A Comparison on Parameter Estimation in the First-Order Autoregressive Process Having Non-Normal Errors and Additive Outliers

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Abstract

This paper presents an estimator for an unknown mean AR(1) process having non-normal errors and additive outliers and compares this estimator with other existing estimators. We apply the double recursive median adjustment to the weighted symmetric estimator. Five estimators are considered as follows: the weighted symmetric estimator ($\hat{\rho}_W$), the Guo's estimator ($\hat{\rho}_G$), the recursive mean adjusted weighted symmetric estimator ($\hat{\rho}_R$), the recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{RMD}$) and the double recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{DRMD}$). The mean square error (MSE) of estimators is compared via simulation studies. Simulation results have shown that the proposed estimator, $\hat{\rho}_{DRMD}$, provides a MSE lower than those of the other estimators when ρ is in the range of about 0.3 to 0.6, and the errors have the t_3 , exponential(1)-1 and uniform(-1,1) distribution. For the 0.9N(0,1)+0.1N(0,100) errors, the $\hat{\rho}_{DRMD}$ outperforms the others in terms of the MSE when ρ is close to one.

Keywords: parameter estimation, autoregressive process, non-normal errors, additive outliers

1. Introduction

Time series observations are sometimes influenced by interrupting phenomena, such as strikes, outbreak of war, sudden political or economic crises, unexpected hot or cold waves, and even unnoticed errors of typing or recording. Such observations are usually referred to as *outliers*. Because outliers are known to wreck havoc on the parameter estimation, it is therefore important to have procedures that will deal such outliers effects. The detection of time series outliers was first studied by Fox (1972), who introduced two statistical models for times series contaminated by outliers, namely, additive outliers (AO) and innovations outliers (IO). Additive outlier corresponds to the situation in which a gross error of observation or recording error affects a

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single observation (Fox, 1972). Furthermore, innovations outlier affects not only the particular observation but also subsequent observations (Fox, 1972). A time series that does not contain any outliers is called an outlier-free series.

Suppose an outlier-free time series $\{X_t\ ; t = 2,3,...,n\}$ follows AR(1) process:

$$X_t \quad = \quad \mu + \rho(X_{t-1} - \mu) + e_t, \qquad (1)$$

where μ is the population mean, ρ is an autoregressive parameter, $\rho \in$ (-1,1), e_t are unobservable independent errors and identically $N(0, \sigma_e^2)$ distributed. For ρ =1, the model (1) is called the random walk model, otherwise it is called a stationary AR(1) process when $\rho < 1$. For ρ close to one or near a non-stationary process, the mean and variance of this model change over time. Let the observed time series be denoted by $\{Y_t\}$. In the simple case when $\{X_t\}$ has a single additive outlier at time point $T \ \ (1 < T < n)$, model (1) can be modified as

$$Y_t \quad = \quad X_t + \delta I_t^{(T)}, \qquad (2)$$

where δ represents the magnitude of the additive outlier effect and $I_t^{(T)}$ is an indicator variable such that

$$I_t^{(T)} \quad = \quad \begin{cases} 1\ , & t = T, \\ 0\ , & t \neq T. \end{cases}$$

