RG-Homomorphism and Its Properties

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Abstract

In this paper, we investigate RG-homomorphism properties. Moreover, the relations between the quotient RG-algebra and the RG-homomorphism are provided.

Keywords: RG-algebra; RG-homomorphism; RG-antihomomorphism; RG-ideal

1. Introduction

The notions of the two algebraic structures BCK – algebra and BCI – algebra were first introduced by Imai and Iseki (1966). BCK-algebra is now known as a proper subclass of the class of BCI – algebra. Later, Hu and Li (1983) introduced the notion of BCH – algebra. Again, Hu and Li (1985) considered the proper BCH – algebra. More recently, Jun, Roh and Kim (1998) introduced the notion of BH – algebra which is a generalization of (BCK \ BCI) – algebras. The notion of d-algebra, which is another generalization of BCK – algebra, were introduced by Neggers and Kim (1999). Furthermore, Omar (2014) introduced RG – algebra which is a good generalization of the previous algebraic structures and studied some of its basic properties and also derived some straight forward consequences relations between the RG-algebra and the abelian group which is related to it. Moreover, Omar (2014) studied the notion of the homomorphism of RG-algebra, called RG-homomorphism.

In this paper, we shall investigate some of RG-homomorphism properties and derive some straight forward consequences relations between the quotient RG-algebra and the RG-homomorphism.

2. Preliminary Results

This section gathers together results, which we shall use later. We describe the algebraic structure of RG-algebra and then go on to introduce some important results related to it.

Definition 2.1: (Omar, 2014) An algebra $(X;*,0)$ of type (2,0) is called *RG-algebra* if the following axioms are satisfied: for all $x, y, z \in X$,

(i) $x*0 = x$,

(ii) $x*y = (x*z)*(y*z)$,

(iii) $x*y = y*x = 0$ imply $x = y$.

Proposition 2.2: (Omar, 2014) In any RG-algebra $(X;*,0)$, the following hold: for all $x, y, z \in X$,

(i) $0*(y*x) = x*y$,

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(ii) $0*(0*x)=x,$

(iii) $x*(x*y)=y,$

(iv) $x*y=(z*y)*(z*x),$

(v) $x*y=0$ if and only if $y*x=0,$

(vi) $((x*y)*(x*z))*(z*y)=0,$

(vii) $x*x=0,$

(viii) $x*0=0$ implies $x=0.$

Proposition 2.3: (Omar, 2014) In any RG-algebra $(X;*,0)$, the following hold: for all $x,y,z\in X$,

(i) $(x*y)*(0*y)=(x*(0*y))*y=x,$

(ii) $x*(x*(x*y))=x*y,$

(iii) $(x*y)*z=(x*y)*((z*y)*(0*y))$

$=((x*z)*z)*(y*z)$

$=((x*y)*y)*(z*y)$

$=(x*z)*y.$

Definition 2.4: Let $(X;*,0)$ be an RG-algebra. A nonempty subset A of X is called an *RG-sub algebra* of X if $(A;*,0)$ is itself an RG-algebra.

Definition 2.5: (Omar, 2014) Let $(X;*,0)$ be an RG-algebra. A nonempty subset A of X is called an *ideal* or *RG-ideal* of X if:

(i) $0\in A$ and (ii) $x*y\in A$ and $0*x\in A$ imply $0*y\in A$ for all $x,y\in X.$

Remark: If $(X;*,0)$ is an RG-algebra, then $\{0\}$ and X are RG-ideals of X.

Definition 2.6: (Omar, 2014) Let $(X;*,0)$ be an RG-algebra and A be an RG-ideal of X. The relation θ on X defined by $x\theta y$ if and only if $x*y\in A$ and $y*x\in A$ for all $x,y\in X$ is called *the relation defined by the ideal* A.

Remark: It is clear that θ is an equivalence relation on X.

Proposition 2.7: (Omar, 2014) Let $(X;*,0)$ be an RG-algebra. If A is an RG-ideal of X, then A is an RG-sub algebra of X.

