

สมบัติของตัวดำเนินการ \mathcal{R}_m^a ในปริภูมิโครงสร้างเล็กสุดที่มีอุดมคติ

Properties of \mathcal{R}_m^a -operator in minimal structure space with an ideal

ยุทธพงศ์ มนัสทิพารมณ¹, ดรุณี บุญชาร์^{2*}, โชคชัย วิริยะพงษ์³,
Yutthapong Manuttiparom¹, Daruni Boonchari^{2*}, Chokchai Viriyapong³

Received: 19 October 2020 ; Revised: 20 December 2020 ; Accepted: 8 January 2021

บทคัดย่อ

ในบทความนี้ ผู้วิจัยได้นำเสนอเซตเปิดแบบ δ - m เซตเปิดแบบ a - m ฟังก์ชันแบบ δ - m -local ตัวดำเนินการแบบ R_m^a บนปริภูมิโครงสร้างเล็กสุดที่มีอุดมคติพร้อมทั้งศึกษาสมบัติของฟังก์ชัน และตัวดำเนินการนี้

คำสำคัญ: เซตเปิดแบบ δ - m เซตเปิดแบบ a - m ฟังก์ชันแบบ δ - m -local ตัวดำเนินการแบบ R_m^a ปริภูมิโครงสร้างเล็กสุดที่มีอุดมคติ

Abstract

In this article, the concepts of δ - m -open sets, a - m -open sets in a minimal structure space with an ideal are introduced. In addition, we present an a - m -local function and an R_m^a -operator in a minimal structure space with an ideal. We studied the properties of the function and this operator.

Keywords: δ - m -open sets, a - m -open sets, δ - m -local functions, R_m^a -operator, a minimal structure space with an ideal.

¹ นิสิตปริญญาโท, คณะวิทยาศาสตร์ มหาวิทยาลัยมหาสารคาม อำเภอกันทรวิชัย จังหวัดมหาสารคาม 44150

² ผู้ช่วยศาสตราจารย์, คณะวิทยาศาสตร์ มหาวิทยาลัยมหาสารคาม อำเภอกันทรวิชัย จังหวัดมหาสารคาม 44150

³ ผู้ช่วยศาสตราจารย์, คณะวิทยาศาสตร์ มหาวิทยาลัยมหาสารคาม อำเภอกันทรวิชัย จังหวัดมหาสารคาม 44150

¹ Master degree student, Mahasarakham University, Kantharawichai District, Maha Sarakham 44150, Thailand.

² Asst. Prof., Faculty of Science, Mahasarakham University, Kantharawichai District, Maha Sarakham 44150, Thailand.

³ Asst. Prof., Faculty of Science, Mahasarakham University, Kantharawichai District, Maha Sarakham 44150, Thailand.

* Corresponding author ; Daruni Boonchari, Faculty of Science, Mahasarakham University, Kantharawichai District, Maha Sarakham 44150, Thailand. daruni.b@msu.ac.th.

Introduction

In 1945, Vaidyanathaswamy (1945) defined a local function in an ideal topological space and studied some properties of this function. In 1996, Maki, Umehara and Noiri (1996) defined a minimal structure and studied some properties of this structure. In 2014, Al-Omeri *et al.* (2014) defined an a -local function in an ideal topological space and also studied some properties of an a -local function. Later in 2016, Al-Omeri *et al.* (2016) defined an R_a -operator in an ideal topological space and studied some properties of this operator. In this article, we introduce the concepts of δ - m -open sets and δ - m -open sets in a minimal structure space with an ideal and study some fundamental properties. Moreover, we introduce the notions of δ - m -local functions and R_m^a -operators in minimal structure spaces, along with studying some properties related to an δ - m -local function and an R_m^a -operator defined above.

Preliminaries

Definition 2.1⁵ Let X be a nonempty set and $P(X)$ the power set of X . A subfamily m of $P(X)$ is called a minimal structure (briefly MS) on X if $\emptyset \in m$ and $X \in m$.

By (X, m) we denote a nonempty set X with a minimal structure m on X and it is called a minimal structure space. Each member of m is said to be m -open and the complement of m -open is said to be m -closed.

Definition 2.2 (Noiri & Popa, 2009) Let (X, m) be a minimal structure space and $A \subseteq X$. The m -closure of A , denoted by $CI_m(A)$ and the m -interior of A , denoted by $Int_m(A)$, are defined as follows ;

- 1) $CI_m(A) = \bigcap \{F : A \subseteq F, X \setminus F \in m\}$,
- 2) $Int_m(A) = \bigcup \{U : U \subseteq A, U \in m\}$.

