Research Article

Confidence Intervals for the Parameter of a Gaussian First-Order Autoregressive Model with Additive Outliers: A Simulation Study

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Abstract

This paper is concerned with interval estimation of a parameter for a Gaussian first-order autoregressive model, AR(1), when there are additive outliers in a time series. We compared the confidence intervals based on the weighted symmetric estimator ($\hat{\phi}_W$), the recursive mean adjusted weighted symmetric estimator ($\hat{\phi}_{RW}$), the recursive median adjusted weighted symmetric estimator ($\hat{\phi}_{RDW}$), and the improved recursive median adjusted weighted symmetric estimator ($\hat{\phi}_{IRDW}$) by using Monte Carlo simulation. Simulation results have shown that the confidence interval based on the estimator $\hat{\phi}_{IRDW}$ is better than the other confidence intervals with respect to the coverage probability and the length criteria.

Key Words: AR(1) model; Additive outliers; Confidence interval; Coverage probability; Length

Introduction

In time series analysis, outliers or atypical observations can have adverse impacts on model identification, parameter estimation as well as forecasting. Outliers may occur because of human errors, such as typing, recording and measuring mistakes or because of abrupt, short-term changes in the underlying process (Cryer and Chan, 2008). Fox (1972), Abraham and Box (1979), and Martin (1980) discussed two kinds of outliers that can be founded in time series data, namely, additive outliers (AO) and innovational outliers (IO). An additive outlier corresponds to the situation in which a gross error of observation or recording error affects a single observation (Fox, 1972). An innovational outlier affects not only the particular observation but also subsequent observations (Fox, 1972). In this study, we concentrate on the additive outliers because these outliers are more harmful than innovational outliers (Chatfield, 2001). A time series that does not contain any outliers is called an outlier-free series.

Suppose an outlier-free time series $\{X_t\,; t = 2,3,...,n\}$ follows a Gaussian first-order autoregressive model, AR(1), satisfying

$$X_t = \mu + \phi(X_{t-1} - \mu) + \varepsilon_t, \qquad (1)$$

where μ is the mean of the model, ϕ is an autoregressive parameter; $-1 < \phi < 1$, ε_t's are independent and identically distributed (i.i.d.) random variables having normal distribution with zero mean and variance σ_ε^2, i.e., $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. The model (1) will be called a random walk model if $|\phi| = 1$, otherwise it is called a stationary model. In the case of ϕ close to one or near a non-stationary model, the mean and variance of this model change over time. Let the observed time series be denoted by $\{Y_t\}$. An additive outlier model is defined as

$$Y_t = X_t + \omega I_t^{(T)}, \qquad (2)$$

where ω is the magnitude of the additive outliers and $I_t^{(T)}$ is the indicator function such that

$$I_t^{(T)} = \begin{cases} 1\ , & t = T, \\ 0\ , & t \neq T. \end{cases}$$

The model (2) can be interpreted that $\{X_t\}$ has additive outliers at time point T $(1 < T < n)$.

One of the well-known estimators of an autoregressive parameter; ϕ is the ordinary least squares (OLS) estimator. Although the OLS estimator has asymptotic normality for $|\phi| < 1$ (see, Mann and Wald, 1943; Brockwell and Davis, 1991), it has long been known that the OLS estimator can have large bias; see, for example, Marriott and Pope (1954), Shaman and Stine (1988), Newbold and Agiakloglou (1993). In addition, Conover (1980) indicated that the OLS estimator is sensitive to outliers (see Conover, 1980, pp.267). Therefore, there have been the useful improvements in the parameter estimation so as to reduce the bias of the OLS estimator. Park and Fuller (1995) proposed the weighted symmetric estimator (W) of ϕ. The robust estimator for an autoregressive model was presented by Denby and Martin (1979). Guo (2000) developed the simple and robust estimator for an AR(1) model. Apart from that, So and Shin (1999) applied a recursive mean adjustment to the OLS estimator (ROLS) and they found that the mean square error of the ROLS estimator is smaller than the OLS estimator for $\phi \in (0,1)$. They also showed that the ROLS estimator has a coverage probability which is close to the nominal value. Niwitpong (2007) applied the recursive mean adjustment to the weighted symmetric estimator (RW) of Park and Fuller (1995). Panichkitkosolkul (2010a) proposed an estimator for an unknown mean Gaussian AR(1) model with additive outliers by applying the recursive median adjustment to the weighted symmetric estimator (RDW). He found that the RDW estimator provides the efficiency more than the W and RW estimators in terms of the mean square error for almost all situations. Moreover, Panichkitkosolkul (2010b) also improved the RDW estimator by applying the new recursive median adjustment to the weighted symmetric estimator (IRDW). New recursive median adjustment can be derived from computing the double recursive median. Using the simulations, an IRDW estimator performs better than the W, RW and RDW estimators in terms of the mean square error for almost all scenarios. Thus, our major purpose in this paper is to evaluate the efficiency of the confidence intervals for the parameter of a Gaussian AR(1) model based on the W, RW, RDW and IRDW estimators when there are additive outliers in a time series data.

The rest of this paper is structured as follows. In the following section, we describe the confidence intervals of the parameter of a Gaussian AR(1) model based on the W, RW, RDW and IRDW estimators. Simulation results obtained from Monte Carlo simulation are shown in the third section. In the forth section, all confidence intervals are illustrated and compared through empirical application. The discussions of the results and conclusions are presented in the final section.

Methodology

Park and Fuller (1995) proposed the weighted symmetric estimator of ϕ for model (1) given by

$$\hat{\phi}_W = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y})(Y_{t-1} - \bar{Y})}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y})^2}. \qquad (3)$$

Therefore, the $(1-\alpha)100\%$ confidence interval for ϕ of model (1) based on $\hat{\phi}_W$ can be derived as follows

$$CI_W = \left[\hat{\phi}_W - Z_{1-\frac{\alpha}{2}}SE(\hat{\phi}_W)\,, \hat{\phi}_W + Z_{1-\frac{\alpha}{2}}SE(\hat{\phi}_W)\right], \quad (4)$$

where $SE(\hat{\phi}_W) = \dfrac{\hat{\sigma}_W}{\sqrt{\sum_{t=2}^{n}(Y_{t-1} - \bar{Y})^2}}$,

$\hat{\sigma}_W^2 = \dfrac{\sum_{t=2}^{n}\left((Y_t - \bar{Y}) - \hat{\phi}_W(Y_{t-1} - \bar{Y})\right)^2}{n-2}$ and $Z_{1-\frac{\alpha}{2}}$ is a $\left(1-\dfrac{\alpha}{2}\right)$th quantile of the standard normal distribution.

Niwitpong (2007) extended the concepts of So and Shin (1999) by applying the recursive mean adjustment to the weighted symmetric estimator. He replaces $\bar{Y}$ by $\bar{Y}_t = \frac{1}{t}\sum_{i=1}^{t} Y_i$ in (3). The estimator of ϕ obtained as a result of this recursive mean adjustment is

$$\hat{\phi}_{RW} = \frac{\sum_{t=2}^{n}(Y_t - \bar{Y}_t)(Y_{t-1} - \bar{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \bar{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \bar{Y}_t)^2} \qquad (5)$$

Based on $\hat{\phi}_{RW}$, the $(1-\alpha)100\%$ confidence interval for ϕ of model (1) is defined as

$$CI_{RW} = \left[\hat{\phi}_{RW} - Z_{1-\frac{\alpha}{2}}SE(\hat{\phi}_{RW})\,, \hat{\phi}_{RW} + Z_{1-\frac{\alpha}{2}}SE(\hat{\phi}_{RW})\right], (6)$$

where $SE(\hat{\phi}_{RW}) = \dfrac{\hat{\sigma}_{RW}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1} - \bar{Y}_{t-1})^2}}$,

$\hat{\sigma}_{RW}^2 = \dfrac{\sum_{t=2}^{n}\left((Y_t - \bar{Y}_t) - \hat{\phi}_{RW}(Y_{t-1} - \bar{Y}_{t-1})\right)^2}{n-2}$ and

$Z_{1-\frac{\alpha}{2}}$ is a $\left(1-\dfrac{\alpha}{2}\right)$th quantile of the standard normal distribution.

