

## **Optimisation Using a Central Composite Rotatable Design for Lacquer Production Process**

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### **Abstract**

The objective of this research is to study the controllable factors affecting the lacquer quality and to find out the optimum conditions of the controllable factors by Central Composite Rotatable Design. For the crushing in Lacquer process, three controllable factors such as the quantity of solvent ( $x_1$ ), cooling temperature ( $x_2$ ), and the specific time frame of crushing ( $x_3$ ) were investigated. Response factor was the smoothness of lacquer surface. The relationship between response and controllable factors was determined. From the results, it was found that the optimal controllable factors were as followed: solvent of 95.59 kilograms, the cooling water temperature of 14.7 °C, and specific crushing time of 23.25 minutes. These controllable factors led to obtain the optimum smoothness of lacquer surface of 6.5 Hexman. The validation of the experiment by using such optimum setting of controllable factors from the Central Composite Rotatable Design, resulted in the 4.6% error of the lacquer surface smoothness.

**Keywords :** Optimization; Smoothness of lacquer surface; Central composite rotatable design; Lacquer production process

### **Introduction**

Lacquer production process consists of mixing process, crushing process, and quality adjustment process. The obstacle in the production is quality testing of paint which is not acceptable by customer requirement. This makes the operators to rework by adding some substances in case the paint intensity does not correspond to quality requirement. For whole consideration about the production process, crushing process and mixing process are important processes which affect paint quality change. In the mixing process, mixing

components are exactly fixed, they cannot be adjusted and other processes have no effect on paint quality. Therefore, only crushing process is considered in this study.

Crushing process is important process which affects paint quality. There are many factors influencing the production of good paint quality such as solvent quantity, cooling water temperature and crushing time. These factor condition levels would be varied appropriately with paint quality property requirement. In crushing process, the operators have to work with skills and trial and

error for setting factor condition levels. This is the problem to control paint quality, especially when the customer wants new various paint property. If paint property is not qualified by customer, the product is reworked. This makes loss time in production and increases production cost. The application of statistical experimental design (Montgomery, 1997) in lacquer process production can result in improving product quality, reducing process variability, i.e., closer confirmation of the output response to nominal and target requirements and reducing development time and overall costs. Conventional practice of classical method of maintaining other factors involved at an unspecified constant level does not depict the combined effect of all the factors involved. This method is also a time consuming process and requires a number of experiments to determine optimum levels, which are unreliable. These limitations of a classical method process can be eliminated by optimising all the affecting parameters collectively by statistical experimental design using response surface methodology (RSM). Response surface methodology (Myers and Montgomery, 1995) is the statistical and mathematical technique useful for developing, improving and optimising processes. It also has important applications in the design, development, and formulation of new products, as well as in the improvement of existing product designs. This approach can help the crushing process operators in Quality Control area and can control the consistency of product quality with less effort. Moreover, it can help them in new product development in case they do not know the exact optimum crushing process conditions used in the process.

In this study, the Central Composite Rotatable Design (CCRD) was employed as RSM tools for optimising crushing process. The

regression analysis (Wiesberg, 1985) was used as the tool for building relationship between controllable factors and response. The estimated function was in form of polynomial function. The performance measures were the coefficient of Determination ( $R^2$ ) and Mean Square Error (MSE). This research illustrated the optimisation procedure with two stages. In the first stage, the RSM was introduced as powerful method to build the statistical approximation to provide for the description of the relationship between the controllable factors and response. In the second stage, the predictive model would be defined as the objective function of optimisation to accomplish the optimisation procedure using Optimiser in MINITAB.

The objectives of this research are to study the controllable factors which influence on paint quality in crushing process and to find optimum controllable factor conditions by using a Central Composite Rotatable Design for crushing process in lacquer process production.

## Methods

This study was empirical research. The experimental design and analysis and Response Surface Methodology were the tools of procedure design and analysis of experimental data. The relationship between response and controllable factors was developed in form of polynomial model.

Central Composite Rotatable Design is the experimental design used in this research. The Central Composite Design was proposed by Box and Wilson (Box and Wilson, 1951). It consists of  $2^k$  full factorial points or  $2^{k-q}$  resolution V fraction factorial points called cubic points,  $2k$  axial or star points and  $n_0 \geq 2$  runs in the design center (Draper, 1982) (where  $k$  is the number of controllable factors,  $q$  is the number of fraction,

and  $n_0$  is the number of design center runs). CCRD with the rotatability property is conducted by choosing an appropriate axial distance (Myers and Montgomery, 1995). Rotatability property is important for a second-order design to possess a reasonably stable distribution of scaled prediction variance throughout the experimental design region. The reasonably stable scaled prediction variance provides assurance that the quality of the predicted response values is roughly the same throughout the region of interest.

