

Flexible Printed Circuit Process Improvement via Interchangeable Linear Constrained Response Surface Optimisation Models

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Abstract

This paper presents a collection of experimental design and mathematical programming techniques for quality improvement in automotive electronic parts. The quality performance of interest is measured via the relationship of the etched rate of acid solution and circuit width, one of the key failure and break down to LED of lighting vehicles. With lower levels from monitoring the product quality the manufacturer has spent a lot of cost and time for product verification procedures. This brings the production with higher levels of waste and lead time. To validate on processing and to sustain finished goods with the permanent prevention, the precisely etched condition should be optimised. The proper factorial experiments, multiple regression and mathematical programming approaches are applied to investigate the preferable levels of significant process variables in order to improve the quality of etched rate. The interchangeable constrained response surface optimisation models provide the new operating conditions. The experimental results in each part with less than twenty five lines showed that the first model decreases the bottom circuit width deviation from 0.0026 to 0.0024 and the latter model decreases the etching rate from 2.033 to 1.124.

Keywords : Flexible Printed Circuit Process; Circuit Width; Etched Rate; Response Surface Methodology; Multiple Regression; Steepest Descent

Introduction

In the field of an electronic circuitry, the flexible printed circuits (FPC) have been developed for lighting automotive vehicles by assembling with the LED. The emission light and optical properties are mainly relied on the width of an FPC circuit line. An existing process to confirm the correct width of a lead line in an electronic field is a damaged part investigation. The process obviously

causes the high quality cost in FPC manufacturers.

Currently, the circuit width of the FPC is with lower process capability (Cpk) at -3.03 on the top circuit width and 0.85 on the bottom circuit width that comparing to the minimal target at 1.33 as shown in Figure 1. In this case, the deep details of an etching process should be investigated so that the optimal working condition would be determined as a standard process.

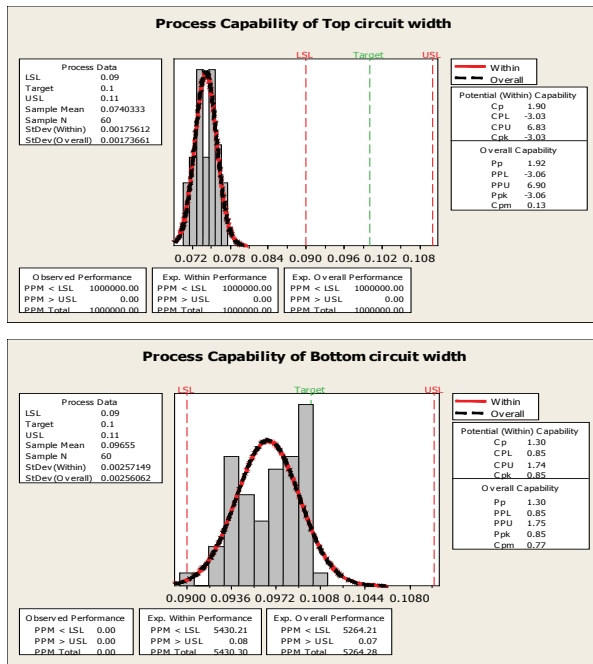


Figure 1 Current performance on top and bottom circuit widths

Process Review

Characteristics of the FPC circuit width based on a crossed section image view as shown in Figure 2 have composed with the top (T) and bottom (B) circuit lines. These are the varieties on the horizontal etching. The principles of the upward acid spray and the use of additives to reduce the etching ability are necessary for successful implementations (Coombs, 1988).