It is well known that if there are outliers in the data set then the mean will be strongly affected. On the other hand, the median is less sensitive to outliers than the mean. Therefore, Panichkitkosolkul (2010a) improved the RW estimator by replacing the recursive mean $\bar{Y}_t$ in (5) by the recursive median. The estimator of ϕ based on the recursive median adjustment called the RDW estimator is given by

$$\hat{\phi}_{RDW} = \frac{\sum_{t=2}^{n}(Y_t - \tilde{Y}_t)(Y_{t-1} - \tilde{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \tilde{Y}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \tilde{Y}_t)^2}, \qquad (7)$$

where $\tilde{Y}_t = \text{median}(Y_1, Y_2, \ldots, Y_t)$. The $(1-\alpha)100\%$ confidence interval for ϕ of model (1) based on $\hat{\phi}_{RDW}$ is

$$CI_{RDW} = \begin{bmatrix} \hat{\phi}_{RDW} - Z_{1-\frac{\alpha}{2}}SE(\hat{\phi}_{RDW})\,, & \\ & \hat{\phi}_{RDW} + Z_{1-\frac{\alpha}{2}}SE(\hat{\phi}_{RDW}) \end{bmatrix}, (8)$$

where $SE(\hat{\phi}_{RDW}) = \dfrac{\hat{\sigma}_{RDW}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1} - \tilde{Y}_{t-1})^2}}$,

$$\hat{\sigma}^2_{RDW} = \frac{\sum_{t=2}^{n}\left((Y_t - \tilde{Y}_t) - \hat{\phi}_{RDW}(Y_{t-1} - \tilde{Y}_{t-1})\right)^2}{n-2}$$ and

$Z_{1-\frac{\alpha}{2}}$ is a $\left(1-\frac{\alpha}{2}\right)$th quantile of the standard normal distribution.

Further, Panichkitkosolkul (2010b) also reduced the effect of outliers on an estimator of ϕ in model (1) by using the new recursive median adjustment. The new recursive median values are derived from computing the double recursive median. There are two steps for computing the new recursive median. Firstly, the recursive median ($\tilde{Y}_t$) are computed by using observed time series data Y_t. Secondly, we calculate the double recursive median by using the recursive median obtained from the first step. Therefore, the recursive median in (7) is replaced by the improved recursive median. This estimator is abbreviated the IRDW estimator. The estimator of ϕ improved by using the improved recursive median is defined as

$$\hat{\phi}_{IRDW} = \frac{\sum_{t=2}^{n}(Y_t - \tilde{\tilde{Y}}_t)(Y_{t-1} - \tilde{\tilde{Y}}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1} - \tilde{\tilde{Y}}_{t-1})^2 + n^{-1}\sum_{t=1}^{n}(Y_t - \tilde{\tilde{Y}}_t)^2}, \quad (9)$$

where $\tilde{\tilde{Y}}_t = \text{median}(\tilde{Y}_1, \tilde{Y}_2, ..., \tilde{Y}_t)$ and $\tilde{Y}_t = \text{median}(Y_1, Y_2, ..., Y_t)$. An R function for computing the estimator in (9) is given in the Appendix of Panichkitkosolkul (2010b). In addition, the $(1-\alpha)100\%$ confidence interval for ϕ of model (1) based on the IRDW estimator is

$$CI_{IRDW} = \begin{bmatrix} \hat{\phi}_{IRDW} - Z_{1-\frac{\alpha}{2}} SE(\hat{\phi}_{IRDW}), \\ \hat{\phi}_{IRDW} + Z_{1-\frac{\alpha}{2}} SE(\hat{\phi}_{IRDW}) \end{bmatrix}, (10)$$

where $SE(\hat{\phi}_{IRDW}) = \dfrac{\hat{\sigma}_{IRDW}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1} - \tilde{\tilde{Y}}_{t-1})^2}}$,

$$\hat{\sigma}^2_{IRDW} = \frac{\sum_{t=2}^{n}\left((Y_t - \tilde{\tilde{Y}}_t) - \hat{\phi}_{IRDW}(Y_{t-1} - \tilde{\tilde{Y}}_{t-1})\right)^2}{n-2}$$ and

$Z_{1-\frac{\alpha}{2}}$ is a $\left(1-\frac{\alpha}{2}\right)$th quantile of the standard normal distribution.

To study the different confidence intervals, we consider their coverage probability and length. For each of the methods considered, we obtain a $(1-\alpha)100\%$ confidence interval denoted by (L, U) (based on $M = 5,000$ replicates) and estimated the coverage probability and the length, respectively, by

$$\widehat{1-\alpha} = \frac{\#(L \le \phi \le U)}{M},$$

and $$\widehat{Length} = \frac{\sum_{i=1}^{M}(U_i - L_i)}{M},$$

where $\#(L \le \phi \le U)$ denotes the number of simulations for which an autoregressive parameter; ϕ lies within the confidence interval (L, U).

In the following section, the simulation results is presented in order to evaluate the performance of the confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} based on their coverage probabilities and lengths.

Simulation Results

This section is devoted to access the performance of the confidence intervals (4), (6), (8) and (10) via simulations. The data sets of five-thousand Gaussian AR(1) time series with AOs were simulated by using R statistical software (The R Development Core Team, 2010a, 2010b). The scope of the simulations is set under $(\mu, \sigma_\varepsilon)$ = (0, 1); ϕ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8

and 0.9; sample sizes $n = 25, 50, 100$ and 250; the magnitudes of the AOs $\omega = 0, 3\sigma_\varepsilon, 5\sigma_\varepsilon, -3\sigma_\varepsilon$ and $-5\sigma_\varepsilon$; the percentages of additive outliers $p = 5\%$ and 10%. The confidence level is fixed at 0.05. Additionally, the additive outliers arose randomly. Our simulation results are summarized in Tables 1-9. Table 1 show the results on coverage probabilities and lengths of prediction intervals when there are no outliers in time series $(\omega = 0)$. Tables 2-5 present the simulation results for $p = 5\%$. Similarly, Tables 6-9 present the results for $p = 10\%$.

We begin with the results in the case when there are no outliers $(\omega = 0)$ shown in Table 1. For small sample size $(n = 25)$, the coverage probability of the CI_{RDW} is the largest compared to other confidence intervals when the parameter is small $(0.1 \le \phi \le 0.5)$. When $\phi = 0.6$ and 0.7, the CI_{RW} provides the coverage probabilities larger than those of other confidence intervals. The CI_{IRDW}, on the other hand, provides the coverage probabilities close to the nominal confidence level (0.95) when the parameter is close to one $(\phi = 0.8$ and $0.9)$. For moderate sample sizes $(n = 50$ and $100)$, the coverage probability of the CI_{RW} is the largest compared to other confidence intervals for almost all ϕ values except when $\phi = 0.9$. For large sample size $(n = 250)$, the CI_{RW} provides the coverage probabilities close to the nominal confidence level for all cases.

The results in case of $\omega = 3\sigma_\varepsilon$ and $-3\sigma_\varepsilon$ at $p = 5\%$ are reported in Tables 2 and 3, respectively. For small sample size $(n = 25)$, the coverage probability of the CI_{RDW} is the largest compared to other confidence intervals when the parameter is small ($\phi = 0.1, 0.2$ and 0.3). In case of $0.4 \le \phi \le 0.9$, the CI_{IRDW} provides the coverage probabilities larger than those of other confidence intervals. For moderate sample size $(n = 50)$, the coverage probability of the CI_{RDW} is the largest compared to other confidence intervals when the parameter is small $(\phi = 0.1$ and $0.2)$. In case of $0.3 \le \phi \le 0.9$, the CI_{IRDW} provides the coverage probabilities larger than those of other confidence intervals. For large sample sizes $(n = 100$ and $250)$, the coverage probability of the CI_{IRDW} is the largest compared to other confidence intervals except when $\phi = 0.1$ and $n = 100$.