Theorem 2.8: (Omar, 2014) Let A be an RG-ideal of an RG-algebra $(X;*,0)$. If $x\theta y\in A$ and $x\in A,$ then $y\in A$ for all $x,y\in X,$ where θ is the relation defined by the ideal A.

Recall that if θ is the relation on the empty-set X, then θ is called a *congruence* on X if and only if (i) θ is an equivalence relation on X and (ii) $x\theta y$ and $u\theta v$ imply $(xu)\theta(yv)$ for all $x,y,u,v\in X.$

Theorem 2.9: (Omar, 2014) Let $(X;*,0)$ be an RG-algebra and A be an RG-ideal of X. If θ is the relation defined by the ideal A, then θ is a congruence on X.

Since θ is an equivalence relation on X, for all $x\in X,$ the equivalence class of x is $C_x=\{y\in X|x\theta y\}$ and the family $\{C_x|x\in X\}$ form a partition of X which is denoted by $X\mid\theta$. We define the operation $\bullet$ on $X|\theta$ by $C_x\bullet C_y=C_{x*y}$ for all $x,y\in X.$

It is easy to verify that $\bullet$ is well–defined on $X|\theta$ and $(X|\theta;\bullet,C_0)$ satisfies all the axioms of the RG-algebra except the axiom (iii) of Definition 2.1. If the axiom holds for all the classes $C_x\in X\mid\theta,$ that is if the system $(X|\theta;\bullet,C_0)$ is an RG-algebra, then the congruence θ is called *regular*.

Theorem 2.10: (Omar, 2014) Let $(X;*,0)$ be an RG-algebra and θ be a congruence on X. Then, $C_0 = \{x \in X | x\theta 0\}$ is an RG-ideal of X.

Corollary 2.11: (Omar, 2014) Let $(X;*,0)$ be an RG-algebra. Then, any RG-ideal in X can be determined by some congruence.

Theorem 2.12: (Omar, 2014) A congruence on an RG-algebra X is regular if and only if it is defined by some RG-ideal.

Corollary 2.13: (Omar, 2014) All congruences of a finite RG-algebra are regular and the theory of universal algebra yields.

Definition 2.14: (Omar, 2014) Let $(X;*,0)$ and $(Y;*',0')$ be two RG-algebras. A mapping $f: X \to Y$ is called an *RG-hommorphism* if $f(x*y) = f(x)*' f(y)$ for all $x, y \in X$ and is called an *RG-antihommorphism* if $f(x*y) = f(y)*' f(x)$ for all $x, y \in X$. If f is an RG-hommorphism or RG-antihommorphism, then $\ker f = \{x \in X : f(x) = 0'\}$.

Example 2.15: Let $X = \{0, a, b, c\}$ and $*$ be a binary operation on X defined by

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then, $(X;*,0)$ is an RG-algebra. Let $f:(X;*,0) \to (X;*,0)$ be a mapping defined by $f(x) = x*x$ for all $x \in X$. Thus, f is an RG-hommorphism and $\ker f = \{x \in X : f(x) = 0\} = \{x \in X : x*x = 0\} = \{0, a, b, c\} = X$.

Definition 2.16: Let $(X;*,0)$ be an RG-algebra. A non empty subset I of X is called a *closed set* of X if $a*b \in I$ for all $a, b \in I$.

Definition 2.17: Let $f: X \to Y$ be an RG-hommorphism, where $(X;*,0)$ and $(Y;*',0')$ are RG-algebras and let $I \subseteq X$ and $A \subseteq Y$. The *image of* I *in* X *under* f is $f(I) = \{f(x) | x \in I\}$ and the *inverse image of* A *in* Y is $f^{-1}(A) = \{x \in X | f(x) \in A\}$.

3. Main Results

We maintain the notation introduced in Section 2. Throughout, we let $(X;*,0)$ and $(Y;*',0')$ be two RG-algebras. The aim of this section is to describe the properties of RG-homomorphism.

Theorem 3.1: Let $f: X \to Y$ be an RG-homomorphism. Then,

(1) $f(0) = 0'$.

(2) If $0*x = x$ for all $x \in X$, then $f(0)*' y = y$ for all $y \in f(X)$.