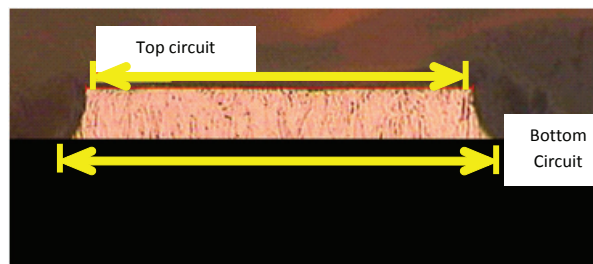


Figure 2 Crossed section view of the circuit width

Methodology

Multiple Regression Analysis

An aim of the simple regression analysis is to adjust the values of slope and intercept to find the expected line that best predicts the dependent variable or response of Y from the independent variable or factor of X. More precisely, the goal of regression is to minimise the sum of the squares of the vertical distances of the design points from the expected line. The slope quantifies the steepness of the expected line. It equals the change in Y for each unit change in X. It is expressed in the units of the Y-axis divided by the units of the X-axis. If the slope is positive, Y increases as X increases. In contrast, if the slope is negative, Y decreases as X increases (Luangpaiboon and Peeraprawit, 2009). In statistics, the most commonly used mathematical formulas for expressing linear relationships among more than two variables are equations of the following form (Luangpaiboon et al., 2010),

$$y = b_0 + b_1x_1 + b_2x_2 + \dots + b_kx_k. \quad (1)$$

In the multivariate case, when there are more than one independent variable, the regression line cannot be visualised in the two dimensional space, but can be computed just as easily. Multiple regression models for k independent variables are usually fitted by using the method of least squares. The least-squares method, published by Legendre and Gauss, minimises the variance of the unbiased estimators of the coefficients. Multiple regression analysis played an important role in the development of regression analysis, with a greater emphasis on issues of design and inference. An aim of multiple regression analysis is again to formulate a model of influential variables (or vector of influential variables) of x's. In the multiple linear regression line, the following model is used,

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \varepsilon. \quad (2)$$

An unobserved random error of ε is with the mean of zero on scalar influential variables of x 's. In this model, for each unit increase in the value of x , the conditional expectation of y increases by β_i units of x_i . Conveniently, these models are all linear from the point of view of estimation, since the regression model is linear in terms of the unknown parameters of $\beta_0, \beta_1, \dots, \beta_k$. Therefore, for least squares analysis, the computational and inferential problems of multiple regressions can be completely addressed using the multiple regression techniques. This is done by treating $x_1, x_2, \dots, x_k$ as being distinct independent variables in a multiple regression model. More details are referred to in many statistical textbooks.

Response Surface Methodology (RSM)

The objective of the RSM is to describe how the response of a process varies with change in k process variable as shown in Figure 3. The process variable determined will depend on the specific problem of the applications (Luangpaiboon, 2010). The RSM is the combination of mathematical and statistical aspects to improve the response. One of the most widely used in the area of Response Surface Methodology (RSM) is the steepest descent (descent) method that an aim is to minimise (minimise) the system of interest. However, various iterative procedures in the field of RSM are proposed to find the appropriate choices of process variables such as the modified simplex (MSM), super modified simplex (SMS), weighted centroid modified simplex (WCMSM), modified complex (MCM) and linear constrained response surface optimisation (LCRSOM) methods (Luangpaiboon, 2011).

On the theory and practice of RSM, it is assumed that the mean response (η) is related to values of the process variables ($\xi_1, \xi_2, \dots, \xi_k$) by an fitted unknown mathematical function f . The functional relationship between the mean response and k process variables can be written as $\eta = f(\xi)$, if ξ denotes a column vector with elements $\xi_1,$

$\xi_2, \dots, \xi_k$. Estimation of such surfaces, and hence identification of near optimal setting for process variables is an important practical issue with interesting theoretical aspects.

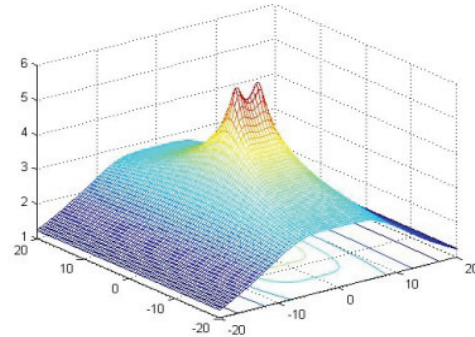


Figure 3 Response surface and its contour plot

The procedure begins with any types of designed experiments around the prevailing operating condition. A sequence of first order model and line searches are conventionally justified on the basis that such a plane would be fitted well as a local approximation to the true response. The estimated coefficients for the first order model are determined using the principle of least square. An algorithm for finding the nearest local minimum of a function which presupposes that the gradient of the function can be generated. The method of steepest descent, also called the gradient descent method, starts at a point P_0 and, as many times as needed, moves from P_i to P_{i+1} by minimising along the line extending from P_i in the direction of $-\nabla f(P_i)$ or the local downhill gradient. When applied to a 1-dimensional function of $f(x)$, the method takes the form of iterating from a starting point x_0 for some small $\varepsilon > 0$ until a fixed point is reached.