The results in case of $\omega = 5\sigma_\varepsilon$ and $-5\sigma_\varepsilon$ at $p = 5\%$ are shown in Tables 4 and 5. For small and moderate sample sizes $(n = 25, 50$ and $100)$, the coverage probability of the CI_{IRDW} is the largest compared to other confidence intervals except when $\phi = 0.1$. For large sample size $(n = 250)$, the coverage probability of the CI_{IRDW} is the largest compared to other confidence intervals.

In Tables 6 and 7, the results in case of $\omega = 3\sigma_\varepsilon$ and $-3\sigma_\varepsilon$ at $p = 10\%$ are reported. For small sample size $(n = 25)$, the coverage probability of the CI_{IRDW} is the largest compared to other confidence intervals when the parameter is small $(\phi = 0.1$ and $0.2)$. In case of $0.3 \le \phi \le 0.9$, the CI_{IRDW} provides the coverage probabilities larger than those of other confidence intervals. For moderate sample sizes $(n = 50$ and $100)$, the coverage probability of the CI_{IRDW} is the largest compared to other confidence intervals except when $\phi = 0.1$. For large sample size $(n = 250)$, the coverage probability of the CI_{IRDW} is the largest compared to other confidence intervals for all autoregressive parameter values considered.

The results in case of $\omega = 5\sigma_{\varepsilon}$ and $-5\sigma_{\varepsilon}$ at $p = 10\%$ are shown in Tables 8 and 9. They are similar to Tables 6 and 7 so that we omit the explanation in the details.

Furthermore, the CI_{IRDW} dominates the other confidence intervals with respect to the length criterion. Namely, the lengths of the CI_{IRDW} are shortest with comparing those of any other confidence intervals except when $n = 25$ and $\phi = 0.1$ of Tables 2, 4, 5, 8 and 9. Additionally, the coverage probabilities decrease as sample sizes get larger. This is intuitive in nature because as n increases it is possible to estimate the standard error of the estimator more accurately. Consequently, the length decreases when the sample increases.

Apart from that, if the magnitude of the AOs increases, then the coverage probability decreases (compare between Table 2 and Table 4) while the length increases. Similarly, if the percentage of additive outliers increases, then the coverage probability decreases (compare between Table 2 and Table 6) while the length increases.

Table 1 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when no outliers $(\omega = 0)$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9410	0.9530	0.9566	0.9048	0.8016	0.8181	0.8164	0.7995
	0.2	0.9382	0.9558	0.9564	0.9026	0.7944	0.8086	0.8058	0.7822
	0.3	0.9360	0.9524	0.9568	0.9090	0.7808	0.7914	0.7875	0.7572
	0.4	0.9350	0.9520	0.9542	0.9166	0.7600	0.7672	0.7640	0.7269
	0.5	0.9276	0.9486	0.9504	0.9224	0.7337	0.7378	0.7348	0.6901
	0.6	0.9248	0.9484	0.9482	0.9288	0.6972	0.6980	0.6963	0.6446
	0.7	0.9090	0.9368	0.9346	0.9362	0.6560	0.6533	0.6523	0.5960
	0.8	0.8918	0.9236	0.9162	0.9426	0.5989	0.5931	0.5937	0.5297
	0.9	0.8650	0.8886	0.8906	0.9376	0.5360	0.5265	0.5276	0.4592
50	0.1	0.9410	0.9490	0.9446	0.8822	0.5586	0.5639	0.5627	0.5551
	0.2	0.9516	0.9568	0.9560	0.8914	0.5524	0.5565	0.5548	0.5435
	0.3	0.9438	0.9536	0.9512	0.8930	0.5405	0.5434	0.5412	0.5258
	0.4	0.9414	0.9478	0.9468	0.8962	0.5227	0.5242	0.5222	0.5026
	0.5	0.9422	0.9516	0.9550	0.8982	0.4994	0.4998	0.4976	0.4730
	0.6	0.9422	0.9520	0.9488	0.9012	0.4682	0.4673	0.4653	0.4363
	0.7	0.9340	0.9448	0.9428	0.9170	0.4287	0.4262	0.4242	0.3911
	0.8	0.9226	0.9332	0.9298	0.9234	0.3787	0.3751	0.3733	0.3359
	0.9	0.9042	0.9104	0.9100	0.9432	0.3157	0.3110	0.3099	0.2694
100	0.1	0.9500	0.9520	0.9464	0.8810	0.3924	0.3941	0.3936	0.3905
	0.2	0.9478	0.9512	0.9518	0.8930	0.3873	0.3884	0.3875	0.3827
	0.3	0.9376	0.9434	0.9430	0.8836	0.3781	0.3790	0.3779	0.3709
	0.4	0.9508	0.9540	0.9532	0.8950	0.3646	0.3651	0.3639	0.3544
	0.5	0.9434	0.9480	0.9476	0.8838	0.3463	0.3463	0.3450	0.3329
	0.6	0.9422	0.9478	0.9452	0.8948	0.3227	0.3221	0.3209	0.3062
	0.7	0.9420	0.9454	0.9426	0.8892	0.2919	0.2909	0.2896	0.2721
	0.8	0.9320	0.9382	0.9324	0.9008	0.2522	0.2507	0.2493	0.2292
	0.9	0.9230	0.9252	0.9260	0.9200	0.1971	0.1952	0.1938	0.1714
250	0.1	0.9496	0.9514	0.9506	0.9010	0.2473	0.2477	0.2475	0.2467
	0.2	0.9492	0.9502	0.9486	0.9036	0.2437	0.2440	0.2437	0.2421
	0.3	0.9510	0.9520	0.9494	0.9000	0.2376	0.2378	0.2374	0.2350
	0.4	0.9430	0.9470	0.9460	0.8888	0.2284	0.2285	0.2281	0.2247
	0.5	0.9484	0.9486	0.9466	0.8930	0.2164	0.2163	0.2159	0.2115
	0.6	0.9494	0.9510	0.9496	0.8988	0.2007	0.2005	0.2000	0.1946
	0.7	0.9528	0.9544	0.9518	0.8938	0.1799	0.1795	0.1790	0.1724
	0.8	0.9400	0.9418	0.9380	0.8812	0.1529	0.1525	0.1519	0.1438
	0.9	0.9380	0.9388	0.9372	0.8970	0.1149	0.1143	0.1137	0.1041