A  $2^3$  full factorial central composite design (Myers and Montgomery, 1995) with five coded levels leading to 19 runs of experiments was performed. There were 8 cubic points of  $2^3$  full factorial points, 6 axial points (star points) and 5 center points in design. The design was rotatable CCRD, using an axial distance  $\alpha = 1.682$ . Response was measured as smoothness of lacquer surface ( $y$ ) in unit of Hexman. There were three controllable factors affecting response, i.e. solvent quantity ( $x_1$ ) in unit of kilograms, cooling water temperature ( $x_2$ ) in unit of degree celcius and crushing time ( $x_3$ ) in unit of minutes.

The coded variable levels and natural variable levels used in this study were illustrated in Table 1.

**Table 1** Coded variable levels and natural variable levels

|              | Coded variable levels |     |      |      |        |
|--------------|-----------------------|-----|------|------|--------|
| Factors      | -1.682                | -1  | 0    | 1    | 1.682  |
| $x_1$ (kg)   | 58.36                 | 72  | 92   | 112  | 125.64 |
| $x_2$ (°C)   | 5.3                   | 7.2 | 10.0 | 12.8 | 14.7   |
| $x_3$ (min.) | 6.35                  | 10  | 15   | 20   | 23.25  |

The experimental data was illustrated in Table 2.

**Table 2** Experimental data of solvent quantity, colling water temperature, crushing time and smoothness of lacquer surface

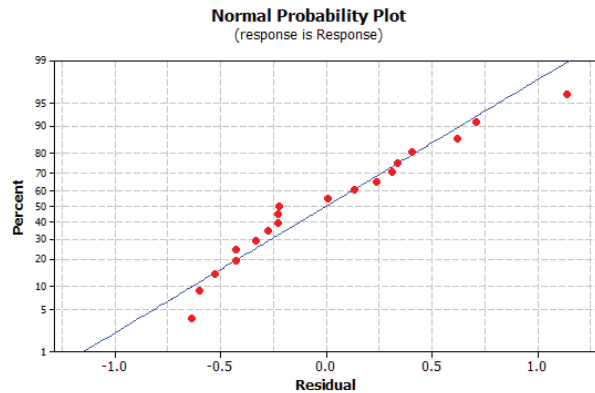
| Run | $x_1$ (kg) | $x_2$ (°C) | $x_3$ (min.) | $y$ (Hexman) |
|-----|------------|------------|--------------|--------------|
| 1   | 72         | 7.2        | 10           | 6.5          |
| 2   | 112        | 7.2        | 10           | 5.5          |
| 3   | 72         | 12.8       | 10           | 5.5          |
| 4   | 112        | 12.8       | 10           | 5.3          |
| 5   | 72         | 7.2        | 20           | 7.5          |
| 6   | 112        | 7.2        | 20           | 6.0          |
| 7   | 72         | 12.8       | 20           | 6.2          |
| 8   | 112        | 12.8       | 20           | 5.8          |
| 9   | 58.36      | 10         | 15           | 5.5          |
| 10  | 125.64     | 10         | 15           | 4.9          |
| 11  | 92         | 5.3        | 15           | 5.6          |
| 12  | 92         | 14.7       | 15           | 6.0          |
| 13  | 92         | 10         | 6.35         | 4.8          |
| 14  | 92         | 10         | 23.25        | 5.5          |
| 15  | 92         | 10         | 15           | 5.1          |
| 16  | 92         | 10         | 15           | 5.4          |
| 17  | 92         | 10         | 15           | 5.2          |
| 18  | 92         | 10         | 15           | 5.2          |
| 19  | 92         | 10         | 15           | 5.4          |

## Results and Discussions

### Results

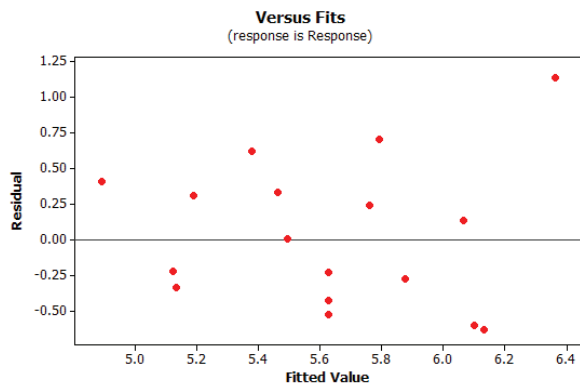
The model adequacy checking consists of 3 components as followed