Lemma 2.3 (Maki & Gani, 1999) Let (X, m) be a minimal structure space and $A, B \subseteq X$, the following properties hold ;

- (1) $CI_m(X \setminus A) = X \setminus Int_m(A)$ and $Int_m(X \setminus A) = X \setminus CI_m(A)$.
- (2) If $X \setminus A \in m$, then $CI_m(A) = A$ and if $A \in m$, then $Int_m(A) = A$.
- (3) $CI_m(\emptyset) = \emptyset$, $CI_m(X) = X$, $Int_m(\emptyset) = \emptyset$, and $Int_m(X) = X$.

(4) If $A \subseteq B$, then $CI_m(A) \subseteq CI_m(B)$ and $Int_m(A) \subseteq Int_m(B)$.

(5) $A \subseteq CI_m(A)$ and $Int_m(A) \subseteq A$.

(6) $CI_m(CI_m(A)) = CI_m(A)$ and $Int_m(Int_m(A)) = Int_m(A)$.

Lemma 2.4 (Maki & Gani, 1999) Let (X, m) be a minimal structure space and $A \subseteq X, x \in X$. Then $x \in CI_m(A)$ if and only if $U \cap A \neq \emptyset$ for every an m -open set U containing x .

Definition 2.5 (Rosas *et al.*, 2009) Let (X, m) be a minimal structure space and $A \subseteq X$.

(1) A is called m -regular open if $A = Int_m(CI_m(A))$

(2) A is called m -regular closed if $X \setminus A$ is m -regular open.

The family of all m -regular open sets of X is denoted by $r(m)$ and the family of all m -regular closed sets of X is denoted by $rc(m)$.

Definition 2.6 (Ozbakir & Yildirim, 2009) An ideal \mathcal{I} on a minimal structure space (X, m) is a nonempty collection of subsets of X which satisfies the following properties ;

- (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$ (heredity),
- (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$ (finite additivity).

The set \mathcal{I} together with a minimal structure space (X, m) is called a minimal structure space with an ideal, denoted by (X, m, \mathcal{I}) .

Main Results

Definition 3.1 Let (X, m) be a minimal structure space. A subset A is said to be δ - m -open if for each $X \in A$ there exists an m -regular open set G such that $X \in G \subseteq A$. The complement of δ - m -open set is called δ - m -closed. The family of all δ - m -closed sets of X , denoted by $\delta C_m(X)$.

Theorem 3.2 Let (X, m) be a minimal structure space and $A \subseteq X$. The arbitrary union of δ - m -open sets is a δ - m -open set.

Proof Let B_α be a δ - m -open set for all $\alpha \in J$ where J is an index set and let $x \in \bigcup_{\alpha \in J} B_\alpha$. There exists $\beta \in J$ such that $x \in B_\beta$. Since B_β is δ - m -open, there exists an m -regular open set G_β such that $x \in G_\beta \subseteq B_\beta$. Then $x \in G_\beta \subseteq B_\beta \subseteq \bigcup_{\alpha \in J} B_\alpha$. Therefore $\bigcup_{\alpha \in J} B_\alpha$ is δ - m -open.

Definition 3.3 Let (X, m) be a minimal structure space and $A \subseteq X$. A point $x \in X$ is called a δ - m -cluster point of A if $U \cap A \neq \emptyset$ for each m -regular open set U containing x .

Definition 3.4 Let (X, m) be a minimal structure space and $A \subseteq X$. The set of all δ - m -cluster points of A is called δ - m -closure of A and is denoted by $C_{\delta m}(A)$ and the union m -regular open sets contained in A is called the δ - m -interior of A , denoted by $I_{\delta m}(A)$.

Theorem 3.5 Let (X, m) be a minimal structure space and $A \subseteq X$. Then A is δ - m -open if and only if $I_{\delta m}(A) = A$.

Proof (\Rightarrow) Suppose that A is δ - m -open. By definition of δ - m -interior, $I_{\delta m}(A) = A$. Let $x \in A$. Since A is δ - m -open, there exists an m -regular open set O such that $x \in O \subseteq A$. This implies that $x \in I_{\delta m}(A)$. Then $A \subseteq I_{\delta m}(A)$. Hence $A = I_{\delta m}(A) = A$. (\Leftarrow) It follows from Theorem 3.2.

Theorem 3.6 Let (X, m) be a minimal structure space and $A, B \subseteq X$. The following property hold ;

- (1) If $A \subseteq B$, then $I_{\delta m}(A) \subseteq I_{\delta m}(B)$,
- (2) If $A \subseteq B$, then $C_{\delta m}(A) \subseteq C_{\delta m}(B)$.

Proof (1) Assume that $A \subseteq B$ and $x \in I_{\delta m}(A)$. Then, there exists an m -regular open set G such that $x \in G \subseteq A$. Since $A \subseteq B$, we have $x \in G \subseteq A \subseteq B$. This implies that $x \in I_{\delta m}(B)$. Hence $I_{\delta m}(A) \subseteq I_{\delta m}(B)$.