Table 2 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 5\%$ and $\omega = 3\sigma_\varepsilon$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9374	0.9550	0.9640	0.9298	0.8024	0.8150	0.8144	0.8027
	0.2	0.9238	0.9466	0.9500	0.9348	0.7996	0.8108	0.8088	0.7923
	0.3	0.9002	0.9290	0.9358	0.9342	0.7927	0.8016	0.7997	0.7783
	0.4	0.8690	0.9074	0.9164	0.9346	0.7808	0.7884	0.7862	0.7581
	0.5	0.8404	0.8832	0.8954	0.9246	0.7646	0.7697	0.7672	0.7324
	0.6	0.7982	0.8434	0.8546	0.9120	0.7426	0.7454	0.7432	0.7011
	0.7	0.7598	0.8104	0.8128	0.8960	0.7127	0.7133	0.7111	0.6601
	0.8	0.7178	0.7726	0.7654	0.8668	0.6720	0.6705	0.6691	0.6095
	0.9	0.6700	0.7100	0.7070	0.8324	0.6161	0.6119	0.6122	0.5415
50	0.1	0.9336	0.9420	0.9486	0.9194	0.5595	0.5635	0.5627	0.5577
	0.2	0.9156	0.9286	0.9370	0.9250	0.5561	0.5594	0.5583	0.5504
	0.3	0.8872	0.9128	0.9188	0.9286	0.5496	0.5523	0.5510	0.5400
	0.4	0.8476	0.8704	0.8814	0.9072	0.5397	0.5416	0.5399	0.5251
	0.5	0.8096	0.8408	0.8444	0.9028	0.5242	0.5253	0.5237	0.5041
	0.6	0.7534	0.7892	0.7986	0.8884	0.5045	0.5046	0.5029	0.4784
	0.7	0.7314	0.7666	0.7708	0.8740	0.4739	0.4729	0.4711	0.4409
	0.8	0.6924	0.7252	0.7268	0.8508	0.4340	0.4317	0.4297	0.3926
	0.9	0.6620	0.6770	0.6834	0.8258	0.3745	0.3712	0.3688	0.3252
100	0.1	0.9334	0.9442	0.9512	0.9310	0.3934	0.3946	0.3942	0.3923
	0.2	0.8904	0.9046	0.9180	0.9296	0.3908	0.3918	0.3912	0.3880
	0.3	0.8330	0.8508	0.8702	0.9072	0.3860	0.3868	0.3859	0.3811
	0.4	0.7542	0.7820	0.7982	0.8750	0.3786	0.3790	0.3781	0.3715
	0.5	0.6586	0.6848	0.7036	0.8172	0.3687	0.3689	0.3679	0.3589
	0.6	0.6010	0.6272	0.6434	0.7928	0.3530	0.3528	0.3518	0.3399
	0.7	0.5448	0.5722	0.5876	0.7510	0.3314	0.3308	0.3296	0.3141
	0.8	0.5258	0.5458	0.5596	0.7428	0.2983	0.2973	0.2959	0.2758
	0.9	0.5576	0.5664	0.5754	0.7616	0.2462	0.2447	0.2429	0.2174
250	0.1	0.9186	0.9258	0.9392	0.9396	0.2479	0.2482	0.2480	0.2475
	0.2	0.8390	0.8516	0.8714	0.9088	0.2461	0.2463	0.2460	0.2450
	0.3	0.7000	0.7190	0.7508	0.8366	0.2428	0.2430	0.2426	0.2410
	0.4	0.5634	0.5854	0.6126	0.7498	0.2377	0.2378	0.2374	0.2351
	0.5	0.4154	0.4332	0.4660	0.6360	0.2306	0.2306	0.2301	0.2269
	0.6	0.3286	0.3430	0.3634	0.5596	0.2199	0.2198	0.2193	0.2149
	0.7	0.2612	0.2756	0.3012	0.5178	0.2050	0.2048	0.2043	0.1983
	0.8	0.2790	0.2876	0.3100	0.5392	0.1819	0.1815	0.1810	0.1728
	0.9	0.3856	0.3886	0.4068	0.6380	0.1438	0.1434	0.1425	0.1317

Table 3 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 5\%$ and $\omega = -3\sigma_\varepsilon$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9406	0.9544	0.9614	0.9304	0.8031	0.8158	0.8149	0.8029
	0.2	0.9246	0.9456	0.9512	0.9454	0.7998	0.8111	0.8093	0.7935
	0.3	0.8964	0.9322	0.9394	0.9402	0.7937	0.8032	0.8011	0.7794
	0.4	0.8752	0.9118	0.9214	0.9354	0.7821	0.7893	0.7874	0.7596
	0.5	0.8428	0.8896	0.8942	0.9284	0.7636	0.7688	0.7663	0.7310
	0.6	0.8006	0.8516	0.8576	0.9098	0.7428	0.7457	0.7432	0.7021
	0.7	0.7572	0.8082	0.8152	0.8874	0.7121	0.7129	0.7112	0.6606
	0.8	0.7222	0.7702	0.7684	0.8668	0.6703	0.6681	0.6664	0.6070
	0.9	0.6832	0.7234	0.7206	0.8424	0.6159	0.6114	0.6115	0.5399
50	0.1	0.9386	0.9490	0.9544	0.9248	0.5598	0.5637	0.5630	0.5581
	0.2	0.9216	0.9386	0.9446	0.9292	0.5562	0.5593	0.5582	0.5505
	0.3	0.8912	0.9132	0.9204	0.9276	0.5496	0.5522	0.5507	0.5398
	0.4	0.8524	0.8802	0.8894	0.9188	0.5394	0.5413	0.5397	0.5252
	0.5	0.8076	0.8364	0.8472	0.9066	0.5250	0.5260	0.5241	0.5050
	0.6	0.7576	0.7944	0.8070	0.8866	0.5043	0.5043	0.5026	0.4786
	0.7	0.7132	0.7504	0.7564	0.8598	0.4755	0.4745	0.4726	0.4420
	0.8	0.6880	0.7184	0.7240	0.8498	0.4341	0.4320	0.4297	0.3932
	0.9	0.6706	0.6886	0.6914	0.8336	0.3741	0.3707	0.3687	0.3251
100	0.1	0.9368	0.9434	0.9470	0.9260	0.3934	0.3946	0.3942	0.3922
	0.2	0.8922	0.9066	0.9174	0.9288	0.3908	0.3919	0.3912	0.3880
	0.3	0.8308	0.8534	0.8720	0.9134	0.3860	0.3868	0.3859	0.3811
	0.4	0.7546	0.7818	0.8062	0.8780	0.3789	0.3794	0.3784	0.3718
	0.5	0.6662	0.6924	0.7102	0.8316	0.3685	0.3687	0.3677	0.3587
	0.6	0.5834	0.6140	0.6336	0.7860	0.3537	0.3537	0.3526	0.3408
	0.7	0.5274	0.5528	0.5708	0.7544	0.3314	0.3309	0.3296	0.3141
	0.8	0.5126	0.5320	0.5466	0.7364	0.2990	0.2980	0.2966	0.2765
	0.9	0.5524	0.5588	0.5788	0.7664	0.2456	0.2440	0.2421	0.2167
250	0.1	0.9094	0.9164	0.9298	0.9348	0.2479	0.2482	0.2480	0.2475
	0.2	0.8250	0.8392	0.8634	0.9072	0.2461	0.2463	0.2461	0.2451
	0.3	0.7090	0.7238	0.7544	0.8422	0.2428	0.2430	0.2426	0.2410
	0.4	0.5564	0.5778	0.6060	0.7404	0.2378	0.2379	0.2375	0.2352
	0.5	0.4136	0.4308	0.4600	0.6312	0.2306	0.2306	0.2302	0.2270
	0.6	0.3256	0.3394	0.3616	0.5540	0.2199	0.2199	0.2194	0.2149
	0.7	0.2730	0.2858	0.3062	0.5172	0.2047	0.2045	0.2039	0.1979
	0.8	0.2854	0.2946	0.3100	0.5362	0.1820	0.1817	0.1811	0.1729
	0.9	0.3710	0.3732	0.3958	0.6314	0.1444	0.1439	0.1431	0.1322