1. Normal probability plot of residual.
2. Fitted value versus residual plot.
3. Observation order versus residual plot.



**Figure 1** Normal probability plot of residual

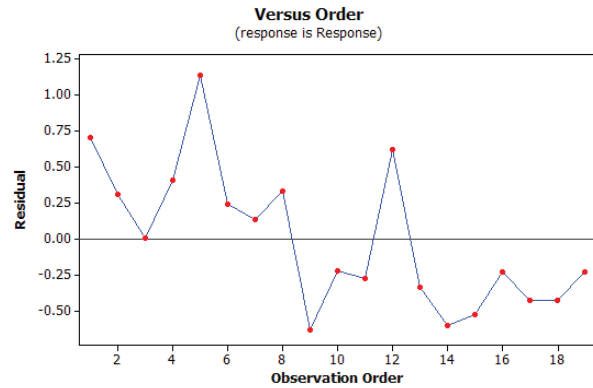
Normal probability plot of residual was illustrated in Figure 1. The data were distributed near the straight line, which indicated that the residual was normal distribution.



**Figure 2** Fitted value versus residual plot

The fitted value versus residual plot was illustrated in Figure 2. The data were randomly scattered around zero-centered line. They had no open-ended funnel patterns. It indicated that the variance of residual was constant.

The observation order versus residual plot was illustrated in Figure 3. The data were randomly scattered around zero-centered line. It indicated that the residual was independently random variable and uncorrelated.



**Figure 3** Observation order versus residual plot

In this study, the influence of solvent quantity ( $x_1$ ), cooling water temperature ( $x_2$ ), and crushing time ( $x_3$ ) were studied on the smoothness of lacquer surface ( $y$ ) at 5% significance level ( $\alpha$ ). This was done by hypothesis testing. The hypotheses were as followed

(1)  $H_0$  : Solvent quantity affected smoothness of lacquer surface.

$H_1$  : Solvent quantity had no effect on smoothness of lacquer surface.

(2)  $H_0$  : Cooling water temperature affected smoothness of lacquer surface.

$H_1$  : Cooling water temperature had no effect on smoothness of lacquer surface.

(3)  $H_0$  : Crushing time affected smoothness of lacquer surface.

$H_1$  : Crushing time had no effect on smoothness of lacquer surface.

Lack-of-Fit Test was used for consideration of appropriate regression model. The hypotheses were as followed

$H_0$  : The equation model was appropriate.

$H_1$  : The equation model was not appropriate.

The relationship between response and controllable factors was analysed in the form of polynomial model which was expressed in equation (1).

$$y = \beta_0 + \sum_{i=1}^3 \beta_i x_i + \sum_{i=1}^3 \beta_{ii} x_i^2 + \sum_{i=1}^2 \sum_{j=i+1}^3 \beta_{ij} x_i x_j \quad (1)$$

Where  $\beta_0$  was intercept on y-axis,  $\beta_i$  was linear coefficients,  $\beta_{ii}$  was quadratic coefficients,  $\beta_{ij}$  was cross-product coefficients, and  $x_i, x_j$  were uncoded independent variables.

The polynomial equation which represented the relationship between response and three controllable factors was expressed in equation (2)

$$\begin{aligned} \hat{y} = & 20.7946 - 0.134992x_1 - 1.87332x_2 + \\ & 0.165672x_3 + 0.000459871x_1^2 + \\ & 0.0695257x_2^2 - 5.52735 \times 10^{-5}x_3^2 + \\ & 0.00424107x_1x_2 - 8.75 \times 10^{-4}x_1x_3 - \\ & 0.00267857x_2x_3 \end{aligned} \quad (2)$$

This regression equation explained that when  $x_1$  changed 1 unit,  $\hat{y}$  decreased 0.134992 units, when  $x_2$  changed 1 unit,  $\hat{y}$  decreased 1.87332 units and when  $x_3$  changed 1 unit,  $\hat{y}$  increased 0.165672 units with 91.72% of R-squared. This indicated that the polynomial equation was capable to explain the smoothness of lacquer surface well with 91.72%.