(2) Let $A \subseteq B$. Assume that $x \notin C_{\delta m}(B)$. Then there exists an m -regular open set U containing x such that $U \cap B = \emptyset$. Since $A \subseteq B$, we have $U \cap A \subseteq U \cap B = \emptyset$. Thus $x \notin C_{\delta m}(A)$. Therefore $C_{\delta m}(A) \subseteq C_{\delta m}(B)$.

Theorem 3.7 Let (X, m) be a minimal structure space and $A \subseteq X$. The following properties hold ;

- (1) $C_{\delta m}(A) = X \setminus I_{\delta m}(X \setminus A)$,
- (2) $I_{\delta m}(A) = X \setminus C_{\delta m}(X \setminus A)$.

Proof (1) We will show that $C_{\delta m}(A) = X \setminus I_{\delta m}(X \setminus A)$ by contrapositive. Assume that $x \notin X \setminus I_{\delta m}(X \setminus A)$. We get that $x \in I_{\delta m}(X \setminus A)$. So there exists an m -regular open set G such that $x \in G \subseteq X \setminus A$. Then $G \cap A = \emptyset$ and $x \notin C_{\delta m}(A)$. Thus $C_{\delta m}(A) \subseteq X \setminus I_{\delta m}(X \setminus A)$.

Next, we show that $X \setminus I_{\delta m}(X \setminus A) \subseteq C_{\delta m}(A)$ by contrapositive. Assume that $x \notin C_{\delta m}(A)$. Then x is not a δ - m -cluster point of A . There exists an m -regular open set G containing x such that $G \cap A = \emptyset$.

So $x \in G \subseteq X \setminus A$ and we get that $x \in I_{\delta m}(X \setminus A)$. Hence $x \notin X \setminus I_{\delta m}(X \setminus A)$. Thus $X \setminus I_{\delta m}(X \setminus A) \subseteq C_{\delta m}(A)$.

(2) Since $X \setminus A \subseteq X$, we have $C_{\delta m}(X \setminus A) = X \setminus I_{\delta m}(X \setminus (X \setminus A))$ by (1) and we get $C_{\delta m}(X \setminus A) = X \setminus I_{\delta m}(A)$. Therefore $I_{\delta m}(X \setminus A) = X \setminus C_{\delta m}(X \setminus A)$.

Definition 3.8 Let (X, m) be a minimal structure space and $A \subseteq X$.

(1) A is called a - m -open if $A \subseteq \text{Int}_m(CI_m(I_{\delta m}(A)))$. The family of all a - m -open sets of X is denoted by \mathcal{M}^a .

(2) A is called a - m -closed if $CI_m(\text{Int}_m(C_{\delta m}(A))) \subseteq A$.

Theorem 3.9 Let (X, m) be a minimal structure space and $A \subseteq X$. Then A is a - m -open if and only if $X \setminus A$ is a - m -closed.

Proof Assume that A is a - m -open. Then $A \subseteq \text{Int}_m(CI_m(I_{\delta m}(A)))$. and $X \setminus A \supseteq X \setminus (\text{Int}_m(CI_m(I_{\delta m}(A))))$. By Lemma 2.3 and Theorem 3.7, $X \setminus A \supseteq CI_m(\text{Int}_m(C_{\delta m}(X \setminus A)))$. Therefore, $X \setminus A$ is a - m -closed.

Conversely, assume that $X \setminus A$ is a - m -closed. Then $CI_m(\text{Int}_m(C_{\delta m}(X \setminus A))) \subseteq X \setminus A$ and $X \setminus CI_m(\text{Int}_m(C_{\delta m}(X \setminus A))) \supseteq X \setminus (X \setminus A)$. By Lemma 2.3 and Theorem 3.7, $\text{Int}_m(CI_m(I_{\delta m}(A))) \supseteq A$. Hence A is a - m -open.

Example 3.10 Let $X = \{a, b, c, d\}$ with a minimal structure $m = \{\emptyset, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, x\}$. Then $r(m) = \{\emptyset, \{a, b\}, \{a, d\}, \{b, c\}, \{c, d\}, x\}$, and $\delta O_m(x) = \{\emptyset, \{a, b\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, x\}$, $\mathcal{M}^a = \{\emptyset, \{a, b\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, x\}$. In this example $\{a, b\}, \{a, d\} \in \mathcal{M}^a$ but $\{a, b\} \cap \{a, d\} = \{a\} \notin \mathcal{M}^a$, that means \mathcal{M}^a does not have the property that any finite intersection of a - m -open sets is a - m -open.

Definition 3.11 Let (X, m) be a minimal structure space and $A \subseteq X$. The a - m -closure of A , denoted by $aC_m(A)$ and the a - m -interior of A , denoted by $aI_m(A)$, are defined as follows ;

- (1) $aC_m(A) = \bigcap \{F : X \setminus F \in \mathcal{M}^a \text{ and } A \subseteq F\}$,
- (2) $aI_m(A) = \bigcup \{U : U \in \mathcal{M}^a \text{ and } U \subseteq A\}$.

