

อิทธิพลกลุ่มเพื่อนของนักดื่มหน้าใหม่ที่มีต่อตัวแบบคณิตศาสตร์การระบาดของโรคติดสุรา

The influence of new drinkers' peer groups on a mathematical model of alcoholism epidemics

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บทคัดย่อ

การวิจัยครั้งนี้มีวัตถุประสงค์เพื่อพัฒนาและวิเคราะห์ตัวแบบคณิตศาสตร์การระบาดของโรคติดสุราของนักดื่มหน้าใหม่โดยใช้วิธีมาตรฐานในการศึกษาได้แบ่งประชากรคนออกเป็น 5 กลุ่ม ประกอบด้วยกลุ่มที่เสี่ยงต่อการเป็นนักดื่มสุรา (S) กลุ่มที่เป็นนักดื่มหน้าใหม่ (D) กลุ่มที่เป็นนักดื่ม (H) กลุ่มที่ได้รับการบำบัดโรคติดสุรา (T) และกลุ่มที่หายจากโรคติดสุรา (R) ซึ่งแบบจำลองที่ได้อยู่ในรูปของระบบสมการเชิงอนุพันธ์สามัญไม่เชิงเส้น และวิเคราะห์แบบจำลองโดยใช้วิธีมาตรฐาน หาค่าจุดสมดุล คำนวณค่าระดับการติดเชื้อ (R_0) โดยใช้วิธีรันถัดไป และ Spectral Radius วิเคราะห์เสถียรภาพของจุดสมดุลโดยใช้เกณฑ์ของ Routh – Hurwitz criteria และหาคำตอบเชิงตัวเลขเพื่อสนับสนุนผลลัพธ์ที่ได้ พบว่าผลเฉลยเชิงตัวเลขของระบบเข้าสู่จุดสมดุลที่ไม่มีผู้ดื่มแอลกอฮอล์ $E_0(1.45, 0, 0, 0, 0)$ เป็น Local Asymptotically Stable เมื่อกำหนดให้ $\alpha_1 = 0.01, \alpha_2 = 0.5$ และ $\alpha_3 = 0.01$ จะได้ $R_0 = 0.08787878786 < 1$ และจุดสมดุลที่มีผู้ดื่มแอลกอฮอล์ $E_1(1.20339, 0.01511, 0.00548, 0.04876, 0.17732)$ เป็น Local Asymptotically Stable เมื่อกำหนดให้ $\alpha_1 = 0.5, \alpha_2 = 0.01$ และ $\alpha_3 = 0.5$ จะได้ $R_0 = 1.106870229 > 1$ ผลการวิจัยแสดงให้เห็นว่าอิทธิพลกลุ่มเพื่อนของนักดื่มหน้าใหม่มีผลต่อตัวแบบคณิตศาสตร์การระบาดของโรคติดสุรา ดังนั้นควรลดอัตราการมีปฏิสัมพันธ์กับเพื่อนกลุ่มที่เป็นนักดื่มเพื่อทำให้การระบาดของโรคลดลง

คำสำคัญ : ตัวแบบคณิตศาสตร์, การระบาดของโรคติดสุรา, จุดสมดุล, ค่าระดับการติดเชื้อ

Abstract

The objective of this research is to develop and analyze a mathematical model of the alcohol addiction epidemic among new drinkers using standard methods. In this study, the population is divided into five compartments: susceptible drinkers (S), new drinkers (D), heavy drinkers (H), drinkers in treatment (T), and recovered drinkers (R). The formulated model is in the form of a system of nonlinear ordinary differential equations and is analyzed using standard methods. We obtained the equilibrium points and calculated the basic reproduction number (R_0) using the next-generation matrix method and spectral radius. The stability of the equilibrium points is analyzed using the Routh-Hurwitz criteria, and numerical solutions are provided to support the results. It was found that the numerical solution of the system converges to the alcohol-free equilibrium point, $E_0(1.45, 0, 0, 0, 0)$, which is locally asymptotically stable when given $\alpha_1 = 0.01, \alpha_2 = 0.5$, and $\alpha_3 = 0.01$, yielding $R_0 = 0.08787878786 < 1$. Furthermore, the alcohol-present equilibrium point, $E_1(1.20339, 0.01511, 0.00548, 0.04876, 0.17732)$, is locally asymptotically stable when given $\alpha_1 = 0.5, \alpha_2 = 0.01$, and $\alpha_3 = 0.5$, yielding $R_0 = 1.106870229 > 1$. The research results demonstrate that the influence of new drinkers' peer groups affects the mathematical model of the alcoholism epidemic. Therefore, reducing the rate of interaction with peer groups of drinkers will help decrease the alcoholism epidemic.