#### ANOVA result

| Source         | DF | Seq SS  | Adj SS  | Adj MS  | F     | P     |
|----------------|----|---------|---------|---------|-------|-------|
| Regression     | 9  | 6.76449 | 6.76449 | 0.75161 | 20.69 | 0.000 |
| Linear         | 3  | 3.32432 | 1.03834 | 0.34611 | 9.53  | 0.007 |
| Square         | 3  | 2.91643 | 2.91643 | 0.97214 | 26.76 | 0.000 |
| Interaction    | 3  | 0.52375 | 0.52375 | 0.17458 | 4.81  | 0.040 |
| Residual Error | 7  | 0.25433 | 0.25433 | 0.03633 |       |       |
| Lack-of-Fit    | 3  | 0.18233 | 0.18233 | 0.06078 | 3.38  | 0.135 |
| Pure Error     | 4  | 0.07200 | 0.07200 | 0.01800 |       |       |
| Total          | 16 | 7.01882 |         |         |       |       |

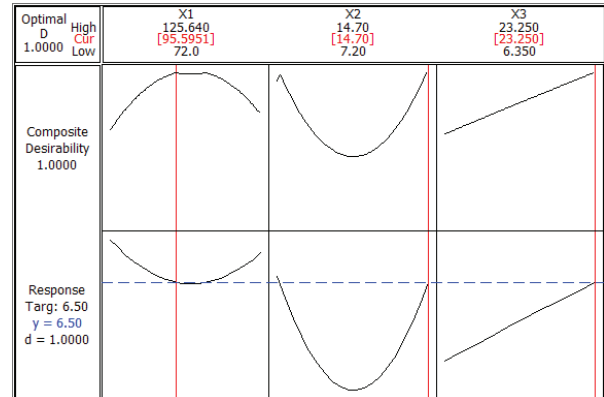


Figure 4 Parameter optimization graph

The analysis results were illustrated as below.

#### Estimated Regression Coefficients for Response

| Term     | Coef     | SE Coef | T      | P     |
|----------|----------|---------|--------|-------|
| Constant | 5.08413  | 0.08333 | 61.013 | 0.000 |
| X1       | -0.28464 | 0.09032 | -3.152 | 0.016 |
| X2       | 0.10781  | 0.09027 | 1.194  | 0.271 |
| X3       | 0.40761  | 0.10285 | 3.963  | 0.005 |
| X1*X1    | 0.33079  | 0.12691 | 2.606  | 0.035 |
| X2*X2    | 0.97770  | 0.12687 | 7.707  | 0.000 |
| X3*X3    | -0.00395 | 0.15161 | -0.026 | 0.980 |
| X1*X2    | 0.42655  | 0.12103 | 3.524  | 0.010 |
| X1*X3    | -0.19830 | 0.15273 | -1.298 | 0.235 |
| X2*X3    | -0.08488 | 0.15253 | -0.556 | 0.595 |

$$S = 0.190612 \quad \text{PRESS} = 2.76367$$

$$\text{R-Sq} = 96.38\% \quad \text{R-Sq(pred)} = 60.62\%$$

$$\text{R-Sq(adj)} = 91.72\%$$

The controllable factors which affected paint quality in crushing process were optimised using CCRD as shown in Figure 4. It was found that the optimised values of solvent quantity ( $x_1$ ), cooling water temperature ( $x_2$ ) and crushing time ( $x_3$ ) were 95.59 kg., 14.7 °C and 23.25 min., respectively. These led to the response of 6.5 Hexman which was the target response requirement.

### The Validation of Experiment

The crushing process operators used the optimum factor conditions of solvent quantity ( $x_1$ ) 95.59 kg, cooling water temperature ( $x_2$ ) 14.7 °C and crushing time ( $x_3$ ) 23.25 min. to produce the lacquer. It was found that the smoothness of lacquer surface (y) was 6.2 Hexman. It had 4.6% error of smoothness of lacquer surface compared to the results from CCRD.

### Discussions

Crushing time was the most influenced factor to smoothness of lacquer surface. This might be caused by the homogeneous paint quality requirement in crushing process. Therefore, the crushing time was the most significant factor in crushing process with p-value 0.005 (less than  $\alpha = 0.05$ ).

### Conclusions

The model adequacy checking was approved corresponding to the assumption. The Student-t hypothesis testing of regression coefficients indicated that solvent quantity ( $x_1$ ) affected smoothness of lacquer surface (y) with p-value 0.016 (less than  $\alpha = 0.05$ ), cooling water temperature ( $x_2$ ) had no effect on smoothness of lacquer surface (y) with

p-value 0.271 (more than  $\alpha = 0.05$ ) and crushing time ( $x_3$ ) affected smoothness of lacquer surface (y) with p-value 0.005 (less than  $\alpha = 0.05$ ). For Lack-of-Fit test, it indicated that the regression model was appropriate with p-value 0.135 (more than  $\alpha = 0.05$ ).

The optimum factor conditions were as followed: solvent quantity ( $x_1$ ) was 95.59 kg., cooling water temperature ( $x_2$ ) was 14.7 °C and crushing time ( $x_3$ ) was 23.25 min. with fitted value of smoothness of lacquer surface 6.5 Hexman and it made 4% error of smoothness of lacquer surface.

### Acknowledgement

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