Theorem 3.12 Let (X, m) be a minimal structure space and $A \subseteq X$, $x \in X$. Then $x \in aC_m(A)$ if and only if $U \cap A \neq \emptyset$ for every a - m -open set U containing x .

Proof (\Rightarrow) Suppose that there exists an a - m -open set U containing x such that $U \cap A = \emptyset$. So $A \subseteq X \setminus U$ and $X \setminus U$ is a - m -closed. Since $aC_m(A)$ is the intersection of

all a - m -closed sets containing A , $aC_m(A) \subseteq X \setminus U$. Since $x \notin X \setminus U$, we have $x \notin aC_m(A)$.

(\Leftarrow) Assume that $x \notin aC_m(A)$. Then there exists an a - m -closed set F such that $A \subseteq F$ and $x \notin F$. Choose $U = X \setminus F$. Then U is a - m -open and $x \in X \setminus F = U$. Moreover, $U \cap A \subseteq (X \setminus F) \cap F = \emptyset$.

Theorem 3.13 Let (X, m) be a minimal structure space and $A, B \subseteq X$. The following properties hold ;

(1) If $A \subseteq B$, then $aC_m(A) \subseteq aC_m(B)$.

(2) If $A \subseteq B$, then $aI_m(A) \subseteq aI_m(B)$.

Proof (1) Assume that $A \subseteq B$ and $x \notin aC_m(B)$. Then there exists an a - m -open set U containing x such that $U \cap B = \emptyset$. Since $A \subseteq B$, $U \cap A = \emptyset$. Hence $x \notin aC_m(A)$.

(2) Let $A \subseteq B$ and $x \in aI_m(A)$. Then there exists an a - m -open set U such that $x \in U \subseteq A$. Since $A \subseteq B$, $x \in U \subseteq B$. Therefore $x \in aI_m(B)$.

Proposition 3.14 Let (X, m) be a minimal structure space. Then $\emptyset \in \mathcal{M}^a$ and $X \in \mathcal{M}^a$.

Proof Since $\emptyset \subseteq \text{Int}_m(CI_m(I_{\delta m}(\emptyset)))$, \emptyset is a - m -open, and so $\emptyset \in \mathcal{M}^a$. Clearly $X = \text{Int}_m(CI_m(X))$, so X is an m -regular open. Then X is δ - m -open, that is $I_{\delta m}(X) = X$, and so $X \subseteq \text{Int}_m(CI_m(I_{\delta m}(X)))$. Therefore $X \in \mathcal{M}^a$.

Theorem 3.15 Let (X, m) be a minimal structure space. Then the arbitrary union of elements of \mathcal{M}^a belongs to \mathcal{M}^a .

Proof Let V_α be a - m -open for all $\alpha \in J$ and $G = \bigcup_{\alpha \in J} V_\alpha$. Then $V_\alpha \subseteq \text{Int}_m(CI_m(I_{\delta m}(V_\alpha)))$ for all $\alpha \in J$. Since $V_\alpha \subseteq G$, it follows that $I_{\delta m}(V_\alpha) \subseteq I_{\delta m}(G)$ and so $CI_m(I_{\delta m}(V_\alpha)) \subseteq CI_m(I_{\delta m}(G))$. Then $\text{Int}_m(CI_m(I_{\delta m}(V_\alpha))) \subseteq \text{Int}_m(CI_m(I_{\delta m}(G)))$. This implies that $V_\alpha \subseteq \text{Int}_m(CI_m(I_{\delta m}(G)))$ for all $\alpha \in J$. Thus $\bigcup_{\alpha \in J} V_\alpha \subseteq \text{Int}_m(CI_m(I_{\delta m}(G)))$. Therefore $G \subseteq \text{Int}_m(CI_m(I_{\delta m}(G)))$.

Corollary 3.16 Let (X, m) be a minimal structure space. Then the arbitrary intersection of a - m -closed sets is an a - m -closed set.

Proof Let G_α be a - m -closed for all $\alpha \in J$. Then $X \setminus G_\alpha$ is a - m -open and so $\bigcup_{\alpha \in J} (X \setminus G_\alpha)$ is a - m -open. Since $X \setminus \bigcap_{\alpha \in J} G_\alpha = \bigcup_{\alpha \in J} (X \setminus G_\alpha)$, $\bigcap_{\alpha \in J} G_\alpha$ is a - m -closed.

Remark 3.17 In a minimal structure space, by Corollary 3.16, $aC_m(A)$ is a - m -closed.