Keywords: Mathematical model, alcoholism epidemic, equilibrium point, basic reproduction number

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Introduction

It is well known that alcohol consumption leads to various negative consequences, including a range of health issues such as liver diseases, heart and cardiovascular conditions, cancer, and mental health disorders (World Health Organization, 2018; Assanangkornchai & Chittrakan, 2019; Wichaidit et al., 2019). Furthermore, alcohol consumption contributes to road accidents when individuals consume alcohol and then drive. It also leads to conflicts, violence, and physical harm. Despite these risks, a considerable number of people in Thailand still favor alcohol consumption. According to the report on alcohol consumption in each province in 2021, the prevalence of alcohol consumption in the Thai population is 28.0%, which means nearly 3 out of 10 individuals aged 15 and above have consumed alcohol within the past 12 months. Specifically, among adolescents, the prevalence of alcohol consumption is 9.0% (Vichitkunakorn & Tanaree, 2022). Moreover, a significant number of new drinkers, representing 5.93% of the Thai population aged 15 and above, started drinking alcohol for the first time within the past 3 years prior to the survey in 2021 (Center for Alcohol Studies, 2023; National Statistical Office, 2021).

New drinkers are a population group that requires close attention in each survey round because they might transition into regular drinkers, heavy drinkers, alcohol-dependent individuals, or even quit drinking. If there is a significant increase in the number of new drinkers during a certain period without a corresponding decrease in previous drinkers, the prevalence of current drinkers will also rise. Based on the trend analysis of alcohol consumption behavior among the Thai population, the prevalence of new drinkers aged 15 and above has been increasing since 2011 until 2021. Specifically focusing on new drinkers among adolescents, it was found that the prevalence has notably increased in the 2021 survey (Chirapradispol et al., 2020; Vichitkunakorn & Tanaree, 2022). Moreover, the study on the initiation of alcohol consumption found that the main reasons for starting drinking were being influenced by friends or peer pressure (38.85%), curiosity (29.12%), and socialization (20.82%),

with other reasons accounting for 11.21% (Wichaidit et al., 2019). It is evident that the most influential factor is peer pressure (Noiphon, 2017; Hashim et al., 2017; Gutiérrez et al., 2022), especially among adolescents who tend to seek peer approval. Therefore, they often follow the drinking behavior of their peers. Given this context, addressing the issue of alcohol consumption among current youth and adolescents is essential as they are at high risk of alcohol-related problems and consequences. Consequently, strategies for controlling the expansion of alcohol consumption and minimizing economic and social losses, particularly among adolescents, are necessary (Hashim et al., 2017).

Mathematical modeling regarding alcohol consumption has been frequently conducted, employing the principles and approaches used in infectious and contagious diseases (Poonpanang et al., 2015; Sharma & Samanta, 2015; Khajji et al., 2020; Reyes, 2020). Alcohol consumption behavior is considered a “disease” that can spread through peer influence. The interconnection of consumption patterns typically follows a sequence of transfer: current drinkers, at-risk drinkers, harmful drinkers, and recovering drinkers, with the possibility of transitioning back or being removed depending on interventions (Gutiérrez et al., 2022). For instance, Sharma and Samanta (2015) developed a mathematical model of alcohol dependence by categorizing the student population into four classes: moderate and occasional drinkers, heavy drinkers, treatment drinkers, and recovered drinkers. The researchers calculated the basic reproduction number (R_0) and examined the stability of equilibrium points. The study found that when $R_0 > 1$, the epidemic of alcohol consumption reaches an equilibrium point. Reyes (2020) used a mathematical model to analyze and explain the trend of alcohol consumption among university students in Colombia. The population was divided into three classes: regular drinkers, at-risk drinkers, and current alcohol-dependent individuals. The researchers found that when the transmission rate exceeds $R_0 > 1$, the epidemic of alcohol consumption reaches an equilibrium point.

In this study, we aim to examine the spread of alcohol dependence among new drinkers using a mathematical model. We analyze the causes and factors resulting from the influence of peers, which drive the formulation of control measures to monitor and prevent the spread of alcohol dependence.

Mathematical Model and Basic Properties

In this section we have developed a mathematical model of alcohol abuse.

1. Mathematical Model

This model is developed based on the framework proposed by Sharma and Samanta (2015), in which the population is divided into four compartments: moderate and occasional drinkers, heavy drinkers, treatment drinkers, and recovered drinkers. However, recent literature has emphasized the significant role of peer influence in the initiation of alcohol consumption, particularly among adolescents and young adults (Gutierrez et al., 2022; Chirapradispol et al., 2020).

Accordingly, the present model introduces an additional compartment representing new drinkers, comprising individuals who have downgraded from the heavy drinker group by reducing their alcohol intake. This behavioral transition is governed by the parameter α_2 , which denotes the proportion of heavy drinkers who shift to new drinkers without undergoing formal treatment. At the same time, peer influence from existing heavy drinkers is modeled through the parameter α_1 , representing the rate at which susceptible individuals are encouraged to initiate alcohol consumption and enter the heavy drinker class. Importantly, individuals in the new drinker class are not yet considered fully recovered and remain vulnerable to relapse. This risk is captured by the parameter α_3 , which quantifies the transition rate from new drinkers back to heavy drinking due to social, psychological, or environmental triggers.

As a result, the human population in this model is divided into five distinct classes: susceptible drinkers (S), heavy drinkers (H), new drinkers (D), treatment drinkers (T) and recovered drinkers (R). The transitions

among these groups, driven by peer influence, behavioral downgrading, and relapse dynamics, are illustrated in Figure 1.