Table 4 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 5\%$ and $\omega = 5\sigma_\varepsilon$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9484	0.9618	0.9704	0.9640	0.8046	0.8140	0.8138	0.8071
	0.2	0.9212	0.9474	0.9562	0.9624	0.8036	0.8117	0.8112	0.8015
	0.3	0.8552	0.8940	0.9074	0.9366	0.8012	0.8086	0.8079	0.7955
	0.4	0.7830	0.8404	0.8552	0.9128	0.7966	0.8028	0.8016	0.7847
	0.5	0.6950	0.7580	0.7730	0.8604	0.7886	0.7933	0.7925	0.7698
	0.6	0.6034	0.6668	0.6724	0.7940	0.7763	0.7798	0.7793	0.7499
	0.7	0.5238	0.5942	0.5932	0.7532	0.7579	0.7607	0.7594	0.7193
	0.8	0.4398	0.5060	0.5034	0.6892	0.7308	0.7325	0.7312	0.6814
	0.9	0.4108	0.4630	0.4566	0.6442	0.6879	0.6870	0.6864	0.6234
50	0.1	0.9402	0.9502	0.9578	0.9554	0.5610	0.5639	0.5633	0.5605
	0.2	0.8784	0.9028	0.9168	0.9368	0.5592	0.5617	0.5609	0.5564
	0.3	0.7770	0.8156	0.8374	0.8902	0.5559	0.5580	0.5570	0.5503
	0.4	0.6664	0.7026	0.7198	0.8236	0.5510	0.5527	0.5516	0.5425
	0.5	0.5564	0.6018	0.6160	0.7534	0.5423	0.5433	0.5420	0.5286
	0.6	0.4378	0.4800	0.4934	0.6808	0.5315	0.5320	0.5308	0.5134
	0.7	0.3670	0.4126	0.4136	0.6112	0.5129	0.5125	0.5111	0.4876
	0.8	0.3292	0.3584	0.3686	0.5832	0.4828	0.4818	0.4799	0.4472
	0.9	0.3392	0.3570	0.3712	0.5798	0.4337	0.4318	0.4293	0.3857
100	0.1	0.9072	0.9174	0.9304	0.9302	0.3938	0.3947	0.3942	0.3932
	0.2	0.7910	0.8156	0.8534	0.8958	0.3928	0.3935	0.3929	0.3912
	0.3	0.5776	0.6090	0.6630	0.7624	0.3908	0.3914	0.3906	0.3879
	0.4	0.3880	0.4186	0.4650	0.6092	0.3875	0.3880	0.3870	0.3832
	0.5	0.2248	0.2454	0.2792	0.4500	0.3824	0.3827	0.3816	0.3761
	0.6	0.1288	0.1468	0.1642	0.3364	0.3744	0.3745	0.3734	0.3655
	0.7	0.0928	0.1058	0.1178	0.2756	0.3606	0.3604	0.3592	0.3474
	0.8	0.0828	0.0910	0.1024	0.2774	0.3380	0.3375	0.3361	0.3188
	0.9	0.1346	0.1418	0.1574	0.3720	0.2934	0.2923	0.2902	0.2647
250	0.1	0.8502	0.8624	0.9042	0.9190	0.2482	0.2484	0.2482	0.2479
	0.2	0.5756	0.5946	0.6850	0.7608	0.2474	0.2476	0.2473	0.2467
	0.3	0.2714	0.2894	0.3594	0.4838	0.2459	0.2461	0.2456	0.2447
	0.4	0.0924	0.1032	0.1364	0.2382	0.2435	0.2436	0.2431	0.2417
	0.5	0.0218	0.0240	0.0366	0.0910	0.2398	0.2399	0.2392	0.2373
	0.6	0.0048	0.0056	0.0076	0.0380	0.2341	0.2341	0.2334	0.2305
	0.7	0.0018	0.0020	0.0026	0.0258	0.2245	0.2244	0.2237	0.2192
	0.8	0.0028	0.0030	0.0036	0.0404	0.2076	0.2074	0.2067	0.1996
	0.9	0.0116	0.0122	0.0180	0.1150	0.1739	0.1735	0.1726	0.1613

Table 5 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 5\%$ and $\omega = -5\sigma_\varepsilon$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9570	0.9680	0.9738	0.9622	0.8053	0.8142	0.8138	0.8068
	0.2	0.9130	0.9374	0.9506	0.9602	0.8041	0.8122	0.8116	0.8017
	0.3	0.8670	0.9040	0.9098	0.9414	0.8013	0.8083	0.8074	0.7944
	0.4	0.7862	0.8378	0.8458	0.9084	0.7954	0.8015	0.8007	0.7834
	0.5	0.6960	0.7598	0.7618	0.8586	0.7885	0.7937	0.7925	0.7699
	0.6	0.5898	0.6588	0.6648	0.7924	0.7764	0.7805	0.7793	0.7504
	0.7	0.5226	0.5962	0.5986	0.7422	0.7568	0.7587	0.7575	0.7182
	0.8	0.4398	0.5078	0.5018	0.6804	0.7311	0.7321	0.7309	0.6819
	0.9	0.4034	0.4570	0.4516	0.6400	0.6882	0.6874	0.6866	0.6242
50	0.1	0.9382	0.9476	0.9540	0.9500	0.5606	0.5634	0.5629	0.5601
	0.2	0.8690	0.8934	0.9100	0.9328	0.5590	0.5615	0.5607	0.5563
	0.3	0.7916	0.8274	0.8444	0.8984	0.5561	0.5582	0.5572	0.5506
	0.4	0.6690	0.7128	0.7322	0.8276	0.5509	0.5525	0.5514	0.5420
	0.5	0.5486	0.5944	0.6114	0.7544	0.5433	0.5444	0.5432	0.5305
	0.6	0.4296	0.4716	0.4854	0.6644	0.5320	0.5327	0.5315	0.5139
	0.7	0.3598	0.3954	0.4076	0.6158	0.5129	0.5127	0.5115	0.4877
	0.8	0.3186	0.3534	0.3690	0.5770	0.4833	0.4824	0.4806	0.4481
	0.9	0.3288	0.3500	0.3616	0.5794	0.4338	0.4318	0.4292	0.3859
100	0.1	0.9112	0.9216	0.9350	0.9350	0.3938	0.3947	0.3943	0.3932
	0.2	0.7838	0.8090	0.8440	0.8914	0.3929	0.3937	0.3930	0.3913
	0.3	0.5724	0.6044	0.6604	0.7546	0.3909	0.3916	0.3907	0.3881
	0.4	0.3890	0.4184	0.4626	0.6118	0.3875	0.3879	0.3870	0.3832
	0.5	0.2306	0.2584	0.2844	0.4516	0.3823	0.3826	0.3815	0.3761
	0.6	0.1342	0.1490	0.1712	0.3340	0.3744	0.3745	0.3734	0.3652
	0.7	0.0872	0.0990	0.1120	0.2610	0.3613	0.3611	0.3600	0.3485
	0.8	0.0834	0.0926	0.1018	0.2792	0.3376	0.3370	0.3357	0.3185
	0.9	0.1202	0.1252	0.1424	0.3528	0.2943	0.2932	0.2912	0.2659
250	0.1	0.8540	0.8646	0.9080	0.9178	0.2482	0.2484	0.2482	0.2478
	0.2	0.5806	0.5980	0.6864	0.7636	0.2474	0.2476	0.2472	0.2466
	0.3	0.2586	0.2744	0.3464	0.4672	0.2460	0.2462	0.2457	0.2448
	0.4	0.0898	0.0962	0.1294	0.2270	0.2436	0.2437	0.2431	0.2418
	0.5	0.0258	0.0284	0.0376	0.0976	0.2399	0.2399	0.2393	0.2374
	0.6	0.0060	0.0064	0.0116	0.0432	0.2340	0.2340	0.2333	0.2304
	0.7	0.0034	0.0042	0.0042	0.0290	0.2245	0.2244	0.2237	0.2192
	0.8	0.0030	0.0034	0.0044	0.0348	0.2078	0.2076	0.2069	0.1999
	0.9	0.0138	0.0142	0.0208	0.1224	0.1740	0.1736	0.1727	0.1611