(3) $\ker f$ is an RG-ideal of X.

(4) $\ker f$ is an RG-sub algebra of X.

(5) $\ker f$ is a closed set of X.

(6) $\ker f = \{0\}$ if and only if f is an injective.

(7) If $x*y = 0$, then $f(x)*' f(y) = 0'$, where $x, y \in X$.

Proof: Let $f: X \to Y$ be an RG-homomorphism. Then, $f(x*y) = f(x)*' f(y)$

for all $x, y \in X$.

(1) Since $0*0=0$, $f(0)=f(0*0)=f(0)*' f(0)$. Thus, $f(0)*' f(0)=0'$, because Y is an RG-algebra, it follows that $f(0)=0'$.

(2) Assume that $0*x=x$ for all $x \in X$. Let $y \in f(X)$ Then, $y=f(x)$ for some $x \in X$. Since $f(0)=0'$, $f(0)*' y=f(0)*' y$ $=f(0)*' f(x)=f(0*x)=f(x)=y$.

(3) Since $f(0)=0'$, $0 \in \ker f$. Now, we let $x, y \in X$ be such that $x*y \in \ker f$ and $0*x \in \ker f$. Then, $f(x*y)=0'$ and $f(0*x)=0'$. Therefore, $f(x)*' f(y)=f(x*y)$ $=0'=f(0)$. It follows that $(f(x)*' f(y))*'$ $f(x)=f(0)*' f(x)$. By Proposition 2.3 (iii), $(f(x)*' f(x))*' f(y)=f(0)*' f(x)$ and we have $f(x*x)*' f(y)=f(0*x)$. Since $x*x=0$, we get $0'=f(0*x)=f(x*x)*'$ $f(y)=f(0)*' f(y)=f(0*y)$. Thus, $0*y \in$ $\ker f$. Hence, $\ker f$ is an RG-ideal of X.

(4) By Proposition 2.7, we have that $\ker f$ is an RG-sub algebra of X.

(5) Since $\ker f$ is an RG-sub algebra of X, $\ker f$ is a closed set of X.

(6) Assume that $\ker f=\{0\}$. Let $f(x)$ $=f(y)$, where $x, y \in X$. Then, $f(x*y)$ $=f(x)*' f(y)=f(y)*' f(y)=0'$. It implies that $x*y \in \ker f$. Since $\ker f=\{0\}$, we then have $x*y=0$. Therefore, $(x*y)*x=0*x$. By Proposition 2.3 (iii), $(x*x)*y=0*x$. Since $x*x=0$, we get $0*y=0*x$. It follows that $0*(0*y)=0*(0*x)$. By Proposition 2.2 (ii), $y=x$. Hence, f is an injective. Conversely, suppose that f is an injective and let $m \in \ker f$. Then, $f(m)=0'=f(0)$. It follows that $m=0$ and $\ker f=\{0\}$.

(7) Suppose that $x*y=0$, where $x, y \in X$. By Proposition 2.2 (v), $y*x=0$. Thus, $x=y$. Since f is a function, we get $f(x)=f(y)$. Therefore, $f(x)*' f(y)=f(x)*' f(x)=0'$. □

Theorem 3.2: Let $f: X \rightarrow Y$ be an RG-antihomomorphism. Then,

(1) $f(0)=0'$.

(2) $f(0)*' y=y$ for all $y \in f(X)$.

(3) $\ker f$ is an RG-ideal of X.

(4) $\ker f$ is an RG-sub algebra of X.

(5) $\ker f$ is a closed set of X.

(6) $\ker f=\{0\}$ if and only if f is an injective.

(7) If $x*y=0$, then $f(y)*' f(x)=0'$, where $x, y \in X$.

Proof: Let $f: X \rightarrow Y$ be an RG-antihomomorphism. Then, $f(x*y)=f(y)*' f(x)$ for all $x, y \in X$.

(1) Since $0*0=0$, so $f(0)=f(0*0)$ $=f(0)*' f(0)$. We see that $f(0)*' f(0)=0'$, because Y is an RG-algebra, it follows that $f(0)=0'$.

