
100	0.266	0.170	0.342	0.459	0.476
200	0.248	0.161	0.313	0.472	0.399
500	0.219	0.156	0.321	0.476	0.408
1000	0.212	0.149	0.258	0.499	0.407

Table 1 shows the reasonableness in order to use the Allan variance in prediction because the minimum RMSE is 0.149 in the Flicker Phase Modulation and the other not access 0.5 and with the reasons before, the Allan variance suitable for predict the noise model efficiency.

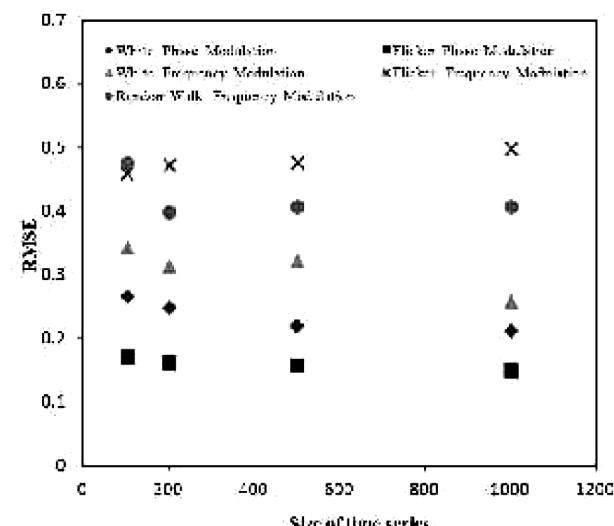


Figure 2 Compare the RMSE values of all noise models and the trend of noise models.

References

1. Baran O, Kasal M, "Allan variances calculation and simulation," Proceedings of 19 th International Conference Radioelektronika 2009;09:187-190.
2. Baran O, Kasal M, "Study of oscillators frequency stability in satellite communication links," Proceedings of 4th Recent Advances in Space Technologies 2009;09:535-540.
3. El-Sheimy N, Haiying H, Xiaoji N, "Analysis and Modeling of Inertial Sensors Using Allan Variance," IEEE Transactions on Instrumentation and Measurement, Vol. 57, No. 1, pp 140-149, JANUARY 2008.

4. M. S. McCorquodale, "On modern and historical short-term frequency stability metrics for frequency sources," *Proceedings of Frequency Control Symposium, Joint with the 22nd European Frequency and Time forum 2009*;09:328-333.