It is well-known that the ordinary least squares (OLS) estimator of ρ suffers from excessive bias problems; especially for ρ close to one (see for example, Marriott and Pope, 1954; Shaman and Stine, 1988; Newbold and Agiakloglou, 1993). In the case of additive outliers, the OLS estimator of ρ is not robust (Barnett and Lewis, 1984). Therefore, statisticians have suggested methods to reduce the bias. Denby and Martin (1979) suggested a generalized M-estimator to estimate the first-order autoregressive parameter. Even though Denby and Martin's M-estimator is robust, the calculation of this estimator is very complicated. Park and Fuller (1995) proposed the easy-to-compute estimator, namely, the weighted symmetric estimator of ρ, which is denoted by $\hat{\rho}_W$. In addition, Guo (2000) presented the simple and robust estimator of ρ, which is denoted by $\hat{\rho}_G$. So and Shin (1999) applied recursive mean adjustment in the OLS estimator (abbreviated, ROLS) and they concluded that the mean square error of the ROLS estimator, which is denoted by $\hat{\rho}_{ROLS}$, is smaller than the OLS estimator for $\rho \in (0,1)$. They also showed that the $\hat{\rho}_{ROLS}$ estimator has a coverage probability which is close to the nominal value. Niwitpong (2007) applied the recursive mean adjustment to the weighted symmetric estimator of Park and Fuller (1995) (abbreviated, R). The recent work of Panichkitkosolkul (2010) has proposed an estimator for an unknown mean Gaussian AR(1) process with additive outliers by applying the recursive median adjustment to the weighted symmetric estimator (abbreviated, RMD). He found that the $\hat{\rho}_{RMD}$ estimator provides mean square error lower than those of $\hat{\rho}_W$ and $\hat{\rho}_R$ for almost all situations. We, therefore, apply the double recursive median adjustment to the weighted symmetric estimator (abbreviated, DRMD) for model (1) when there are additive outliers in time series data. Because the outliers do not affect the median values, we replace the recursive mean adjustment with the improved recursive median adjustment to the weighted symmetric estimator. Furthermore, in practice, we cannot know the distribution of error, e_t in model (1), i.e., the distribution of e_t may not be normal distribution. Thus, our aim in this paper is to compare five estimators, $\hat{\rho}_W$, $\hat{\rho}_G$, $\hat{\rho}_R$, $\hat{\rho}_{RMD}$ and $\hat{\rho}_{DRMD}$, in terms of mean square error (MSE) of estimators when there are additive outliers in time series data and the error distributions are non-normal distribution.

Following this introduction, this paper is organized into three main sections. In Section 2, the details of all estimators $\hat{\rho}_W$, $\hat{\rho}_G$, $\hat{\rho}_R$, $\hat{\rho}_{RMD}$ and $\hat{\rho}_{DRMD}$ are described. Simulation results obtained from simulation studies are shown in Section 3. Finally, a discussion of the results and conclusions are presented in Section 4.

2. Methodology

Park and Fuller (1995) proposed the weighted symmetric estimator of ρ given by

$$\hat{\rho}_W = \frac{\sum_{t=2}^{n}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})}{\sum_{t=3}^{n}(Y_{t-1}-\bar{Y})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\bar{Y})^2} \quad (3)$$

The Guo's estimator is defined as follows:

$$\hat{\rho}_G = \text{median}(A_i), \quad i=2,3,\ldots,n, \quad (4)$$

where $A_i = Y_i / Y_{i-1}$. Guo (2000) also showed that the estimator $\hat{\rho}_G$ is unbiased.

Niwitpong (2007) replaces $\bar{Y}$ by $\bar{Y}_t = \frac{1}{t}\sum_{i=1}^{t} Y_i$ in (3). The estimator of ρ obtained as a result of this recursive mean adjustment is

$$\hat{\rho}_R = \frac{\sum_{t=2}^{n}(Y_t-\bar{Y}_t)(Y_{t-1}-\bar{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1}-\bar{Y}_{t-1})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\bar{Y}_t)^2} \quad (5)$$

When there are outliers in time series data, it affects the recursive mean $\bar{Y}_t$ in (5). Panichkitkosolkul (2010) replaced the recursive mean in (5) by the recursive median. The estimator of ρ obtained as a result of the recursive median adjustment is

$$\hat{\rho}_{RMD} = \frac{\sum_{t=2}^{n}(Y_t-\tilde{Y}_t)(Y_{t-1}-\tilde{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1}-\tilde{Y}_{t-1})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\tilde{Y}_t)^2}, \quad (6)$$

Where $\tilde{Y}_t = \text{median}(Y_1, Y_2, \ldots, Y_t)$.