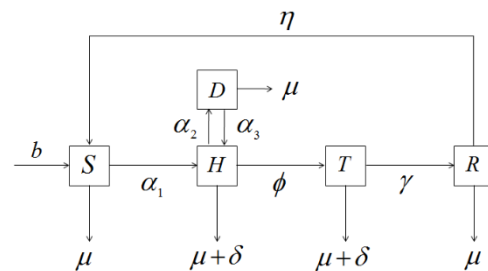


Figure 1 The schematic diagram of the alcohol abuse model

The model can be presented by the following set of ordinary differential equations:

$$\begin{aligned}
 \frac{dS}{dt} &= b + \eta R - \alpha_1 SH - \mu S \\
 \frac{dH}{dt} &= \alpha_1 SH + \alpha_3 DH - \alpha_2 H - (\phi + \mu + \delta)H \\
 \frac{dD}{dt} &= \alpha_2 H - \alpha_3 DH - \mu D \\
 \frac{dT}{dt} &= \phi H - (\gamma + \mu + \delta)T \\
 \frac{dR}{dt} &= \gamma T - (\eta + \mu)R
 \end{aligned} \tag{1}$$

with initial densities

$$S(0) > 0, H(0) > 0, D(0) > 0, T(0) > 0, R(0) > 0.$$

$$\text{So } N(t) = S(t) + H(t) + D(t) + T(t) + R(t).$$

denotes the total number of high-risk human population at time t .

The model parameters are described below:

- b : natural birth rate of the human population
- μ : Natural death rate of each population class
- δ : Drinking related death rate of heavy drinkers
- η : The proportion of recovered class who re-enter into susceptible drinkers
- α_1 : The transmission coefficient from susceptible drinkers to heavy drinkers
- α_2 : The proportion of the heavy drinker class who re-enter into new drinkers
- α_3 : The transmission coefficient from new drinkers to heavy drinkers

ϕ : The proportion of heavy drinkers who enter new in treatment

γ : Recovery rate of drinkers in treatment

The model is based on the following assumptions :

(i) The natural birth rate and death rate of the human population are assumed to be constant, denoted by b and μ , respectively.

(ii) Individuals who cease alcohol consumption enter the recovered drinkers class. A fraction of these individuals may relapse into the susceptible class due to waning immunity, occurring at a rate denoted by η (immune loss rate).

From system (1), by adding all equation, we get

$$\frac{dN}{dt} = b - \mu N - \delta H - \delta T \quad \text{as } t \rightarrow +\infty. \text{ So, the total}$$

population is $N = \frac{b - \delta(H + T)}{\mu}$, which is not constant. Let us scale the state variables with respect to the total population. Then system (1) becomes :

$$\begin{aligned} \frac{dN}{dt} &= b - \mu N - \delta H - \delta T \\ \frac{dH}{dt} &= \alpha_1 (N - H - D - T - R)H + \alpha_3 DH - \alpha_2 H - (\phi + \mu + \delta)H \\ \frac{dD}{dt} &= \alpha_2 H - \alpha_3 DH - \mu D \end{aligned} \quad (2)$$

$$\frac{dT}{dt} = \phi H - (\gamma + \mu + \delta)T$$

$$\frac{dR}{dt} = \gamma T - (\eta + \mu)R$$

where $S = N - H - D - T - R$.

2. Basic Properties

It is assumed by the model that all the state variables are non-negative at time $t = 0$. It then follows from the differential equations of system (1) that the variables are non-negative for all $t \geq 0$. That result can be summarized in the theorem 1.

Theorem 1 Let (S, H, D, T, R) be the solution of the system (1) with initial conditions $S(0) > 0$, $H(0) > 0$, $D(0) > 0$, $T(0) > 0$ and $R(0) > 0$.

$$\text{Then } \Omega = \left\{ \begin{array}{l} S(t), H(t), D(t), T(t) \in \mathbb{R}_+^5, \\ S(t) + H(t) + D(t) + T(t) \leq \frac{b}{\mu} \end{array} \right\} \text{ is a positively}$$

invariant and attracting under the flow described by the system (1)

Proof By the Lyapunov function (Bacciotti & Rosier, 2001) Let $S(t), H(t), D(t), T(t)$ and $R(t)$ are non-negative with $N = S + H + D + T + R$.

The time derivative of this equation is given by:

$$\begin{aligned} \frac{dN}{dt} &= b - \mu N - \delta H - \delta T \\ &\leq b - \mu N. \end{aligned}$$

$$\text{Since } \frac{dN}{dt} + \mu N \leq b,$$

we obtained $0 \leq N(t) \leq \frac{b}{\mu} (1 - e^{-\mu t}) + N(0)e^{-\mu t}$. where $N(0)$ represents the initial values of the total population.

$$\text{Then } \lim_{t \rightarrow \infty} \text{Sup } N(t) = \frac{b}{\mu}.$$

Thus Ω is positively - invariant and attracting under $[0, \infty)$.

Results

1. Basic Reproduction Number

In the following, we will find the basic reproduction number (R_0) of system (2) with the next generation matrix method formulation (Driessche & Watmough, 2002).