Theorem 3.18 Let (X, m) be a minimal structure space and $A \subseteq X$. The following properties hold ;

(1) $aC_m(aC_m(A)) = aC_m(A)$,

(2) $aI_m(aI_m(A)) = aI_m(A)$.

Proof (1) Clearly $aC_m(A) \subseteq aC_m(aC_m(A))$. Since $aC_m(A)$ is a - m -closed, $aC_m(aC_m(A)) \subseteq aC_m(A)$. Therefore $aC_m(aC_m(A)) = aC_m(A)$.

(2) Clearly $aI_m(aI_m(A)) = aI_m(A)$. Since $aI_m(A)$ is a - m -open, $aI_m(A) \subseteq aI_m(aI_m(A))$. Therefore $aI_m(aI_m(A)) = aI_m(A)$.

Let (X, m, \mathcal{I}) be a minimal structure space with an ideal. For each $x \in X$, let $\mathcal{M}^a(x) = \{U : x \in U, U \in \mathcal{M}^a\}$ be the family of all a - m -open sets that contain x .

Definition 3.19 Let (X, m, \mathcal{I}) be a minimal structure space with an ideal and $A \subseteq X$. Then $A_m^a(\mathcal{I}, m) = \{x \in X : U \cap A \notin \mathcal{I}, \text{ for every } U \in \mathcal{M}^a(x)\}$ is called **a - m -local function of A** with respect to \mathcal{I} and m . We denote simply A_m^a for $A_m^a(\mathcal{I}, m)$.

Remark 3.20 The minimal ideal is $\{\emptyset\}$ and the maximal ideal is $P(x)$ in any minimal structure space with an ideal (X, m, \mathcal{I}) . It can be deduced that $A_m^a(\{\emptyset\}, m) = aC_m(A)$ and $A_m^a(P(x), m) = \emptyset$ for every $A \subseteq X$.

Remark 3.21 In general, $A \not\subseteq A_m^a$ and $A_m^a \not\subseteq A$. The next example shows that $A \not\subseteq A_m^a$.

Example 3.22 Let $X = \{a, b, c, d\}$ with a minimal structure $m = \{\emptyset, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, $A = \{a, b\}$. Then $\mathcal{M}^a = \{\emptyset, \{a, b\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ and $A_m^a = \emptyset$.

Theorem 3.23 Let (X, m, \mathcal{I}) be a minimal structure space with an ideal and $A, B \subseteq X$. The following properties hold ;

(1) $(\emptyset)_m^a = \emptyset$.

(2) If $A \subseteq B$, then $A_m^a \subseteq B_m^a$.

(3) $(A_m^a)_m^a \subseteq A_m^a$.

(4) $A_m^a \cup B_m^a \subseteq (A \cup B)_m^a$.

(5) $(A \cap B)_m^a \subseteq A_m^a \cap B_m^a$.

(6) $(A \setminus B)_m^a \setminus (B)_m^a \subseteq A_m^a \setminus B_m^a$.

Proof (1) Assume $(\emptyset)_m^a \neq \emptyset$. Then there exists $x \in (\emptyset)_m^a$. Since $X \in \mathcal{M}^a(X)$, $X \cap \emptyset \notin \mathcal{I}$. It contradicts with $X \cap \emptyset = \emptyset \in \mathcal{I}$. Therefore $(\emptyset)_m^a = \emptyset$.

(2) Assume that $A \subseteq B$. We will show that $A_m^a \subseteq B_m^a$ by contrapositive. Suppose that $x \notin B_m^a$. Then there exists $U \in \mathcal{M}^a(X)$ such that $U \cap B \in \mathcal{I}$. From $A \subseteq B$ and the property of \mathcal{I} , $U \cap A \in \mathcal{I}$. Therefore $x \notin A_m^a$.

(3) Assume that $x \in (A_m^a)_m^a$, and $U \in \mathcal{M}^a(X)$. Then $A_m^a \cap U \notin \mathcal{I}$ and so $A_m^a \cap U \neq \emptyset$. Thus there exists $y \in A_m^a \cap U$, and so $y \in U \in \mathcal{M}^a(y)$. This implies that $A \cap U \notin \mathcal{I}$. Therefore $x \in A_m^a$.

(4) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, by (2) $A_m^a \subseteq (A \cup B)_m^a$ and $B_m^a \subseteq (A \cup B)_m^a$. So $A_m^a \cup B_m^a \subseteq (A \cup B)_m^a$.

(5) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, by (2) $(A \cap B)_m^a \subseteq A_m^a$ and $(A \cap B)_m^a \subseteq B_m^a$. So $(A \cap B)_m^a \subseteq A_m^a \cap B_m^a$.

(6) Since $A \setminus B \subseteq A$, by (2) $(A \setminus B)_m^a \subseteq A_m^a$. So $(A \setminus B)_m^a \setminus B_m^a \subseteq A_m^a \setminus B_m^a$.