Table 6 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 10\%$ and $\omega = 3\sigma_\varepsilon$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9354	0.9508	0.9570	0.9336	0.8024	0.8134	0.8118	0.8022
	0.2	0.9034	0.9288	0.9406	0.9342	0.8001	0.8100	0.8081	0.7950
	0.3	0.8670	0.9074	0.9212	0.9404	0.7965	0.8050	0.8029	0.7861
	0.4	0.8192	0.8622	0.8802	0.9168	0.7892	0.7966	0.7942	0.7719
	0.5	0.7506	0.8120	0.8234	0.8868	0.7787	0.7843	0.7820	0.7531
	0.6	0.6730	0.7372	0.7424	0.8442	0.7618	0.7663	0.7636	0.7279
	0.7	0.6134	0.6812	0.6894	0.8134	0.7391	0.7418	0.7387	0.6941
	0.8	0.5564	0.6216	0.6286	0.7722	0.7081	0.7085	0.7060	0.6514
	0.9	0.5172	0.5710	0.5712	0.7344	0.6600	0.6583	0.6557	0.5903
50	0.1	0.9252	0.9396	0.9512	0.9296	0.5600	0.5633	0.5621	0.5582
	0.2	0.8860	0.9080	0.9266	0.9324	0.5586	0.5614	0.5598	0.5542
	0.3	0.7994	0.8294	0.8562	0.9060	0.5545	0.5570	0.5551	0.5469
	0.4	0.7060	0.7476	0.7812	0.8630	0.5487	0.5506	0.5483	0.5379
	0.5	0.5990	0.6480	0.6824	0.8036	0.5405	0.5419	0.5394	0.5248
	0.6	0.5122	0.5610	0.5784	0.7508	0.5261	0.5268	0.5243	0.5047
	0.7	0.4460	0.4880	0.5098	0.6856	0.5061	0.5060	0.5031	0.4780
	0.8	0.4084	0.4414	0.4600	0.6582	0.4740	0.4727	0.4699	0.4365
	0.9	0.3972	0.4196	0.4288	0.6320	0.4225	0.4201	0.4172	0.3740
100	0.1	0.9174	0.9292	0.9438	0.9330	0.3937	0.3948	0.3942	0.3926
	0.2	0.8184	0.8410	0.8762	0.9108	0.3922	0.3931	0.3921	0.3897
	0.3	0.6950	0.7226	0.7652	0.8408	0.3892	0.3900	0.3888	0.3851
	0.4	0.5520	0.5812	0.6272	0.7584	0.3843	0.3848	0.3834	0.3782
	0.5	0.3974	0.4288	0.4652	0.6322	0.3773	0.3776	0.3761	0.3690
	0.6	0.2882	0.3154	0.3496	0.5452	0.3662	0.3662	0.3647	0.3547
	0.7	0.2340	0.2546	0.2832	0.4902	0.3494	0.3491	0.3474	0.3338
	0.8	0.2238	0.2396	0.2572	0.4758	0.3226	0.3219	0.3201	0.3010
	0.9	0.2770	0.2852	0.3030	0.5418	0.2736	0.2723	0.2703	0.2448
250	0.1	0.8744	0.8830	0.9172	0.9270	0.2481	0.2484	0.2481	0.2476
	0.2	0.6870	0.7032	0.7792	0.8390	0.2470	0.2472	0.2467	0.2460
	0.3	0.4308	0.4514	0.5252	0.6490	0.2449	0.2451	0.2445	0.2432
	0.4	0.2088	0.2230	0.2796	0.4252	0.2418	0.2419	0.2412	0.2394
	0.5	0.0870	0.0958	0.1292	0.2538	0.2368	0.2369	0.2361	0.2337
	0.6	0.0366	0.0404	0.0524	0.1554	0.2293	0.2293	0.2285	0.2249
	0.7	0.0222	0.0236	0.0302	0.1186	0.2178	0.2177	0.2170	0.2117
	0.8	0.0208	0.0224	0.0280	0.1378	0.1985	0.1983	0.1975	0.1899
	0.9	0.0690	0.0696	0.0786	0.2706	0.1625	0.1621	0.1612	0.1497

Table 7 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 10\%$ and $\omega = -3\sigma_\varepsilon$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9404	0.9550	0.9580	0.9374	0.8026	0.8132	0.8117	0.8024
	0.2	0.9086	0.9292	0.9444	0.9434	0.8006	0.8106	0.8087	0.7958
	0.3	0.8694	0.9050	0.9130	0.9352	0.7960	0.8047	0.8023	0.7852
	0.4	0.8100	0.8566	0.8736	0.9146	0.7894	0.7967	0.7937	0.7719
	0.5	0.7602	0.8140	0.8236	0.8920	0.7780	0.7835	0.7811	0.7533
	0.6	0.6980	0.7586	0.7678	0.8558	0.7611	0.7654	0.7625	0.7266
	0.7	0.6238	0.6924	0.6976	0.8202	0.7387	0.7412	0.7373	0.6928
	0.8	0.5670	0.6312	0.6374	0.7760	0.7069	0.7071	0.7043	0.6505
	0.9	0.5146	0.5706	0.5700	0.7374	0.6609	0.6592	0.6558	0.5904
50	0.1	0.9308	0.9438	0.9538	0.9332	0.5601	0.5633	0.5623	0.5586
	0.2	0.8792	0.8990	0.9204	0.9296	0.5581	0.5609	0.5594	0.5535
	0.3	0.7992	0.8312	0.8576	0.9036	0.5548	0.5572	0.5553	0.5473
	0.4	0.7038	0.7434	0.7716	0.8568	0.5490	0.5508	0.5485	0.5376
	0.5	0.6018	0.6506	0.6742	0.8030	0.5402	0.5417	0.5394	0.5247
	0.6	0.5106	0.5578	0.5900	0.7466	0.5262	0.5268	0.5242	0.5049
	0.7	0.4416	0.4850	0.5098	0.6938	0.5071	0.5069	0.5038	0.4787
	0.8	0.3954	0.4298	0.4426	0.6442	0.4747	0.4736	0.4708	0.4374
	0.9	0.3906	0.4074	0.4200	0.6348	0.4235	0.4211	0.4184	0.3756
100	0.1	0.9092	0.9198	0.9396	0.9292	0.3936	0.3947	0.3941	0.3925
	0.2	0.8212	0.8426	0.8744	0.9048	0.3921	0.3930	0.3921	0.3895
	0.3	0.6926	0.7208	0.7700	0.8498	0.3891	0.3899	0.3887	0.3850
	0.4	0.5324	0.5636	0.6170	0.7438	0.3845	0.3851	0.3838	0.3786
	0.5	0.4032	0.4330	0.4758	0.6436	0.3773	0.3776	0.3761	0.3689
	0.6	0.2986	0.3256	0.3538	0.5424	0.3663	0.3664	0.3648	0.3550
	0.7	0.2276	0.2512	0.2768	0.4962	0.3495	0.3492	0.3476	0.3341
	0.8	0.2166	0.2332	0.2538	0.4830	0.3220	0.3212	0.3196	0.3006
	0.9	0.2742	0.2794	0.2986	0.5388	0.2743	0.2730	0.2710	0.2454
250	0.1	0.8782	0.8892	0.9274	0.9270	0.2481	0.2484	0.2481	0.2476
	0.2	0.6698	0.6876	0.7676	0.8360	0.2470	0.2473	0.2468	0.2460
	0.3	0.4206	0.4426	0.5220	0.6428	0.2450	0.2452	0.2445	0.2433
	0.4	0.2100	0.2260	0.2816	0.4260	0.2417	0.2418	0.2411	0.2394
	0.5	0.0870	0.0952	0.1258	0.2606	0.2368	0.2368	0.2361	0.2335
	0.6	0.0384	0.0428	0.0540	0.1606	0.2293	0.2293	0.2286	0.2250
	0.7	0.0228	0.0252	0.0328	0.1268	0.2175	0.2174	0.2166	0.2114
	0.8	0.0266	0.0278	0.0352	0.1412	0.1983	0.1980	0.1973	0.1897
	0.9	0.0756	0.0768	0.0890	0.2692	0.1623	0.1619	0.1610	0.1495