Let $x = (N, H, D, T, R)$, then the system (2) can be written as

$$\frac{dx}{dt} = F(x) - V(x)$$

$$= \begin{bmatrix} 0 \\ \alpha_1 (N - H - D - T - R)H + \alpha_3 DH \\ 0 \\ 0 \\ 0 \end{bmatrix} -$$

$$\begin{bmatrix} \mu N + \delta H + \delta T - b \\ \alpha_2 H + (\phi + \mu + \delta)H \\ -\alpha_2 H + \alpha_3 D H + \mu D \\ (\gamma + \mu + \delta)T - \phi H \\ (\eta + \mu)R - \gamma T \end{bmatrix}$$

where $F(x)$ is the new infection terms, $V(x)$ is the other terms. The corresponding linearized matrices of $F(x)$ and $V(x)$ evaluated at disease-free equilibrium

$$E_0(N, H, D, T, R) = \left(\frac{b}{\mu}, 0, 0, 0, 0 \right) \text{ are respectively.}$$

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \alpha_1 H & \alpha_1 \begin{pmatrix} N- \\ H- \\ D- \\ T- \\ R \end{pmatrix} + \begin{pmatrix} -\alpha_1 H + \\ \alpha_3 H \end{pmatrix} & -\alpha_1 H & -\alpha_1 H & -\alpha_1 H \\ \alpha_3 D & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$V = \begin{bmatrix} \mu & \delta & 0 & \delta & 0 \\ 0 & \alpha_2 + \phi + \mu + \delta & 0 & 0 & 0 \\ 0 & -\alpha_2 + \alpha_3 D & \alpha_3 H + \mu & 0 & 0 \\ 0 & -\phi & 0 & \gamma + \mu + \delta & 0 \\ 0 & 0 & 0 & -\gamma & \eta + \mu \end{bmatrix}$$

Now,

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha_1 b}{\begin{pmatrix} \alpha_2 + \\ \phi + \\ \mu + \\ \delta \end{pmatrix} \mu} & \frac{-\alpha_1 \alpha_2 b}{\begin{pmatrix} \alpha_2 + \\ \phi + \\ \mu + \\ \delta \end{pmatrix} \mu^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is the next generation matrix of system (2) and basic reproduction number of the system will be the spectral radius of matrix FV^{-1} denoted and defined as :

$$R_0 = \rho(FV^{-1}) = \frac{\alpha_1 b}{\mu(\alpha_2 + \phi + \mu + \delta)} > 0$$

2. Equilibrium Points

In this section, we will study the existence of equilibrium points of system (2). It has mainly two equilibrium points:

1) Alcohol- Free Equilibrium (AFE) :

$$E_0(N, H, D, T, R) = \left(\frac{b}{\mu}, 0, 0, 0, 0 \right)$$

2) Alcohol- Present Equilibrium (APE) :

$$E_1(N^*, H^*, D^*, T^*, R^*) = \left(\frac{b + \delta H^* (1 - a_1)}{\mu}, H^*, \frac{\alpha_2 H^*}{\alpha_3 H^* + \mu}, a_1 H^*, \frac{\gamma a_1 H^*}{a_2} \right)$$

where $a_1 = \frac{\phi}{\gamma + \mu + \delta}$ and $a_2 = \eta + \mu$

Theorem 2 System (2) has an alcohol - free

equilibrium $E_0\left(\frac{b}{\mu}, 0, 0, 0, 0\right)$ and if $R_0 > 1$, then it admits an alcohol-present equilibrium $E_1(N^*, H^*, D^*, T^*, R^*)$.

Proof Let $E_1(N^*, H^*, D^*, T^*, R^*)$ is alcohol-present equilibrium. At equilibrium, all derivatives in System (2) are equal to zero, and the system satisfies the following set of algebraic equations :

$$b - \mu N^* - \delta H^* - \delta T^* = 0 \quad (2a)$$

$$\alpha_1(N^* - H^* - D^* - T^* - R^*)H^* + \alpha_3 D^* H^* -$$

$$\alpha_2 H^* - (\phi + \mu + \delta)H^* = 0 \quad (2b)$$

$$\alpha_2 H^* - \alpha_3 D^* H^* - \mu D^* = 0 \quad (2c)$$

$$\phi H^* - (\gamma + \mu + \delta)T^* = 0 \quad (2d)$$

$$\gamma T^* - (\eta + \mu)R^* = 0 \quad (2e)$$

From (2c) and (2d), we obtain

$$D^* = \frac{\alpha_2 H^*}{\alpha_3 H^* + \mu} \quad (3)$$

$$T^* = \frac{\phi H^*}{\gamma + \mu + \delta} = a_1 H^* \quad (4)$$

Substituting (4) into (2a) and (2e) yields

$$N^* = \frac{b - \delta H^* (1 + a_1)}{\mu} \quad (5)$$

$$R^* = \frac{\gamma a_1 H^*}{\eta + \mu} = \frac{\gamma a_1 H^*}{a_2} \quad (6)$$