The effect of outliers to an estimator of ρ in model (1) can be reduced by using the double recursive median adjustment. The proposed recursive median values are derived from computing the double recursive median. Therefore, the recursive median in (6) is replaced by the double recursive median. The proposed estimator of ρ obtained as a result of this double recursive median adjustment is given by

$$\hat{\rho}_{DRMD} = \frac{\sum_{t=2}^{n}(Y_t-\ddot{Y}_t)(Y_{t-1}-\ddot{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1}-\ddot{Y}_{t-1})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\ddot{Y}_t)^2}, \quad (7)$$

where $\ddot{Y}_t = \text{median}(\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_t)$ and $\tilde{Y}_t = \text{median}(Y_1, Y_2, \ldots, Y_t)$.

In the next section, we present the Monte Carlo simulation results to estimate the mean square error (MSE) of these estimators, $\hat{\rho}_W$, $\hat{\rho}_G$, $\hat{\rho}_R$, $\hat{\rho}_{RMD}$ and $\hat{\rho}_{DRMD}$.

3. Simulation results

In this section, the results of using the simulation to investigate the estimated mean square error (MSE) of estimators (3) to (7) described in the previous section are reported. Time series from an unknown mean AR(1) process having non-normal errors and additive outliers are generated. The following parameter values were used; (μ, σ_e^2) = (0, 1); ρ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9; sample sizes n = 25, 50, 100 and 250; The error distributions were t_3, exponential(1)-1, uniform(-1,1) and 0.9N(0,1)+0.1N(0,100); the magnitude of the AOs effect $\delta = 3\sigma_e$; the percentage of additive outliers p =10 %. All simulations were performed using programs written in the R statistical software (The R Development Core Team, 2012a, 2012b) with the number of simulation runs, M= 5,000. In addition, the additive outliers occurred randomly. Simulation results are summarized in Table 1 for the four error distributions. The following information is presented here: the estimated mean square error (MSE) of all estimators, $\hat{\rho}_W$, $\hat{\rho}_G$, $\hat{\rho}_R$, $\hat{\rho}_{RMD}$ and $\hat{\rho}_{DRMD}$.

We begin with the results for the t_3 errors. The MSE of the $\hat{\rho}_W$ is larger than the MSEs of the other estimators. These values decrease as sample sizes get larger. The $\hat{\rho}_W$ performs well for $n \geq 50$. The $\hat{\rho}_{RMD}$ provides the lowest MSE when parameters are small (ρ = 0.1 and/or 0.2) for all sample sizes. On the other hand, the $\hat{\rho}_{DRMD}$ provides the lowest MSE when ρ = 0.3, 0.4, 0.5, 0.6 and 0.7 for small and moderate samples (n = 25, 50 and 100) and when ρ = 0.2, 0.3 and 0.4 for large samples (n = 250). Additionally, the $\hat{\rho}_G$ outperforms the others when ρ is close to one for all sample sizes. For the rest, the MSE of the $\hat{\rho}_{RMD}$ is less than that of $\hat{\rho}_R$ and $\hat{\rho}_W$ for almost all situations.

For the exponential(1)-1 errors, when ρ is equal to 0.1 and 0.2, the MSE of the $\hat{\rho}_{RMD}$ are lowest for all sample sizes. Further, the MSE of the $\hat{\rho}_{DRMD}$ is lower than those of the others when ρ = 0.3, 0.4, 0.5 and/or 0.6 for all sample sizes. When an autoregressive parameter value is close to unity, the $\hat{\rho}_G$ does perform better than other estimators in the sense of the MSE.

For the uniform(-1,1) errors, the $\hat{\rho}_{RMD}$ performs well with respect to the MSE criterion when the autoregressive parameter value is small (ρ = 0.1) and $n \leq 50$. On the contrary, the $\hat{\rho}_{DRMD}$ predominates the others for almost all scenarios when $n \geq 100$.