In contrast to this other algorithmic processes search the system approximation via the systematic searches or the measurement of the response in the design points. When curvature is detected, another factorial experiment is conducted. This is used either to estimate the position of the optimum or

the systematic searches to specify a new direction of steepest descent or the new design point with the better yields (Luangpaiboon, 2010). In this study, the interchangeable linear constrained response surface optimisation model (IC-LCRSOM) is deployed to set up a relationship of the linear constrained responses and influential process variables. Originally, linear programs are problems that can be expressed in canonical form:

$$\begin{aligned} &\text{Minimise } C^T X \\ &\text{Subject to } AX \geq B \\ &\text{And } X \geq 0 \end{aligned}$$

where X represents the vector of process variables (to be determined), C and B are vectors of (known) coefficients and A is a (known) matrix of coefficients of problem constraints. The expression to be maximised or minimised is called the objective function ($C^T X$ in this case). The constraints $AX \geq B$ specify a convex polytope over which the objective function is to be optimised.

In this problem, some of the expected regression equations of process responses are interchangeable. The problem is then called an interchangeable (IC) problem. Sequential procedures of IC-LCRSOM are able to switch upon the circumstances of interest. A factorial experiment design is used to investigate the optimal responses of process of interest. When the model is formulated, analysis of variance (ANOVA) is applied to find statistically significant process variables and determine the most effective levels. Regression analysis is used to fit a relationship equation of the response and its process responses. Interchangeable functions of process variables and various responses are considered as the objective and also the constraint of the LCRSOM. Those possible models are the representatives of the system. The optimal levels in each process variable from a mathematical programming model are determined via a generalised reduced gradient algorithm.

Experimental Results and Analyses

The responses of the system are measured as the top and bottom circuit widths. The lower and upper specifications of both circuit widths are shown in Table 1. There are three steps of experimental analyses which consist of a base line analysis, an etched rate analysis and a circuit width analysis.

Table 1 Responses and their feasible specifications

Response	Specification	
	Lower	Upper
Top Circuit Width	0.09	0.110
Bottom Circuit Width	0.09	0.110

Base Line Analysis

In this first step, the experiments aim to analyse the current data of the circuit width (R_{cw}) by using a completely randomised design or one-way analysis of variance (ANOVA). The experimental designs were performed to determine the statistically significant process conditions or the capability of measurement system which consist of the pattern and sheet positions. The process positions and feasible ranges are provided in Table 2.

Table 2 Process positions and their feasible ranges

Position	Level
Pattern	MT, Cen1, Cen2, OP
Sheet	S1 – S15

In this study, at 95% confidence interval sources of variance and P-value were shown in Table 3. On the numerical results, the significant factor on both circuit widths is the pattern position. The pattern position is then applied as the design factor for the next two steps throughout.

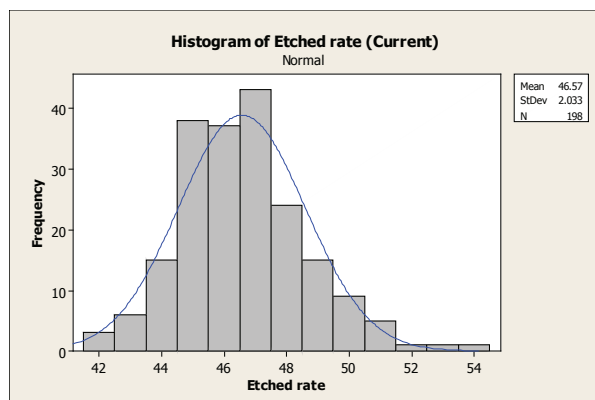
Table 3 ANOVA for base line analysis

Source or Position	P-Value	
	Top Circuit Width	Bottom Circuit Width
Pattern	0.00	0.00
Sheet	0.881	0.954

Etched Rate Analysis

According to the results from the base line analysis the circuit width is unbalanced so the response in the second step is the etched rate (R_{ER}). Currently the etched rate is with the deviation of 0.033 and the three sigma level of 6.1 as shown in Figure 4. A two level experimental design with additional two centre design points was performed to determine the statistically significant process variables of A, B and C (an attribute factor). The levels of process variables (A, B, C) on the centre design points are (45, 3.0, -1) and (45, 3.0, 1). Low and high levels including centre points are selected cover values of feasible ranges in a production line (Table 4).