Then, substituting (3) – (6) into (2b), we have a cubic equation of H^* as follows:

$$a_4 H^{*3} + a_5 H^{*2} + a_6 H^* = 0 \quad (7)$$

where

$$a_1 = \frac{\phi}{\gamma + \mu + \delta},$$

$$a_2 = \eta + \mu,$$

$$a_3 = \phi + \mu + \delta,$$

$$a_4 = \alpha_1 \alpha_3 a_2 \delta - a_1 a_2 \alpha_1 \alpha_3 \delta - a_1 a_2 \mu \alpha_1 \alpha_3 - a_1 \gamma \mu \alpha_1 \alpha_3,$$

$$a_5 = \alpha_1 \alpha_3 a_2 b - a_2 \alpha_1 \delta \mu - a_1 a_2 \alpha_1 \delta \mu - a_2 \alpha_1 \alpha_2 \mu - a_1 a_2 \alpha_1 \mu^2 - \gamma a_1 \alpha_1 \mu^2 - a_2 a_3 \alpha_3 \mu,$$

$$a_6 = b \mu a_2 \alpha_1 - a_2 \alpha_2 \mu^2 - a_2 a_3 \mu^2.$$

Obviously, $a_4 < 0$ and $a_6 < 0$ holds naturally when $R_0 > 1$. Hence (7) has a positive solution, we can get

positive solutions N^*, H^*, D^*, T^* and R^* , respectively.

So, System (2) has an alcohol-present equilibrium

$$E_1(N^*, H^*, D^*, T^*, R^*) \text{ if } R_0 > 1.$$

3. Stability Analysis

In this section, we analyze the local stability of system (2) at the alcohol-free equilibrium and the alcohol-present equilibrium.

The Jacobian matrix of this system is

$$J = \begin{bmatrix} -\mu & -\delta & 0 & -\delta & 0 \\ \alpha_1 H & c_1 & c_2 & -\alpha_1 H & -\alpha_1 H \\ 0 & c_3 & c_4 & 0 & 0 \\ 0 & \phi & 0 & -(\gamma + \mu + \delta) & 0 \\ 0 & 0 & 0 & \gamma & -(\eta + \mu) \end{bmatrix} \quad (8)$$

where $c_1 = \alpha_1(N - H - D - T - R) - \alpha_2 + \alpha_3 D - (\phi + \mu + \delta)$,

$c_2 = -\alpha_1 H + \alpha_3 H$, $c_3 = \alpha_2 - \alpha_3 D$ and $c_4 = -\alpha_3 H - \mu$.

3.1 Local Stability of E_0

Theorem 3 The alcohol-free equilibrium E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof From (8), the Jacobian matrix corresponding to E_0 is given by

$$J(E_0) = \begin{bmatrix} -\mu & -\delta & 0 & -\delta & 0 \\ 0 & C & 0 & 0 & 0 \\ 0 & \alpha_2 & -\mu & 0 & 0 \\ 0 & \phi & 0 & -(\gamma + \mu + \delta) & 0 \\ 0 & 0 & 0 & \gamma & -(\eta + \mu) \end{bmatrix} \quad (9)$$

where $C = \alpha_1 \left(\frac{b}{\mu} \right) - \alpha_2 - (\phi + \mu + \delta)$.

So, eigenvalue of the characteristic equation of $J(E_0)$ are $\lambda_{1,2} = -\mu, \lambda_3 = C, \lambda_4 = -(\gamma + \mu + \delta)$ and $\lambda_5 = -(\eta + \mu)$. It is evident that $\lambda_1, \lambda_2, \lambda_4$ and λ_5 are real and negative due to the positivity of all parameters. Now, E_0 is stable

if $\lambda_3 < 0$, $\alpha_1 \left(\frac{b}{\mu} \right) < \alpha_2 + \phi + \mu + \delta$, $R_0 < 1$. Therefore,

the system (2) is locally asymptotically stable at E_0 if $R_0 < 1$ and unstable if $R_0 > 1$.

3.2 Local Stability of E_1

Theorem 4 If $R_0 > 1$, the alcohol-present equilibrium E_1 is locally asymptotically stable.

Proof From (8), the Jacobian matrix corresponding to E_1 is given by

$$J(E_1) = \begin{bmatrix} D_1 & D_2 & 0 & D_2 & 0 \\ D_3 & D_4 & D_5 & D_6 & D_6 \\ 0 & D_7 & D_8 & 0 & 0 \\ 0 & D_9 & 0 & D_{10} & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} \end{bmatrix} \quad (10)$$

where $D_1 = -\mu$, $D_2 = -\delta$, $D_3 = \alpha_1 H^*$,

$$D_4 = \alpha_1 \left(\frac{b + \delta H^* (1 - a_1)}{\mu} - \frac{\alpha_2 H^*}{\alpha_3 H^* + \mu} - \frac{a_1 H^* - \frac{\gamma a_1 H^*}{a_2}}{\alpha_2} \right),$$

$$\alpha_2 + \frac{\alpha_2 H^*}{H^* + \mu} - (\phi + \mu + \delta)$$

$D_5 = -\alpha_1 H^* + \alpha_3 H^*$, $D_6 = -\alpha_1 H^*$, $D_7 = \alpha_2 - \frac{\alpha_2 H^*}{H^* + \mu}$,
 $D_8 = -\alpha_3 H^* - \mu$, $D_9 = \phi$, $D_{10} = -(\gamma + \mu + \delta)$, $D_{11} = \gamma$
 and $D_{12} = -(\eta + \mu)$.