For the 0.9N(0,1)+0.1N(0,100) errors, the $\hat{\rho}_{RMD}$ gives lower MSEs than its competitors for $\rho \leq 0.5$ and small samples (n = 25 and 50). Furthermore, when $n \geq 100$, the MSE of the $\hat{\rho}_{RMD}$ performs very well with respect to other four estimators for almost all of parameter values except an autoregressive parameter value approaches one. In addition, the $\hat{\rho}_{DRMD}$ provides the lowest MSE when $\rho \to 1$.

Table 1. The estimated mean square error (MSE) of $\hat{\rho}_W$, $\hat{\rho}_G$, $\hat{\rho}_R$, $\hat{\rho}_{RMD}$ and $\hat{\rho}_{DRMD}$ when $e_t \sim t_3$, exponential(1)-1, uniform(-1,1) and 0.9N(0,1)+0.1N(0,100).

n	ρ	t_3 errors					exponential(1)-1 errors				
		W	G	R	RMD	DRMD	W	G	R	RMD	DRMD
25	0.1	0.043	0.071	0.040	**0.036**	0.042	0.043	0.105	0.038	**0.034**	0.042
	0.2	0.048	0.070	0.041	**0.038**	0.038	0.055	0.103	0.047	**0.035**	0.036
	0.3	0.058	0.070	0.049	0.046	**0.041**	0.073	0.103	0.062	0.046	**0.040**
	0.4	0.067	0.071	0.056	0.054	**0.042**	0.093	0.099	0.079	0.063	**0.047**
	0.5	0.076	0.068	0.063	0.061	**0.044**	0.118	0.095	0.101	0.084	**0.058**
	0.6	0.091	0.063	0.076	0.075	**0.050**	0.143	0.089	0.124	0.112	**0.076**
	0.7	0.096	0.056	0.081	0.080	**0.051**	0.168	**0.080**	0.148	0.139	0.090
	0.8	0.103	**0.044**	0.089	0.087	0.055	0.173	**0.061**	0.153	0.151	0.097
	0.9	0.105	**0.031**	0.093	0.093	0.055	0.184	**0.047**	0.166	0.165	0.105
50	0.1	0.022	0.033	0.020	**0.019**	0.021	0.024	0.053	0.022	**0.021**	0.027
	0.2	0.025	0.032	0.023	0.021	**0.021**	0.034	0.055	0.030	**0.020**	0.022
	0.3	0.031	0.034	0.027	0.025	**0.021**	0.048	0.057	0.043	0.027	**0.023**
	0.4	0.038	0.037	0.033	0.031	**0.024**	0.065	0.055	0.059	0.041	**0.031**
	0.5	0.042	0.036	0.037	0.036	**0.025**	0.082	0.056	0.074	0.056	**0.040**
	0.6	0.047	0.035	0.042	0.041	**0.027**	0.099	0.054	0.091	0.076	**0.051**
	0.7	0.050	0.031	0.045	0.044	**0.027**	0.107	**0.047**	0.099	0.089	0.057
	0.8	0.048	**0.024**	0.044	0.042	0.025	0.108	**0.039**	0.101	0.094	0.060
	0.9	0.042	**0.015**	0.040	0.039	0.022	0.097	**0.028**	0.091	0.089	0.055
100	0.1	0.010	0.015	0.010	**0.009**	0.010	0.013	0.029	0.013	**0.012**	0.017
	0.2	0.014	0.016	0.013	0.012	**0.011**	0.020	0.030	0.019	**0.010**	0.012
	0.3	0.017	0.018	0.016	0.015	**0.012**	0.032	0.030	0.030	0.016	**0.014**
	0.4	0.021	0.019	0.020	0.019	**0.014**	0.045	0.033	0.043	0.025	**0.020**
	0.5	0.024	0.019	0.023	0.021	**0.015**	0.060	0.031	0.057	0.039	**0.029**
	0.6	0.027	0.018	0.025	0.024	**0.016**	0.069	**0.030**	0.066	0.052	0.037
	0.7	0.027	0.017	0.025	0.025	**0.015**	0.073	**0.027**	0.070	0.060	0.040
	0.8	0.024	**0.012**	0.023	0.022	0.013	0.068	**0.022**	0.065	0.060	0.038
	0.9	0.017	**0.006**	0.017	0.016	0.009	0.050	**0.015**	0.048	0.046	0.027
250	0.1	0.005	0.006	0.004	**0.004**	0.004	0.006	0.012	0.008	**0.006**	0.010
	0.2	0.007	0.007	0.006	0.006	**0.005**	0.013	0.013	0.012	**0.004**	0.006
	0.3	0.009	0.008	0.009	0.008	**0.006**	0.022	0.013	0.022	0.008	**0.007**
	0.4	0.012	0.009	0.012	0.011	**0.008**	0.034	0.015	0.033	0.016	**0.013**
	0.5	0.015	**0.010**	0.014	0.014	0.010	0.045	0.016	0.044	0.027	**0.022**
	0.6	0.015	**0.009**	0.015	0.014	0.010	0.053	**0.016**	0.052	0.038	0.030
	0.7	0.015	**0.008**	0.015	0.014	0.010	0.054	**0.015**	0.053	0.044	0.033
	0.8	0.012	**0.006**	0.012	0.011	0.007	0.046	**0.012**	0.046	0.041	0.029
	0.9	0.007	**0.003**	0.007	0.006	0.004	0.026	**0.007**	0.025	0.024	0.015