(2) Let $y \in f(X)$. Then, $y=f(x)$ for some $x \in X$. Since $f(0)=0'$, $f(0)*' y=$ $f(0)*' f(x)=f(x*0)=f(x)=y$.

(3) Since $f(0)=0'$, $0 \in \ker f$. Now, we let $x, y \in X$ be such that $x*y \in \ker f$ and $0*x \in \ker f$. It follows that $f(x*y)=0'$ and $f(0*x)=0'$. Therefore,

$f(y)*' f(x) = f(x*y) = 0' = f(0)$. Then, $(f(y)*' f(0))*'(f(x)*' f(0)) = 0'$ and we have $f(0*y)*' f(0*x) = 0'$. Since $f(0*x) = 0'$, $f(0*y)*' 0' = 0'$. It follows that $f(0*y) = 0'$. Thus, $0*y \in \ker f$. Hence, $\ker f$ is an RG-ideal of X.

(4) By Proposition 2.7, we have that $\ker f$ is an RG-sub algebra of X.

(5) Since $\ker f$ is an RG-sub algebra of X, $\ker f$ is a closed set of X.

(6) Assume that $\ker f = \{0\}$. Let $f(x) = f(y)$, where $x, y \in X$. Then, $f(y*x) = f(x)*' f(y) = f(x)*' f(x) = 0'$. It implies that $y*x \in \ker f$. Since $\ker f = \{0\}$, we then have $y*x = 0$. Therefore, $(y*x)*y = 0*y$. By Proposition 2.3 (iii), $(y*y)*x = 0*y$. Since $y*y = 0$, we then get $0*x = 0*y$. Thus, $0*(0*x) = 0*(0*y)$. By Proposition 2.2 (ii), $x = y$. Hence, f is an injective. Conversely, suppose that f is an injective and let $m \in \ker f$. Therefore, $f(m) = 0' = f(0)$. It follows that $m = 0$ and $\ker f = \{0\}$.

(7) Suppose that $x*y = 0$, where $x, y \in X$. By Proposition 2.2 (v), $y*x = 0$. Thus, $x = y$. Since f is a function, we get $f(x) = f(y)$. Therefore, $f(y)*' f(x) = f(y)*' f(y) = 0'$. □

Theorem 3.3: Let $f: X \to Y$ be an RG-homomorphism. Then,

(1) If I is a closed set of X, then $f(I)$ is a closed set of Y.

(2) If I is an RG-ideal of X, then $f(I)$ is an RG-ideal of Y.

(3) If f is an injective and I is an RG-sub algebra of X, then $f(I)$ is an RG-sub algebra of Y.

(4) If A is a closed set of Y, then $f^{-1}(A)$ is a closed set of X.

(5) If A is an RG- ideal of Y, then $f^{-1}(A)$ is an RG-ideal of X.

(6) $\operatorname{Im} f$ is a closed set of Y, where $\operatorname{Im} f = f(X) = \{f(x) | x \in X\}$.

Proof: Let $f: X \to Y$ be an RG-homomorphism. Then, $f(x*y) = f(x)*' f(y)$ for all $x, y \in X$.

(1) Let I be a closed set of X and $a, b \in f(I)$. Then, $a = f(x)$ and $b = f(y)$ for some $x, y \in I$. Since $x*y \in I$, $a*' b = f(x)*' f(y) = f(x*y) \in f(I)$ and hence, $f(I)$ is a closed set of Y.

(2) Let I be an RG-ideal of X. Then, $0 \in I$. By Theorem 3.1 (1), $0' = f(0) \in f(I)$. Let $a*' b \in f(I)$ and $0*' a \in f(I)$, where $a, b \in Y$. Therefore, $a = f(x)$ and $b = f(y)$ for some $x, y \in I$. Since $f(x*y) = f(x)*' f(y) = a*' b \in f(I)$, we then have $f(0*x) = f(0)*' f(x) = 0*' a \in f(I)$ and thus $x*y \in I$ and $0*x \in I$. Since I is an RG-ideal of X, $0*y \in I$ and implies that $f(0*y) \in f(I)$. Therefore, $f(I)$ is an RG-ideal of Y.