The characteristic equation of system (2) at E_1 is $\det(\lambda I_5 - J_1) = 0$, where I_5 is the identity matrix of size 5 and λ is the eigenvalue of matrix J_1 .

The eigenvalue of matrix J_1 is given by

$$\lambda^5 + H_1 \lambda^4 + H_2 \lambda^3 + H_3 \lambda^2 + H_4 \lambda + H_5 = 0 \quad (11) \text{ where}$$

$$H_1 = D_1 + D_4 + D_8 + D_{10} + D_{12},$$

$$H_2 = (D_5 D_7 + D_6 D_9 + D_2 D_3) -$$

$$\left(D_1 D_4 + D_1 D_8 + D_1 D_{10} + D_1 D_{12} + D_4 D_8 + D_8 D_{12} + D_4 D_{12} + D_{10} D_{12} + D_8 D_{10} + D_4 D_{10} \right),$$

$$H_3 = \left(D_1 D_4 D_8 + D_1 D_8 D_{12} + D_1 D_4 D_{12} + D_1 D_{10} D_{12} + D_1 D_8 D_{10} + D_1 D_4 D_{10} + D_4 D_8 D_{12} + D_4 D_8 D_{10} + D_8 D_{10} D_{12} + D_4 D_{10} D_{12} + D_6 D_9 D_{11} + D_2 D_3 D_8 + D_2 D_3 D_{10} + D_2 D_3 D_{12} \right) - \left(D_1 D_5 D_7 + D_1 D_6 D_9 + D_5 D_7 D_{12} + D_5 D_7 D_{10} + D_6 D_8 D_9 + D_6 D_9 D_{12} \right),$$

$$H_4 = \left(D_1 D_5 D_7 D_{12} + D_1 D_5 D_7 D_{10} + D_1 D_6 D_8 D_9 + D_1 D_6 D_9 D_{12} + D_2 D_3 D_8 D_{10} + D_2 D_3 D_8 D_{12} + D_2 D_3 D_{10} D_{12} + D_5 D_7 D_{10} D_{12} + D_6 D_8 D_9 D_{12} \right) - \left(D_1 D_4 D_8 D_{12} + D_1 D_4 D_8 D_{10} + D_1 D_8 D_{10} D_{12} + D_1 D_4 D_{10} D_{12} + D_1 D_6 D_9 D_{11} + D_4 D_8 D_{10} D_{12} + D_6 D_8 D_9 D_{11} \right),$$

$$H_5 = D_1 D_4 D_8 D_{10} D_{12} - D_1 D_5 D_7 D_{10} D_{12} + D_1 D_6 D_8 D_9 D_{11} - D_1 D_6 D_8 D_9 D_{12} - D_2 D_3 D_8 D_{10} D_{12}$$

Thus, by Routh-Hurwitz Criterion (Kot, 2001), we obtain that all the eigenvalues of the characteristic Equation (11) have negative real parts if and only if the conditions:

- (i) $H_i > 0$ for $i = 1, 2, 3, 4, 5$,
- (ii) $H_1 H_2 - H_3 > 0$,
- (iii) $H_1 H_2 H_3 - H_1 H_5 - H_1^2 H_4 - H_3^2 > 0$,

$$(iv) \left(\frac{H_1 H_2 H_3 H_4 - H_1 H_2 H_5 - H_1^2 H_4^2 - H_3^2 H_4 - H_2 H_5 - H_1 H_4 H_5}{H_3^2 H_4 - H_2 H_5 - H_1 H_4 H_5} \right) > 0.$$

Hence, the alcohol-present equilibrium E_1 of system (2) is local asymptotically stable if $R_0 > 1$.

4. Numerical Simulation

In this section, we present some numerical results of system (2), which are presented to support the analytic results obtained above. We have taken these based on the range provided in Table 1 and the different initial values for each variable of state.

Table 1 Values of the parameters used in the model.

Parameter	Values (Person/Day)	Reference
b	0.029	Bhunu and Mushayabasa (2012)
μ	0.02	
δ	0.035	
η	0.5	
ϕ	0.1	
γ	0.005	

First, we choose $\alpha_1 = 0.01$, $\alpha_2 = 0.5$ and $\alpha_3 = 0.01$, numerical simulation gives $R_0 = 0.08787878786 < 1$, then alcohol-free equilibrium $E_0(N, H, D, T, R) = E_0(1.45, 0, 0, 0, 0)$ is local asymptotically stable (Figure 2).