Bold font represents the lowest MSE.

Table 1. (continued)

n	ρ	uniform(-1,1) errors					0.9N(0,1)+0.1N(0,100) errors				
		W	G	R	RMD	DRMD	W	G	R	RMD	DRMD
25	0.1	0.045	0.098	0.040	**0.035**	0.036	0.042	0.096	0.039	**0.038**	0.053
	0.2	0.066	0.102	0.057	0.049	**0.041**	0.042	0.093	0.038	**0.036**	0.046
	0.3	0.101	0.108	0.087	0.076	**0.058**	0.044	0.089	0.038	**0.036**	0.041
	0.4	0.138	0.118	0.120	0.110	**0.082**	0.045	0.085	0.038	**0.036**	0.037
	0.5	0.190	0.128	0.169	0.157	**0.115**	0.044	0.076	0.037	0.035	**0.033**
	0.6	0.236	**0.124**	0.211	0.200	0.146	0.049	0.069	0.040	0.039	**0.030**
	0.7	0.286	**0.121**	0.258	0.248	0.179	0.048	0.058	0.040	0.040	**0.027**
	0.8	0.326	**0.112**	0.295	0.286	0.205	0.051	0.050	0.043	0.043	**0.025**
	0.9	0.347	**0.082**	0.315	0.308	0.215	0.053	0.030	0.046	0.048	**0.028**
50	0.1	0.027	0.045	0.024	**0.020**	0.021	0.020	0.049	0.019	**0.019**	0.028
	0.2	0.045	0.049	0.041	0.032	**0.026**	0.020	0.049	0.019	**0.019**	0.027
	0.3	0.076	0.060	0.069	0.057	**0.044**	0.020	0.044	0.018	**0.018**	0.023
	0.4	0.113	0.072	0.105	0.089	**0.068**	0.020	0.042	0.018	**0.017**	0.021
	0.5	0.155	**0.079**	0.145	0.128	0.097	0.019	0.039	0.018	**0.017**	0.018
	0.6	0.196	**0.088**	0.184	0.168	0.128	0.018	0.035	0.016	0.016	**0.015**
	0.7	0.227	**0.083**	0.214	0.200	0.149	0.018	0.027	0.016	0.016	**0.012**
	0.8	0.247	**0.069**	0.234	0.222	0.159	0.016	0.021	0.015	0.015	**0.010**
	0.9	0.235	**0.051**	0.222	0.215	0.147	0.015	0.013	0.014	0.014	**0.008**
100	0.1	0.016	0.021	0.015	**0.012**	0.012	0.010	0.025	0.010	**0.010**	0.015
	0.2	0.033	0.026	0.031	0.023	**0.018**	0.010	0.023	0.009	**0.009**	0.013
	0.3	0.061	**0.036**	0.058	0.046	0.037	0.010	0.023	0.010	**0.010**	0.013
	0.4	0.092	**0.045**	0.088	0.073	0.058	0.009	0.021	0.009	**0.009**	0.011
	0.5	0.131	**0.056**	0.127	0.111	0.089	0.009	0.019	0.008	**0.008**	0.009
	0.6	0.165	**0.058**	0.160	0.144	0.115	0.008	0.017	0.008	**0.007**	0.008
	0.7	0.192	**0.057**	0.186	0.173	0.134	0.007	0.013	0.007	0.007	**0.006**
	0.8	0.192	**0.046**	0.186	0.176	0.131	0.006	0.010	0.006	0.005	**0.004**
	0.9	0.152	**0.028**	0.147	0.141	0.096	0.005	0.006	0.005	0.005	**0.003**
250	0.1	0.010	0.009	0.009	**0.006**	0.006	0.004	0.010	0.004	**0.004**	0.005
	0.2	0.025	**0.012**	0.024	0.016	0.014	0.004	0.010	0.004	**0.004**	0.005
	0.3	0.049	**0.020**	0.048	0.037	0.031	0.004	0.009	0.004	**0.004**	0.005
	0.4	0.080	**0.029**	0.079	0.065	0.056	0.004	0.008	0.003	**0.003**	0.004
	0.5	0.114	**0.038**	0.112	0.097	0.085	0.003	0.008	0.003	**0.003**	0.004
	0.6	0.145	**0.041**	0.143	0.129	0.111	0.003	0.006	0.003	**0.003**	0.003
	0.7	0.165	**0.037**	0.163	0.151	0.128	0.002	0.005	0.002	**0.002**	0.003
	0.8	0.161	**0.029**	0.158	0.150	0.122	0.002	0.004	0.002	0.002	**0.002**
	0.9	0.109	**0.018**	0.108	0.104	0.076	0.001	0.002	0.001	0.001	**0.001**