Theorem 3.24 Let (X, m) be a minimal structure space and \mathcal{I}, \mathcal{J} are ideals on X where $\mathcal{I} \subseteq \mathcal{J}$. Then $A_m^a(\mathcal{I}, m) \subseteq A_m^a(\mathcal{J}, m)$ for all $A \subseteq X$.

Proof Let $A \subseteq X$. Assume that $x \in A_m^a(\mathcal{I}, m)$. Then $U \cap A \notin \mathcal{I}$ for every $U \in \mathcal{M}^a(x)$. Since $\mathcal{I} \subseteq \mathcal{J}$, $U \cap A \notin \mathcal{J}$ for every $U \in \mathcal{M}^a(x)$. Thus $x \in A_m^a(\mathcal{J}, m)$. Hence $A_m^a(\mathcal{I}, m) \subseteq A_m^a(\mathcal{J}, m)$.

Theorem 3.25 Let (X, m, \mathcal{I}) be a minimal structure space with an ideal and $A \subseteq X$. The following properties hold ;

- (1) $A_m^a \subseteq aC_m(A)$,
- (2) $A_m^a = aC_m(A)$, (i.e., A_m^a is an a - m -closed subset).

Proof (1) Assume that $x \notin aC_m(A)$. Then there exists an a - m -closed set F such that $A \subseteq F$ and $x \notin F$. Thus $x \in X \setminus F$, and so $X \setminus F \in \mathcal{M}^a(x)$. Hence $(X \setminus F) \cap A = \emptyset \in \mathcal{I}$, and so $x \notin A_m^a$. This implies that $A_m^a \subseteq aC_m(A)$.

(2) It is clear that $A_m^a \subseteq aC_m(A_m^a)$. Next, we will prove that $aC_m(A_m^a) \subseteq A_m^a$. Let $x \in aC_m(A_m^a)$ and $U \in \mathcal{M}^a(x)$. Then $A_m^a \cap U \neq \emptyset$. Therefore there exists $y \in A_m^a \cap U$, so $U \in \mathcal{M}^a(y)$. Since $y \in A_m^a$, $A \cap U \notin \mathcal{I}$, and so $x \in A_m^a$. Then $A_m^a = aC_m(A_m^a)$.

Theorem 3.26 Let (X, m, \mathcal{I}) be a minimal structure space with an ideal and $A \subseteq X$. The following properties hold ;

- (1) If $A \in \mathcal{I}$, then $A_m^a = \emptyset$.
- (2) If $U \in \mathcal{I}$, then $A_m^a = (A \cup U)_m^a$.
- (3) If $U \in \mathcal{I}$, then $A_m^a = (A \setminus U)_m^a$.

Proof (1) Assume that $A_m^a \neq \emptyset$. Then there exists $x \in A_m^a$. Since $X \in \mathcal{M}^a(x)$, $A = X \cap A \in \mathcal{I}$.

(2) Assume that $U \in \mathcal{I}$. Since $A \subseteq A \cup U$ by Theorem 3.23(2), we get $A_m^a \subseteq (A \cup U)_m^a$. Next, we will prove that $(A \cup U)_m^a \subseteq A_m^a$ by contrapositive. Suppose that $x \notin A_m^a$. Then there exists $V \in \mathcal{M}^a(x)$ such that $A \cap V \in \mathcal{I}$. Since $(A \cup U) \cap V = (A \cap V) \cup (U \cap V) \in \mathcal{I}$, $(A \cup U) \cap V \in \mathcal{I}$. Therefore $x \notin (A \cup U)_m^a$.

(3) Assume that $U \in \mathcal{I}$. Since $A_m^a = (A \cap X)_m^a = (A \cap ((X \setminus U) \cup U))_m^a = ((A \setminus U) \cup (A \cap U))_m^a$ and $A \cap U \subseteq U \in \mathcal{I}$, by (2) $A_m^a = (A \setminus U)_m^a$.

Definition 3.27 Let (X, m, \mathcal{I}) be a minimal structure with an ideal. An operator $\mathfrak{R}_m^a : P(X) \rightarrow P(X)$ is defined as follows ; for every $A \in P(X)$, $\mathfrak{R}_m^a(A) = \{x \in X : \text{there exists } U \in \mathcal{M}^a(x) \text{ such that } U \setminus A \in \mathcal{I}\}$.

Theorem 3.28 Let (X, m, \mathcal{I}) be a minimal structure space with an ideal and $A \in P(X)$. Then $\mathfrak{R}_m^a(A) = X \setminus (X \setminus A)_m^a$.

Proof Let $x \in \mathfrak{R}_m^a(A)$. Then there exists an a - m -open set U containing x such that $U \setminus A \in \mathcal{I}$. Thus $U \cap (X \setminus A) \in \mathcal{I}$. So $x \notin (X \setminus A)_m^a$ and hence $x \in X \setminus (X \setminus A)_m^a$. Therefore $\mathfrak{R}_m^a(A) \subseteq X \setminus (X \setminus A)_m^a$.