The objective at this step is to analyse main and interaction effects via 20 experimental data. The analysis of variance revealed that the main effects of A and B are significant, but there was no statistically significant on the interaction effect at 95% confidence interval.

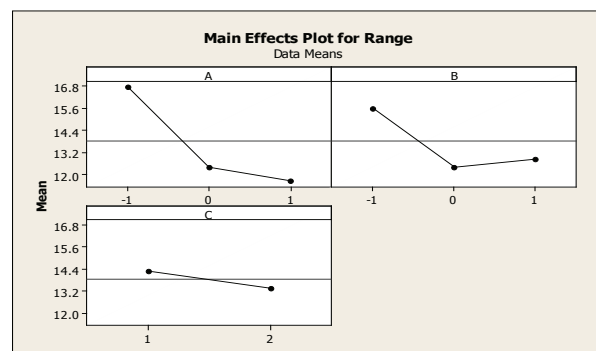
**Figure 4** Current etched rate performance measure**Table 4** Process variables and their feasible and tested levels for etched rate analysis

Process Variable	Feasible Level	Tested Levels	
		Low	High
A	30 – 60	30	60
B	2.0 – 4.0	2.9	3.1
C	Attribute	-1	1

Table 5 ANOVA with all main effects and interactions

Sources	P-Value for the Etched Rate
A	0.001
B	0.029
C	0.371
A*B	0.791
A*C	0.675
B*C	0.169
A*B*C	0.201
Centre Point	0.162

In order to determine the appropriate setting of the process variables, the main effects were plotted in Figure 5. The appropriate levels of process variables A and B are set at 60 and 3.1, respectively. After an implementation of the new operating condition, the response of the etched rate is improved with the deviation of 1.365 and the three sigma level of 4.1 (Figure 6).

**Figure 5** Main effect plots of the etched rate analysis

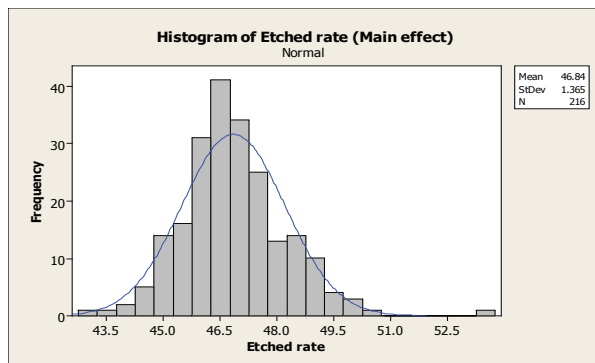


Figure 6 Etched rate performance measure at new process condition from the factorial design

The method of multiple regression analysis at 95% confidence interval is then applied for statistically significant process variables to determine the most preferable fitted equation of associated process variables of A and B to the response of the etched rate (Table 6). The relationship of the process variables and the response (R_{ER}) in terms of the path of steepest descent is

Expected Response of

$$R_{ER} = 63.1 - 0.171A - 13.9B. \quad (3)$$

Table 6 Regression model including its significant coefficients and ANOVA table

Predictor	Coef	SE Coef	T	P-Value	
Constant	63.14	17.31	3.65	0.002	
A	-0.1708	0.0382	-4.47	0.000	
B	-13.875	5.738	-2.42	0.027	
Source	DF	SS	MS	F	P-Value
Regression	2	135.86	67.932	12.90	0.000
Residual	17	89.557	5.268		
Total	19	225.42			

The preferable levels of process variables A and B from the path of steepest descent are 60 and 3.8, respectively (Table 5). When the new levels of process variables have been applied, the new

etched rate is improved with the deviation of 1.124 and the three sigma level of 3.4 (Figure 7).