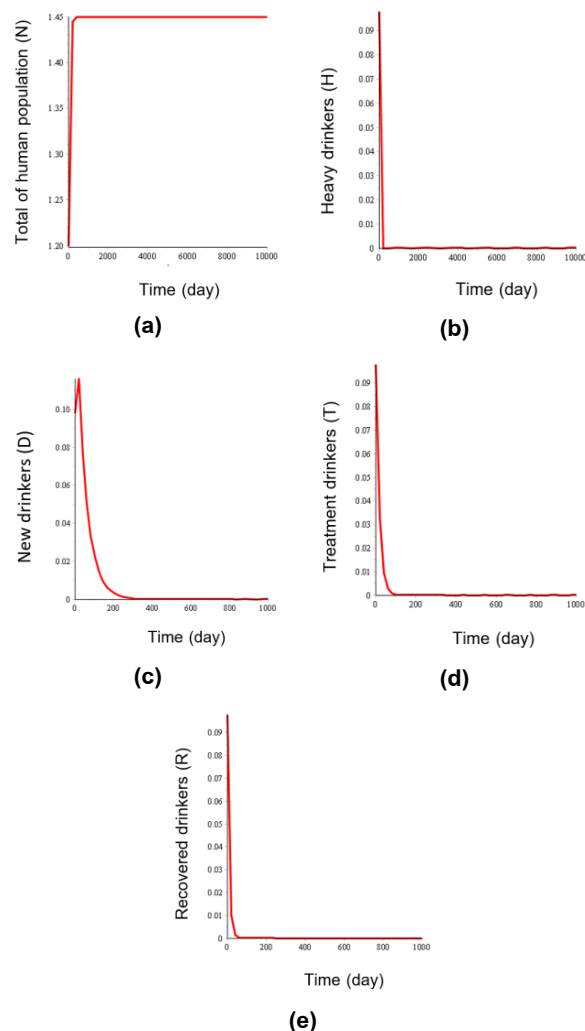


Figure 2 Time series of the (a) total of human population, (b) Heavy drinkers, (c) New drinkers, (d) Treatment drinkers and (e) Recovered drinkers. The state variable approach alcohol-free equilibrium.

From Figure 2(a)–2(e), the simulation results illustrate the temporal evolution of all population compartments under a specific parameter setting: the transmission rate from susceptible to heavy drinkers (α_1) is reduced to 1%, the proportion of heavy drinkers who transition to new drinkers (α_2) is increased to 50%, and the relapse rate from new drinkers back to heavy drinkers (α_3) is also reduced to 1%. Under these conditions, the system gradually evolves toward the alcohol-free equilibrium. In Figure 2(a), the total human population $N(t)$ increases due to natural demographic growth and stabilizes at approximately 1.45 million people after around 2,000 days (~5.5 years). Figures 2(b)–2(e) show that the subpopulations of drinkers including heavy drinkers (H), new drinkers (D), treatment drinkers (T) and recovered drinkers (R) gradually decline and converge to zero at approximately days 300, 500, 600 and 700, respectively. These results suggest that targeted interventions focusing on reducing peer influence and minimizing relapse can effectively eliminate alcohol use within a relatively short period, approximately two years highlighting the importance of early prevention policies and sustained rehabilitation efforts.

Second, we choose $\alpha_1 = 0.5$, $\alpha_2 = 0.01$ and $\alpha_3 = 0.5$, numerical simulation gives $R_0 = 1.106870229 > 1$, then alcohol presents an equilibrium

$$E_1(N^*, H^*, D^*, T^*, R^*) = E_1(1.20339, 0.01511, 0.00548, 0.04876, 0.17732)$$

is local asymptotically stable (Figure 3).

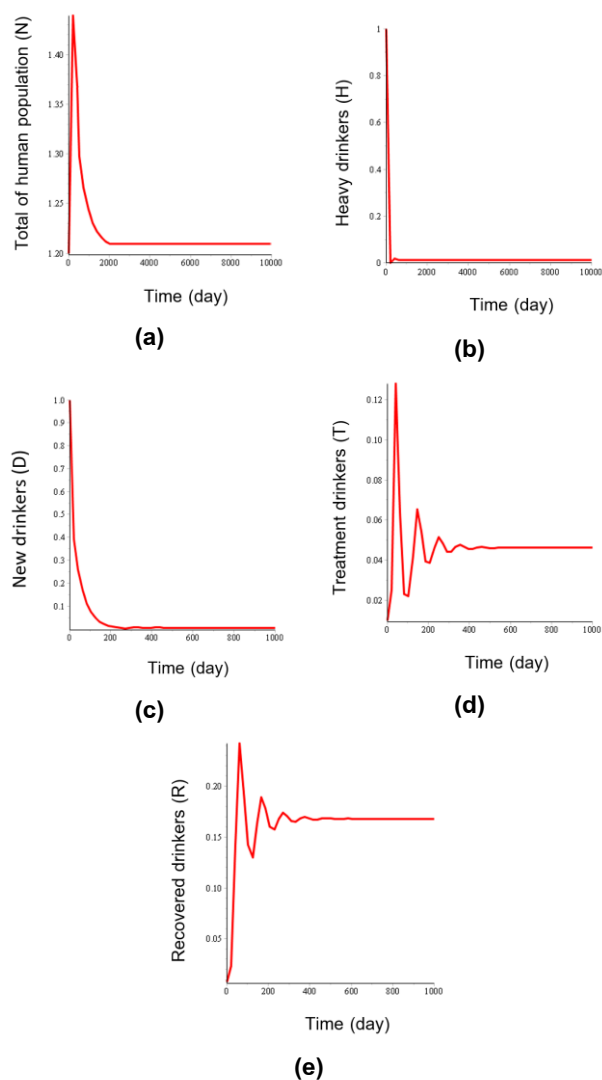


Figure 3 Time series of the (a) total of human population, (b) Heavy drinkers, (c) New drinkers, (d) Treatment drinkers and (e) Recovered drinkers. The state variables evolve toward the alcohol-present equilibrium.