For the reverse inclusion, let $x \in X \setminus (X \setminus A)_m^a$. Then $x \notin (X \setminus A)_m^a$. Thus there exists an a - m -open set U containing x such that $U \cap (X \setminus A) \in \mathcal{I}$. This implies that $U \setminus A \in \mathcal{I}$. Hence $x \in \mathfrak{R}_m^a(A)$. So $X \setminus (X \setminus A)_m^a \subseteq \mathfrak{R}_m^a(A)$. Therefore $\mathfrak{R}_m^a(A) = X \setminus (X \setminus A)_m^a$.

Example 3.29 Let $X = \{a, b, c, d\}$ with a minimal structure $m = \{\emptyset, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, $A = \{a, b\}$. Then $\mathfrak{R}_m^a(A) = \{\emptyset, \{a, b\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ and $\mathfrak{R}_m^a(A) = \{a, b\}$.

Theorem 3.30 Let (X, m, \mathcal{I}) be a minimal structure space with an ideal, and $A \subseteq X$. Then $\mathfrak{R}_m^a(A)$ is a - m -open.

Proof We know that $\mathfrak{R}_m^a(A) = X \setminus (X \setminus A)_m^a$ and $(X \setminus A)_m^a$ is a - m -closed. Therefore $\mathfrak{R}_m^a(A)$ is a - m -open.

Theorem 3.31 Let (X, m, \mathcal{S}) be a minimal structure space with an ideal and $A, B \subseteq X$. Then the following properties hold ;

- (1) If $A \subseteq B$, then $\mathfrak{R}_m^a(A) \subseteq \mathfrak{R}_m^a(B)$.
- (2) If $A \subseteq B$, then $\mathfrak{R}_m^a(A \cap B) \subseteq \mathfrak{R}_m^a(A) \cap \mathfrak{R}_m^a(B)$.
- (3) If $A \subseteq B$, then $\mathfrak{R}_m^a(A) \cup \mathfrak{R}_m^a(B) \subseteq \mathfrak{R}_m^a(A \cup B)$.
- (4) If $A \in \mathcal{M}^a$, then $A \subseteq \mathfrak{R}_m^a(A)$.
- (5) If $A \subseteq B$, then $\mathfrak{R}_m^a(A) \subseteq \mathfrak{R}_m^a(\mathfrak{R}_m^a(A))$.

Proof (1) Assume that $A \subseteq B$. Then $X \setminus B \subseteq X \setminus A$. By Theorem 3.23(2), $(X \setminus B)_m^a \subseteq (X \setminus A)_m^a$ and hence $X \setminus (X \setminus A)_m^a \subseteq X \setminus (X \setminus B)_m^a$. Therefore $\mathfrak{R}_m^a(A) \subseteq \mathfrak{R}_m^a(B)$.

(2) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, $\mathfrak{R}_m^a(A \cap B) \subseteq \mathfrak{R}_m^a(A)$ and $\mathfrak{R}_m^a(A \cap B) \subseteq \mathfrak{R}_m^a(B)$. Therefore $\mathfrak{R}_m^a(A \cap B) \subseteq \mathfrak{R}_m^a(A) \cap \mathfrak{R}_m^a(B)$.

(3) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $\mathfrak{R}_m^a(A) \subseteq \mathfrak{R}_m^a(A \cup B)$ and $\mathfrak{R}_m^a(B) \subseteq \mathfrak{R}_m^a(A \cup B)$. Therefore $\mathfrak{R}_m^a(A) \cup \mathfrak{R}_m^a(B) \subseteq \mathfrak{R}_m^a(A \cup B)$.

(4) Assume that $A \in \mathcal{M}^a$. Then $X \setminus A$ is a - m -closed. By Theorem 3.25(1), we get that $(X \setminus A)_m^a \subseteq \mathcal{C}_m(X \setminus A) = X \setminus A$. Therefore $A = X \setminus (X \setminus A) \subseteq X \setminus (X \setminus A)_m^a = \mathfrak{R}_m^a(A)$.

(5) By Theorem 3.30, we get that $\mathfrak{R}_m^a(A)$ is a - m -open. By (4), we get that $\mathfrak{R}_m^a(A) \subseteq \mathfrak{R}_m^a(\mathfrak{R}_m^a(A))$.