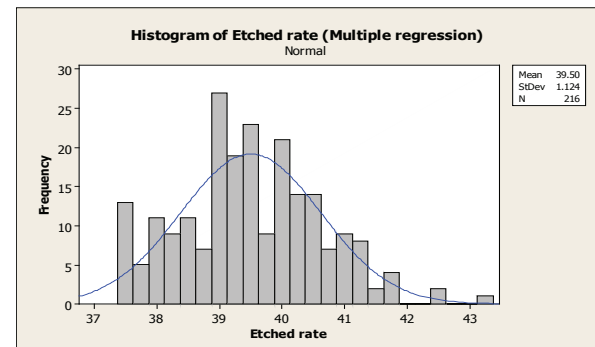


Figure 7 Etched rate performance measure at new process condition from the steepest descent

Table 7 A comparison of the etched rate among various settings

Setting	Deviation	3σ
Current	2.033	6.1
Factorial Design	1.365	4.1
Steepest Descent	1.124	3.4

Circuit Width Analysis

From the previous section, the pattern position brings the lower etched rate deviation when compared to the current operating condition. From the etched rate analysis, the process variable of A is then fixed at the suitable level of 60 and the remaining variable of B returns to be a process variable when focused on the response of the circuit width. The low and high levels of the process variables of B and D including centre points are selected cover values of feasible ranges in a production line to investigate the response of the circuit width (R_{tw}) (Table 8).

Table 8 Process variables and their feasible and tested levels for circuit width analysis

Process Variable	Feasible Level	Tested Level		
		Low	Center	High
B	2.0 – 4.0	2.9	3.1	3.3
D	3.0 – 4.0	3.4	3.5	3.6

The method of multiple regression analysis at 95% confidence interval is then applied for statistically significant process variables to determine the most preferable fitted equation of associated process variables of B and D to the response of the top and bottom circuit widths (Tables 9 and 10). The relationships of the process variables and the responses (R_{CW}) in terms of the paths of steepest descent are then determined.

Table 9 Regression model including its significant coefficients and ANOVA table for top circuit width.

Predictor	Coef	SE Coef	T	P-Value
Constant	0.05392	0.007415	7.27	0.018
B	-0.0162	0.000968	-16.78	0.004
D	0.00750	0.001936	3.87	0.061

Source	DF	SS	MS	F	P-Value
Regression	2	0.000045	22×10^{-6}	148.33	0.007
Residual	2	0.0000003	15×10^{-7}		
Total	4	0.000045			

Table 10 Regression model including its significant coefficients and ANOVA table for bottom circuit width

Predictor	Coef	SE Coef	T	P-Value
Constant	0.1279	0.04287	2.98	0.096
B	-0.0342	0.005598	-6.12	0.026
D	-0.0035	0.01120	-0.31	0.784

Source	DF	SS	MS	F	P-Value
Regression	2	0.000188	94×10^{-6}	18.77	0.05
Residual	2	0.000010	5×10^{-6}		
Total	4	0.000198			

The method of steepest descent is then applied for statistically significant process variables to determine the most preferable fitted equation of associated process variables to the response of R_{rw} at both top and bottom circuits. The actual step size is determined by the experimenter with a consideration of other practicals or the process knowledge. These experiments will be terminated when there is an increase in responses from the last step. Eventually the experiments arrived to the vicinity of the optimum. The mathematical programming model is then formulated to minimise the desired response of the circuit width difference from the target.

From the current operating condition, the relationship of the process variables and the responses are categorised by the top (Figure 8) and bottom (Figure 9) circuit widths. The new levels of process variables via the model are then solved via a generalised reduced gradient algorithm. The former shows that the preferable levels of process variables B and D are at 4 and 3.4, respectively. However, their preferred levels are 3.3 and 3.5 for process variables B and D, respectively. Both new operating conditions from the IC-LCRSOM are different, but a higher level of the circuit width affects the short circuit defect more seriously for FPC processes. So the most proper operating condition could follow the operating condition from the bottom circuit analysis. The preferable levels of all process variables from the IC-LCRSOM are also given in Table 11. The performance measures on top and bottom circuit widths from the new operating condition seem to be better (Figure 10 and Table 12).