Table 8 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 10\%$ and $\omega = 5\sigma_\varepsilon$

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9484	0.9602	0.9618	0.9568	0.8032	0.8107	0.8092	0.8046
	0.2	0.8882	0.9154	0.9336	0.9452	0.8030	0.8097	0.8081	0.8019
	0.3	0.8018	0.8490	0.8750	0.9146	0.8020	0.8083	0.8060	0.7974
	0.4	0.6686	0.7356	0.7748	0.8394	0.7999	0.8054	0.8031	0.7921
	0.5	0.5226	0.5932	0.6358	0.7458	0.7964	0.8012	0.7988	0.7844
	0.6	0.3980	0.4696	0.4960	0.6524	0.7902	0.7949	0.7918	0.7720
	0.7	0.3120	0.3754	0.3904	0.5650	0.7790	0.7826	0.7796	0.7527
	0.8	0.2530	0.3092	0.3196	0.5074	0.7611	0.7640	0.7605	0.7230
	0.9	0.2062	0.2468	0.2596	0.4450	0.7362	0.7381	0.7343	0.6845
50	0.1	0.9212	0.9340	0.9566	0.9482	0.5604	0.5625	0.5612	0.5592
	0.2	0.7968	0.8274	0.8890	0.9134	0.5601	0.5620	0.5603	0.5573
	0.3	0.6134	0.6564	0.7420	0.8100	0.5592	0.5611	0.5589	0.5551
	0.4	0.4310	0.4698	0.5430	0.6576	0.5574	0.5589	0.5565	0.5514
	0.5	0.2736	0.3126	0.3630	0.5078	0.5542	0.5555	0.5526	0.5454
	0.6	0.1656	0.1922	0.2242	0.3808	0.5488	0.5497	0.5465	0.5361
	0.7	0.0972	0.1166	0.1334	0.2842	0.5400	0.5405	0.5375	0.5225
	0.8	0.0678	0.0860	0.0996	0.2462	0.5230	0.5229	0.5193	0.4966
	0.9	0.0868	0.0966	0.1106	0.2858	0.4875	0.4867	0.4824	0.4459
100	0.1	0.8754	0.8914	0.9390	0.9338	0.3939	0.3947	0.3939	0.3930
	0.2	0.6768	0.7028	0.8086	0.8512	0.3936	0.3943	0.3930	0.3917
	0.3	0.4132	0.4372	0.5640	0.6538	0.3927	0.3932	0.3918	0.3901
	0.4	0.1946	0.2138	0.3166	0.4336	0.3909	0.3914	0.3895	0.3869
	0.5	0.0710	0.0800	0.1260	0.2342	0.3885	0.3889	0.3868	0.3831
	0.6	0.0250	0.0302	0.0536	0.1352	0.3837	0.3839	0.3816	0.3762
	0.7	0.0088	0.0100	0.0164	0.0782	0.3763	0.3764	0.3739	0.3656
	0.8	0.0050	0.0060	0.0094	0.0656	0.3609	0.3607	0.3583	0.3448
	0.9	0.0132	0.0150	0.0222	0.1198	0.3269	0.3261	0.3234	0.3005
250	0.1	0.7864	0.7992	0.9212	0.9220	0.2483	0.2485	0.2480	0.2477
	0.2	0.3898	0.4066	0.6232	0.6916	0.2480	0.2482	0.2474	0.2470
	0.3	0.0880	0.0948	0.2096	0.3006	0.2474	0.2475	0.2465	0.2459
	0.4	0.0100	0.0108	0.0322	0.0710	0.2463	0.2464	0.2453	0.2445
	0.5	0.0010	0.0010	0.0020	0.0112	0.2445	0.2445	0.2433	0.2420
	0.6	0.0000	0.0000	0.0000	0.0014	0.2415	0.2416	0.2401	0.2382
	0.7	0.0000	0.0000	0.0000	0.0010	0.2358	0.2358	0.2344	0.2314
	0.8	0.0000	0.0000	0.0000	0.0004	0.2249	0.2248	0.2235	0.2181
	0.9	0.0000	0.0000	0.0002	0.0050	0.1988	0.1985	0.1973	0.1870

Table 9 The estimated coverage probabilities and lengths of confidence intervals CI_W, CI_{RW}, CI_{RDW} and CI_{IRDW} when $p = 10\%$ and $\omega = -5\sigma_\varepsilon$.

n	ϕ	Coverage probabilities				Lengths			
		W	RW	RDW	IRDW	W	RW	RDW	IRDW
25	0.1	0.9494	0.9630	0.9672	0.9620	0.8042	0.8114	0.8103	0.8060
	0.2	0.8956	0.9236	0.9356	0.9470	0.8036	0.8101	0.8085	0.8025
	0.3	0.7950	0.8426	0.8808	0.9146	0.8024	0.8087	0.8069	0.7989
	0.4	0.6864	0.7500	0.7830	0.8464	0.8000	0.8055	0.8028	0.7917
	0.5	0.5244	0.5912	0.6300	0.7538	0.7964	0.8016	0.7989	0.7844
	0.6	0.4102	0.4800	0.5006	0.6524	0.7906	0.7948	0.7920	0.7728
	0.7	0.3246	0.3878	0.4050	0.5782	0.7783	0.7817	0.7783	0.7515
	0.8	0.2456	0.2980	0.3084	0.4954	0.7627	0.7655	0.7615	0.7245
	0.9	0.2100	0.2534	0.2660	0.4520	0.7338	0.7357	0.7315	0.6817
50	0.1	0.9272	0.9370	0.9594	0.9518	0.5608	0.5628	0.5616	0.5596
	0.2	0.7976	0.8284	0.8838	0.9072	0.5600	0.5620	0.5602	0.5573
	0.3	0.6264	0.6674	0.7422	0.8148	0.5590	0.5608	0.5587	0.5548
	0.4	0.4282	0.4698	0.5532	0.6752	0.5573	0.5588	0.5563	0.5511
	0.5	0.2678	0.3036	0.3598	0.5126	0.5542	0.5555	0.5529	0.5455
	0.6	0.1694	0.1974	0.2312	0.3894	0.5487	0.5498	0.5466	0.5358
	0.7	0.1006	0.1216	0.1500	0.2988	0.5399	0.5405	0.5370	0.5220
	0.8	0.0664	0.0780	0.0988	0.2588	0.5226	0.5225	0.5185	0.4951
	0.9	0.0872	0.0972	0.1150	0.2810	0.4873	0.4863	0.4818	0.4459
100	0.1	0.8824	0.8980	0.9416	0.9354	0.3940	0.3947	0.3939	0.3930
	0.2	0.6756	0.7070	0.8144	0.8568	0.3936	0.3942	0.3930	0.3918
	0.3	0.4158	0.4416	0.5712	0.6638	0.3926	0.3931	0.3916	0.3898
	0.4	0.1986	0.2168	0.3060	0.4284	0.3911	0.3915	0.3897	0.3872
	0.5	0.0778	0.0882	0.1258	0.2270	0.3885	0.3889	0.3869	0.3834
	0.6	0.0260	0.0318	0.0466	0.1264	0.3839	0.3841	0.3819	0.3766
	0.7	0.0086	0.0108	0.0184	0.0742	0.3762	0.3762	0.3739	0.3657
	0.8	0.0070	0.0084	0.0110	0.0722	0.3606	0.3603	0.3580	0.3446
	0.9	0.0182	0.0202	0.0278	0.1248	0.3268	0.3260	0.3234	0.3004
250	0.1	0.7900	0.8004	0.9164	0.9166	0.2483	0.2485	0.2480	0.2477
	0.2	0.3878	0.4036	0.6148	0.6814	0.2480	0.2481	0.2474	0.2470
	0.3	0.0884	0.0968	0.2144	0.3026	0.2473	0.2475	0.2465	0.2459
	0.4	0.0122	0.0134	0.0354	0.0782	0.2463	0.2464	0.2452	0.2444
	0.5	0.0008	0.0008	0.0040	0.0132	0.2444	0.2445	0.2432	0.2419
	0.6	0.0000	0.0000	0.0006	0.0024	0.2413	0.2414	0.2400	0.2381
	0.7	0.0000	0.0000	0.0000	0.0008	0.2358	0.2358	0.2344	0.2312
	0.8	0.0000	0.0000	0.0000	0.0012	0.2246	0.2245	0.2233	0.2180
	0.9	0.0000	0.0000	0.0000	0.0060	0.1986	0.1983	0.1970	0.1865