(3) Assume that f is an injective and I is an RG-sub algebra of X. Then, $\varnothing \neq I \subseteq X$ and I is an RG-algebra. We see that $f(I) \subseteq f(X) \subseteq Y$. Because $0 \in I$, $0' = f(0) \in f(I)$, that is $f(I) \neq \varnothing$. Let $a, b, c \in f(I)$. Then,

$a=f(x)$, $b=f(y)$ and $c=f(z)$ for some $x,y,z\in I$. Since $a*'b=f(x)*'f(y)=f(x*y)$ and $x*y\in I$, $a*'b\in f(I)$. Since I is an RG-algebra, we get that

$a*'0'=f(x)*'f(0)=f(x*0)=f(x)=a$,

$$\begin{aligned} a*'b &= f(x)*'f(y) \\ &= f(x*y) \\ &= f((x*z)*(y*z)) \\ &= (f(x)*'f(z))*'(f(y)*'f(z)) \\ &= (a*'c)*'(b*'c), \end{aligned}$$

and if $a*'b=b*'a=0'$, then $f(x)*'f(y)$ $=f(y)*'f(x)=f(0)$ or $f(x*y)=f(y*x)$ $=f(0)$. But f is an injective, thus $x*y=y*x=0$. It follows that $x=y$. Since f is a function, we have that $a=f(x)=f(y)=b$. Hence, $f(I)$ is an RG-algebra and $f(I)$ is also an RG-sub algebra of Y.

(4) Suppose that A is a closed set of Y and $x,y\in f^{-1}(A)$. Then, $f(x)=a$ and $f(y)=b$ for some $a,b\in A$. Since $f(x*y)=f(x)*'f(y)=a*'b\in A$, then $x*y\in f^{-1}(A)$. Therefore, $f^{-1}(A)$ is a closed set of X.

(5) Suppose that A is an RG-ideal of Y. Then, $0'\in A$ and $f(0)=0'\in A$. That is, $0\in f^{-1}(A)$. Let $x,y\in X$ be such that $x*y\in f^{-1}(A)$ and $0*x\in f^{-1}(A)$. It implies that $f(x),f(y)\in Y$ and $f(x*y)\in A$, $f(0*x)\in A$. Thus, $f(x)*'f(y)\in A$ and $f(0)*'f(x)=0'*f(x)\in A$. Since A is an RG- ideal of Y, we have $0'*'f(y)$ $=f(0)*'f(y)=f(0*y)\in A$ and thus $0*y\in f^{-1}(A)$. Hence, $f^{-1}(A)$ is an RG-ideal of X.

(6) Let $a,b\in \mathrm{Im}(f)$. Then, $a=f(x)$ and $b=f(y)$ for some $x,y\in X$. Since $a*'b=f(x)*'f(y)=f(x*y)\in \mathrm{Im}(f)$, $\mathrm{Im}\, f$ is a closed set of Y. □

Recall that if $(X;*,0)$ is a finite RG-algebra and θ is a congruence on X, by Corollary 2.13, then θ is a regular and, using Theorem 2.12, θ is the relation defined by A, where A is an RG–ideal of X. It implies that $(X|\theta;\bullet,C_0)$ is an RG-algebra, which is called the *quotient RG- algebra*, where $X|\theta=\{C_x|x\in X\}$ is a partition of X such that $C_x=\{y\in X|x\theta y\}=\{y\in X|x*y,y*x\in A\}$ is an equivalence class of $x\in X$ and the operation $\bullet$ on $X|\theta$ defined by $C_x\bullet C_y=C_{x*y}$ for all $x,y\in X$ and $A=C_0=\{x\in X|x\theta 0\}$.

Theorem 3.4: Let $(X;*,0)$ be a finite RG-algebra, A be a closed RG-ideal of X and θ be the relation defined by A. If $f:X\to X|\theta$ is the map defined by $f(x)=C_x$ for all $x\in X$, then f is a surjective RG-homomorphism, we call f is the *natural RG-homomorphism*, and $\ker f=A$.