Bold font represents the lowest MSE.

4. Conclusion and Discussion

An estimator for an unknown mean AR(1) process having non-normal errors and additive outliers is proposed in this paper. This proposed estimator of ρ is obtained by applying the double recursive median adjustment to the weighted symmetric estimator. The proposed recursive median values are derived from computing the double recursive median. Furthermore, the simulation experiments were carried out to compare the mean square error (MSE) of proposed estimator with that of the existing one, the weighted symmetric estimator ($\hat{\rho}_W$), the recursive mean adjusted weighted symmetric estimator ($\hat{\rho}_R$), the recursive median adjusted weighted symmetric estimator ($\hat{\rho}_{RMD}$). Based on simulation studies when the error distributions are the t_3, exponential(1)-1 and uniform(-1,1), the proposed estimator, $\hat{\rho}_{DRMD}$, performs better than the other four estimators in sense of the MSE when the autoregressive parameter values is moderate ($0.3 \le \rho \le 0.6$). For $\rho \to 1$, the robust estimator, $\hat{\rho}_G$, provides the minimum MSE. In case of 0.9N(0,1)+0.1N(0,100) errors, MSEs of the $\hat{\rho}_{RMD}$ are lowest when the autoregressive parameter values are less than 0.5. In addition, the $\hat{\rho}_{DRMD}$ is more appropriate than the others when an autoregressive parameter value is close to one. One reason behind this is that the additive outliers do not affect the median values. Moreover, the double recursive median values applied in a formula of $\hat{\rho}_{DRMD}$ in (7) can also reduce the mean square error of estimator. Therefore, the proposed estimator $\hat{\rho}_{DRMD}$ which is based on the double recursive median adjustment is superior to the existing estimators in some situations.

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