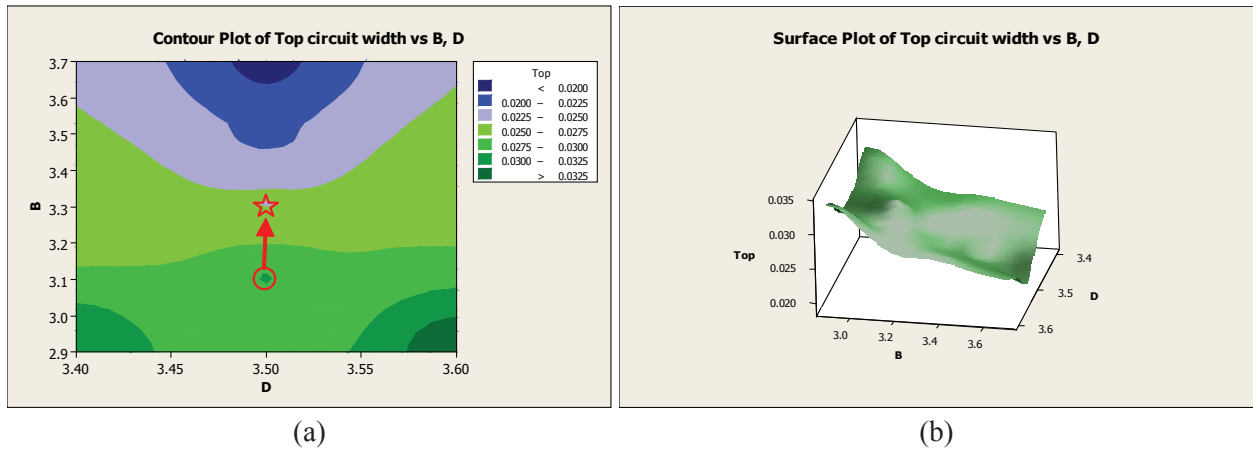


Figure 8 Contour (a) and surface (b) plots of top circuit width

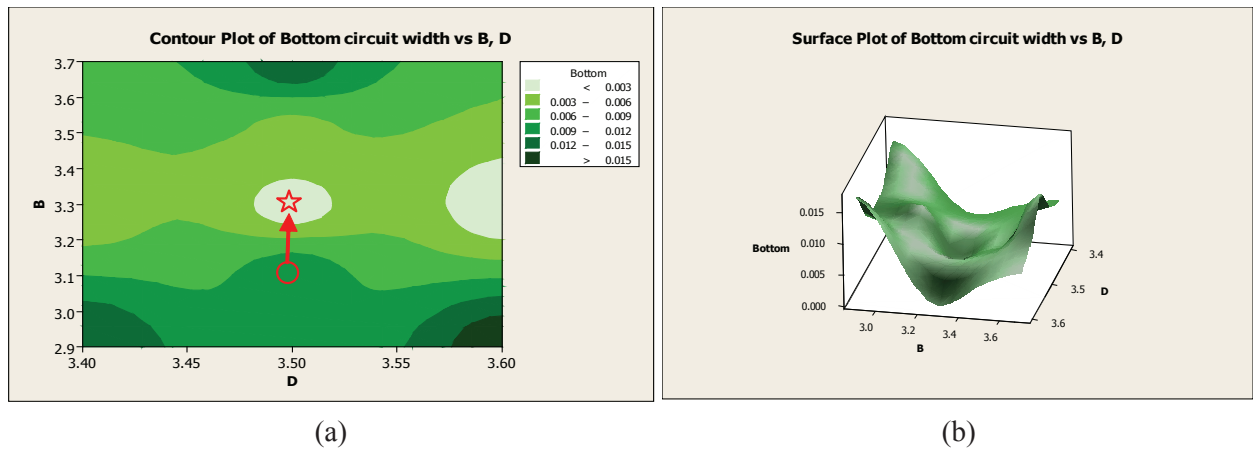


Figure 9 Contour (a) and surface (b) plots of bottom circuit width

Table 11 Process variables and their levels on two scenarios

Process Variable	Scenario	
	Current	New
A	45	60
B	3.1	3.3
C	1	1
D	3.5	3.5

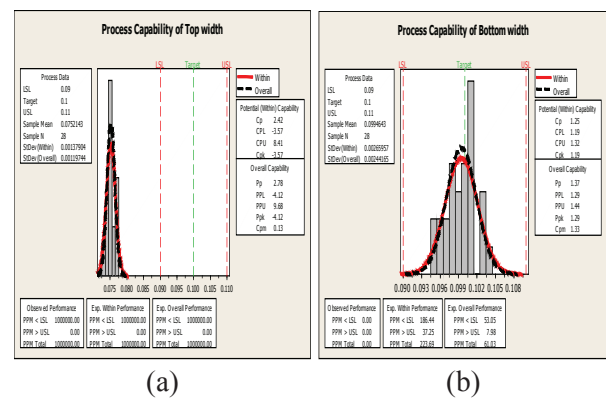


Figure 10 Performance of top (a) and bottom (b) circuit width from new scenario

Table 12 Comparison on circuit width

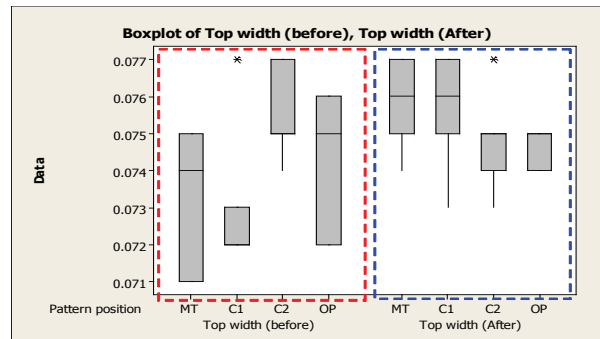
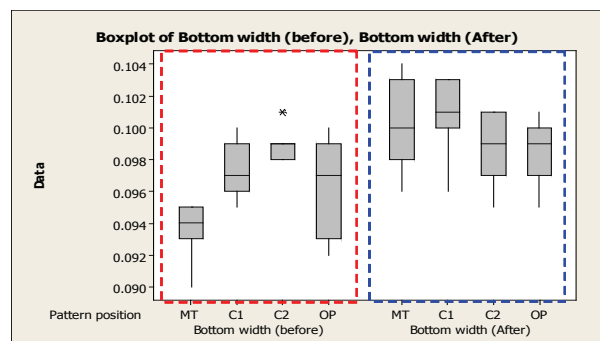
Item	Top Circuit Width		Bottom Circuit Width	
	Before	After	Before	After
Mean	0.074	0.075	0.097	0.099
SD	0.0017	0.0012	0.0026	0.0024
3 σ	0.005	0.004	0.008	0.007
Cpk	-3.03	-3.57	0.85	1.19

Conclusions and Recommendations

In this paper the proper factorial experiments, multiple regression and mathematical programming approaches are applied to investigate the preferable levels of significant process variables in order to improve the quality of etched rate. Firstly, the 2^k factorial design was applied to preliminarily study the effects of those three factors. The responses which consist of circuit widths (R_{CW}) and etching rate (R_{ER}) from the preset experimental designs are measured by Hand-Held Instruments of the Eddy current method. The multiple regression models of those responses were then developed from only significant factors affecting each response. Finally, the regression model of R_{CW} in forms of the path of steepest descent was placed as the objective function of the linear constrained response surface optimisation model to minimise the circuit width subject to the remaining response and the limitation from feasible ranges of two main factors. However, in this study the R_{CW} could be interchangeable to be only the model constraint and the R_{ER} is formulated as the response instead.

After an implementation, the experimental results on top and bottom circuit widths were analysed via t-tests (Table 13). The new condition statistically affects both top and bottom widths at 95% confidence interval. There is a decrease in the deviation from a customer requirement as appeared

in the Box-Whisker plots (Figure 11 and 12). This research was scoped only on one the product and product layout. Consequently conclusions may not be globally optimal. However, the sequential procedures can be applied to the FPC manufactures with many circuit width designs and limited machine capabilities.

**Figure 11** Box-whisker plot of top circuit width**Figure 12** Box-whisker plot of bottom circuit width**Table 13** Comparison via two sample T-tests

Circuit Width	T-Stat	P-Value
Top	-2.76	0.008
Bottom	-4.03	0.000

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