Proof: Let $(X;*,0)$ be a finite RG-algebra, A be a closed RG-ideal of X and θ be the relation defined by A. Thus, $(X|\theta;\bullet,C_0)$ is an RG-algebra. Suppose that $f:X\to X|\theta$ is the map defined by $f(x)=C_x$ for all $x\in X$. Let $x,y\in X$. Then, $C_x,C_y\in X|\theta$. Since $f(x*y)=C_{x*y}=C_x\bullet C_y=f(x)\bullet f(y)$, f is a RG-homomorphism. Let $C_x \in X \mid \theta$. Then, $x \in X$ such that $f(x) = C_x$. It implies that f is a surjective. Next, we will show that $\ker f = A$. Let $x \in \ker f$. We get $f(x) = C_0$. Since $f(x) = C_x$, $C_x = C_0$ and $x\theta 0$. Thus, $x \in C_0 = A$ and we obtain that $\ker f \subseteq A$. Finally, let $x \in A$. Since A is a closed set of X and $0 \in A$, $x * 0 \in A$ and $0 * x \in A$. Therefore, $x\theta 0$ or $C_x = C_0$. It follows that $f(x) = C_x = C_0$ and we have $x \in \ker f$. Thus, $A \subseteq \ker f$. Hence, $\ker f = A$. □

Theorem 3.5: Suppose that $(X; *, 0)$ is a finite RG-algebra. Let $f : X \to Y$ be a surjective RG-homomorphism, A be an RG-ideal of X contain in $\ker f$ and θ be the relation defined by A. If g is the natural RG-homomorphism of X onto $X \mid \theta$, then there exists a unique RG-homomorphism h of $X \mid \theta$ onto Y such that $f = h \circ g$. Furthermore, h is an injective if and only if $A = \ker f$.

Proof: Define the map $h : X \mid \theta \to Y$ by $h(C_x) = f(x)$ for all $C_x \in X \mid \theta$. First, we show that h is well-defined. Let $C_x, C_y \in X \mid \theta$ be such that $C_x = C_y$. Then, $x\theta y$. Thus, $x * y \in A$ and $y * x \in A$. Since $A \subseteq \ker f$, $x * y \in \ker f$ and $y * x \in \ker f$. Thus, $f(x) *' f(y) = f(x * y) = 0'$ and $f(y) *' f(x) = f(y) *' f(x)$ It follows that $f(x) = f(y)$ and hence, h is well-defined.

Next, we will show that h is an RG-homomorphism. Let $C_x, C_y \in X \mid \theta$. Thus, $h(C_x \bullet C_y) = h(C_{x*y}) = f(x * y) = f(x) *' f(y) = h(C_x) *' h(C_y)$. Therefore, h is an RG-homomorphism.

We see that, for any $x \in X$, $(h \circ g)(x) = h(g(x)) = h(C_x) = f(x)$. Hence, $f = h \circ g$. Finally, if $h' : X \mid \theta \to Y$ is another mapping such that $f = h' \circ g$. Let $C_x \in X \mid \theta$. Then, $h(C_x) = f(x) = (h' \circ g)(x) = h'(g(x)) = h'(C_x)$ and thus $h = h'$.

Now, we will show that h is an injective if and only if $A = \ker f$. Suppose that h is an injective and $x \in \ker f$. Then, $h(C_x) = f(x) = 0' = h(C_0)$ and since h is an injective, $C_x = C_0$. It follows that $x\theta 0$, $x * 0 \in A$ and $0 * x \in A$. Since $0 \in A$, $x \in A$, this means $\ker f \subseteq A$. This shows that $A = \ker f$. On the other hand, suppose that $A = \ker f$ and $C_x, C_y \in X \mid \theta$ such that $h(C_x) = h(C_y)$. Then, $f(x) = f(y)$, it follows that $f(x * y) = f(x) *' f(y) = 0'$. Therefore, $x * y \in \ker f$. Since $A = \ker f$, $x * y \in A$. Similarly, $y * x \in A$. Hence, $x\theta y$, proving that $C_x = C_y$. This shows that h is an injective. This completes the proof. □

Studying isomorphisms of the RG-algebras is an interesting task for future research.

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