Figure 3(a)–3(e) illustrates the population dynamics under an alcohol-present scenario. In this simulation, the transmission rate from susceptible individuals to heavy drinkers (α_1) is increased to 50%, the proportion of heavy drinkers who transition back into new drinkers (α_2) is reduced to 1%, and the relapse rate from new drinkers to heavy drinkers (α_3) is increased to 50%, representing a high-risk environment with frequent relapse, weak prevention, and sustained heavy drinking behavior. Figure 3(a) shows that the total human population $N(t)$ initially increases to about 1.45 million,

then declines and stabilizes around 1.20339 million after approximately 2,000 days (5.5 years), reflecting the cumulative effects of alcohol-related morbidity and mortality. Figures 3(b)–3(e) indicate that alcohol-related subpopulations gradually reach equilibrium: heavy drinkers (H) at 0.01511 (15,110 people), new drinkers (D) stabilize at 0.00548 (5,480 people), treatment group (T) at 0.04876 (48,760 people, and recovered individuals (R) at 0.17732 (117,320 people), assuming a total population of 1.45 million. These outcomes indicate that all drinking-related groups persist in the long term, suggesting that the system settles into an alcohol-present equilibrium mirroring real world contexts where insufficient intervention, frequent relapse, and lack of support systems lead to continued alcohol use. Targeted strategies addressing peer influence and relapse prevention are essential to shift the system away from this endemic state.

Discussion

This study developed a mathematical model to examine peer influence on alcohol addiction by introducing a new compartment for new drinkers (D). The analysis shows that the alcohol-free equilibrium is locally stable when $R_0 < 1$, but increases in peer influence (α_1) or relapse rate (α_3) may shift the system toward an alcohol-present equilibrium. Additionally, the model accounts for the downgrading of heavy drinkers to new drinkers (α_2), a behavioral transition that can delay or reverse addiction progression. Unlike earlier models such as Sharma and Samanta (2015), our model incorporates social dynamics, aligning with studies like Gutierrez et al. (2022) and Chirapradispol et al. (2020), which emphasized the critical role of peer and family influence in alcohol initiation among adolescents.

Key parameters such as α_1 (peer influence), α_2 (downgrading) and α_3 (relapse) reflect real-world social triggers. These findings align with Hashim et al. (2017), who identified peer exposure as a major predictor of youth alcohol use in Thailand. Our simulations indicate that while recovery mechanisms (ϕ, γ, η) are relevant, peer-driven initiation (α_1) becomes the dominant driver when

$R_0 > 1$. Under these conditions, the alcohol-present equilibrium is locally stable, suggesting that sustained peer influence and frequent relapse can maintain endemic levels of alcohol use. This supports findings by Reyes (2020), who demonstrated that peer dynamics reinforce long-term alcohol prevalence in university populations. Although the model assumes homogeneous mixing and constant rates, these assumptions can be relaxed in future work. Following Wichaidit et al. (2019) and National Statistical Office (2021), incorporating age structures, geographic factors, and behavioral seasonality would enhance realism. Policy implications include reducing α_1 through peer-led education, minimizing α_3 via long-term relapse prevention programs and enhancing α_2 by encouraging early-stage interventions that transition heavy drinkers to less severe stages. These strategies align with WHO (2018) recommendations. Future directions include adding a moderate drinker class, conducting sensitivity analysis, and validating the model with Thai national data. Most importantly, these results must be understood within the Thai socio-cultural context, where alcohol is embedded in festivals, family life, and peer activities, in order to guide effective and culturally sensitive interventions.

Conclusion

This study developed a compartmental mathematical model to examine the dynamics of alcohol consumption by incorporating peer influence (α_1), the downgrading of heavy drinkers to new drinkers (α_2) and relapse (α_3). Formulated a mathematical model by using the Routh-Hurwitz criteria and constructing Lyapunov functions. The model analysis revealed two non-negative equilibria: an alcohol-free equilibrium and an alcohol-present equilibrium. Numerical simulations showed that when $\alpha_1 = 0.01$, $\alpha_2 = 0.5$ and $\alpha_3 = 0.01$, the basic reproduction number $R_0 = 0.08787878786 < 1$, indicating that the alcohol-free equilibrium is locally asymptotically stable. In contrast, when $\alpha_1 = 0.5$, $\alpha_2 = 0.01$ and $\alpha_3 = 0.5$, the value $R_0 = 1.106870229 > 1$, confirming the local stability of the alcohol-present equilibrium. These findings confirm

that peer-driven initiation and relapse significantly influence the persistence of alcohol use in the population. The model underscores the critical role of minimizing peer influence and relapse in reducing alcohol prevalence. Achieving this objective necessitates the implementation of comprehensive public health strategies, including evidence-based educational campaigns, behavior modification interventions, and sustained relapse prevention programs. Such measures are essential to shift the population dynamics toward a stable alcohol-free equilibrium.

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