An Empirical Application

To illustrate the empirical application of the confidence intervals which have been proposed in the previous section, we have used the yearly real exchange rates between the USA and Mexico from 1970 to 2009 (base year is 2005). The series giving a total of 40 observations was collected from the Economic Research Service, United States Department of Agriculture. The time series plot, the sample autocorrelation function (ACF) and the sample partial autocorrelation function (PACF), as shown in Figures 1 and 2, suggest that an AR(1) model is suitable for this series. We detected the additive outliers of this series by using an iterative detecting procedure proposed by Chang et al. (1988) via the R statistical software (see, for example, Cryer and Chan, 2008, pp.257-259 and pp.455). We found an additive outlier at $t = 12$ (year 1981) and we also compute the estimates and the confidence intervals for an autoregressive parameter which they are displayed in Table 10. As can be seen from Table 10, the CI_{IRDW} provides the length shorter than those of the CI_W, CI_{RW} and CI_{RDW}. The real application in this section confirms that the CI_{IRDW} is much better than the other confidence intervals.

Discussions and Conclusions

In this paper, we have compared the interval estimation of the parameter for a Gaussian first-order autoregressive model with additive outliers. The confidence intervals considered in this study are based on the weighted symmetric estimator ($\hat{\phi}_W$), the recursive mean adjusted weighted symmetric estimator ($\hat{\phi}_{RW}$), the recursive median adjusted weighted symmetric estimator ($\hat{\phi}_{RDW}$), and the improved recursive median adjusted weighted symmetric estimator ($\hat{\phi}_{IRDW}$). Using the simulations, we compare the coverage probability and length of the confidence intervals. For all sample sizes, the percentages of additive outliers and the magnitudes of the additive outliers, the confidence interval based on the estimator $\hat{\phi}_{IRDW}$ assigns more value to coverage probabilities than the confidence intervals based on the estimators $\hat{\phi}_W$, $\hat{\phi}_{RW}$ and $\hat{\phi}_{RDW}$ in almost all ϕ values. So, we can conclude that the confidence interval based on the estimator $\hat{\phi}_{IRDW}$ is superior to the other confidence intervals in terms of the coverage probabilities and lengths when autoregressive time series contaminated with additive outliers.

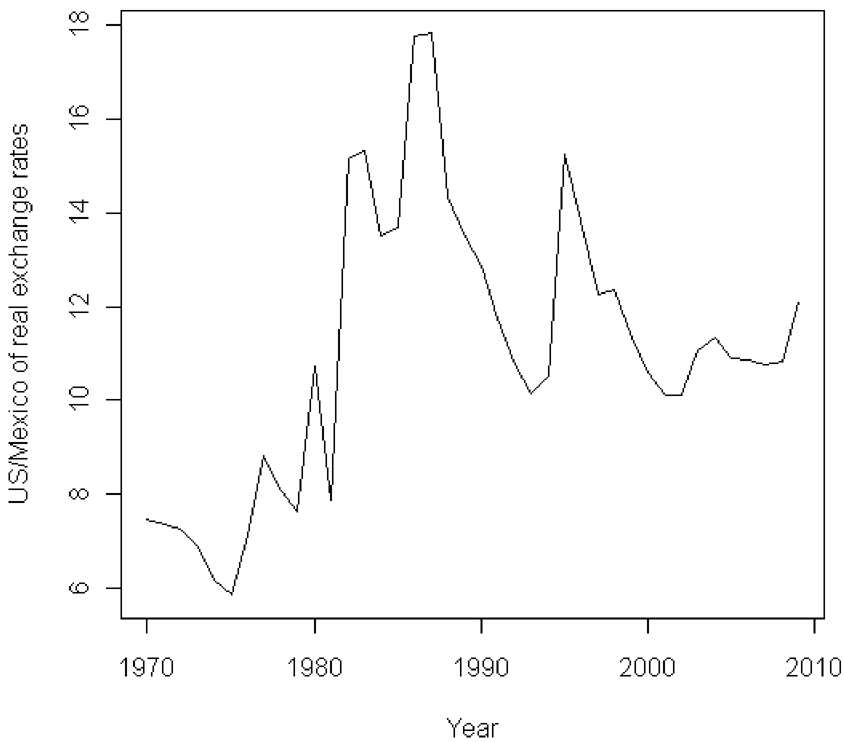

Figure 1 The US/Mexico of real exchange rates; annual from 1970 to 2009

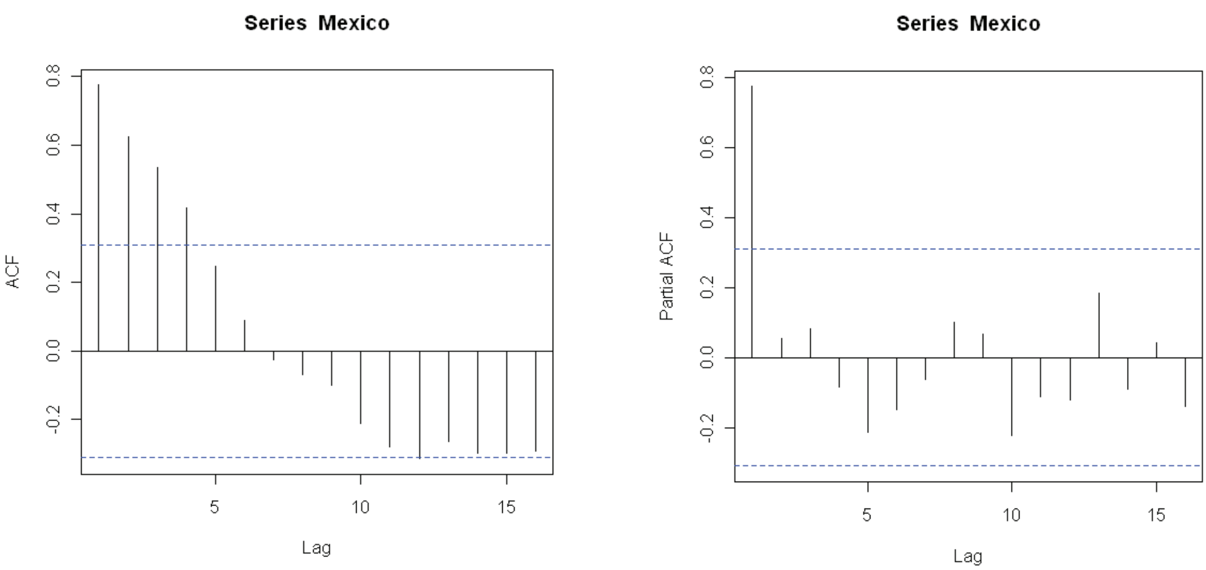

Figure 2 ACF and PACF of the US/Mexico of real exchange rates

Table 10 The estimates and the confidence intervals for the first-order autoregressive parameter of the US/Mexico of real exchange rates series

Methods	Estimates	Confidence intervals		Lengths
		Lower limits	Upper limits	
W	0.7874	0.5964	0.9784	0.3820
RW	0.7821	0.5911	0.9731	0.3820
RDW	0.8378	0.6738	1.0017	0.3279
IRDW	0.8971	0.7663	1.0278	0.2614

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