Theorem 3.32 Let (X, m, \mathcal{S}) be a minimal structure space with an ideal and $A, B, U \subseteq X$. Then the following properties hold ;

- (1) If $U \in \mathcal{S}$, then $\mathfrak{R}_m^a(A \cup U) = \mathfrak{R}_m^a(A)$.
- (2) If $U \in \mathcal{S}$, then $\mathfrak{R}_m^a(A \cap U) = \mathfrak{R}_m^a(A)$.
- (3) If $(A \setminus B) \cup (B \setminus A) \in \mathcal{S}$, then $\mathfrak{R}_m^a(A) = \mathfrak{R}_m^a(B)$.
- (4) If $A \in \mathcal{S}$, then $\mathfrak{R}_m^a(A) = X \setminus X_m^a$.

Proof (1) Assume that $A \subseteq X$, $U \in \mathcal{S}$. By Theorem 3.26(2) and 3.28, we have $\mathfrak{R}_m^a(A \cup U) = X \setminus (X \setminus (A \cup U))_m^a = X \setminus ((X \setminus A) \cup U)_m^a = X \setminus (X \setminus A)_m^a$. Therefore $\mathfrak{R}_m^a(A \cup U) = \mathfrak{R}_m^a(A)$.

(2) Assume that $U \in \mathcal{S}$. By Theorem 3.26(3), we have $\mathfrak{R}_m^a(A \cap U) = X \setminus (X \setminus (A \cap U))_m^a = X \setminus ((X \setminus A) \setminus U)_m^a = X \setminus (X \setminus A)_m^a = \mathfrak{R}_m^a(A)$.

(3) Assume that $(A \setminus B) \cup (B \setminus A) \in \mathcal{S}$.

Thus

$$\begin{aligned}\mathfrak{R}_m^a(A) &= \mathfrak{R}_m^a(A \setminus (A \setminus B)) \\ &= \mathfrak{R}_m^a((A \setminus (A \setminus B)) \cup (B \setminus A)) \\ &= \mathfrak{R}_m^a(B).\end{aligned}$$

(4) Assume that $A \in \mathcal{S}$. By Theorem 3.26(3), we get that $\mathfrak{R}_m^a(A) = X \setminus (X \setminus A)_m^a = X \setminus X_m^a$.

Theorem 3.33 Let (X, m, \mathcal{S}) be a minimal structure space with an ideal and $A \subseteq X$. Then $\mathfrak{R}_m^a(A) = \mathfrak{R}_m^a(\mathfrak{R}_m^a(A))$ if and only if $(X \setminus A)_m^a = ((X \setminus A)_m^a)_m^a$.

Proof It follows from the facts that,

$$\begin{aligned}\text{I) } \mathfrak{R}_m^a(A) &= X \setminus (X \setminus A)_m^a \text{ and } \mathfrak{R}_a \\ \text{II) } \mathfrak{R}_m^a(\mathfrak{R}_m^a(A)) &= X \setminus [X \setminus (X \setminus (X \setminus A)_m^a)]_m^a \\ &= X \setminus ((X \setminus A)_m^a)_m^a.\end{aligned}$$

Therefore $\mathfrak{R}_m^a(A) = \mathfrak{R}_m^a(\mathfrak{R}_m^a(A))$ if and only if $(X \setminus A)_m^a = ((X \setminus A)_m^a)_m^a$.

Discussion and Conclusion

The aim of this article is to introduce the results of properties of some sets in a minimal structure space with an ideal. In addition, we study some properties of δ - m -open sets, a - m -open sets in a minimal structure space with an ideal. Moreover, we define an δ - m -local function and an R_m^a -operator in a minimal structure space with an ideal. Some properties of them are obtained.

Acknowledgements

The authors would like to thank the referees for helpful comments suggestions on the manuscript. It has received help from seniors, juniors, and friends in mathematics as well.

References

- Vaidyanathaswamy, R. (1945). The localization theory in set-topology. *Proc. Indian Acad. Sci.* 20, 51-61.
- Al-Omeri, W, Noorani, M. & Al-Omari, A. (2014). a -local function and it's properties in ideal topological space. *Fasc. Math.* 53, 1-15.
- Al-Omeri, W., Noorani, M. & Al-Omari, A. (2016). The operator in ideal topological space. *Creat. Math.* 25, 1-10.

- Noiri, T. & Popa, V. (2009). A generalization of some forms of g-irresolute functions. *European j. of Pure and Appl. Math*, 2(4): 473-493.
- Maki, H., Nagoor & Gani, K.C. (1999). On generalized semi-open and preopen sets. *Pure Appl. Math*, 49, 17-29.
- Rosas, E., Rajesh, N. & Carpintero, C. (2009). Some new type of open and closed sets in minimal structure-II. *International Mathematical Form*, 44, 2185-2198.
- Ozbakir, O.B. & Yildirim, E.D. (2009). On some closed sets in ideal minimal spaces. *Acta Math. Hungar*, 125(3), 227-235.
- Maki, H, Umehara, T. & Noiri, T. (1996). Every topological space in $\text{pre } T_{1/2}$. *Mem. Fac. Sci. Kochi Univ. Ser. A Math*. 17, 33-42.