บทความวิจัย (Research Article)

RG-Isomorphism and Its Properties

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Abstract

In this paper, we investigate RG-isomorphism properties. Moreover, the relations between the quotient RG-algebra and the RG-isomorphism are provided.

Keywords: RG-algebra; RG-homomorphism; RG-isomomorphism; RG-ideal

1. Introduction

The notions of the two algebraic structures BCK – algebra and BCI – algebra were first introduced by Imai and Iseki (1966). BCK-algebra is now known as a proper subclass of the class of BCI – algebra. Later, Hu and Li (1983) introduced the notion of BCH – algebra. Again, Hu and Li (1985) considered the proper BCH – algebra. More recently, Jun *et al*. (1998) introduced the notion of BH – algebra which is a generalization of (BCK/BCI) – algebras. The notion of d- algebra, which is another generalization of BCK – algebra, were introduced by Neggers and Kim (1999). Furthermore, Omar (2014) introduced RG – algebra which is a good generalization of the previous algebraic structures and studied some of its basic properties and also derived some straight forward consequences relations between the RG-algebra and the abelian group which is related to it. Moreover, Omar (2014) studied the notion of the homomorphism of RG-algebra, called RG-homomorphism. Patthanangkoor (2018) also studied some of RG-homomorphism properties and derived some straight forward consequences relations between the quotient RG-algebra and the RG-homomorphism.

In this paper, we introduce the notion of RG-isomorphism. The purpose of this paper is to derive some straight forward consequences relations between the quotient RG-algebra and the RG-isomorphism and also investigate some of RG-isomorphism properties.

2. Preliminary Results

This section gathers together results, which we shall use later. We describe the algebraic structure of RG-algebra and then go on to introduce some important results related to it.

Definition 2.1: An algebra $(X;*,0)$ of type $(2,0)$ is called *RG-algebra* if the following

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axioms are satisfied (Omar, 2014): For all $x, y, z \in X$,

(i) $x * 0 = x$,

(ii) $x * y = (x * z) * (y * z)$ and

(iii) $x * y = y * x = 0$ imply $x = y$.

Proposition 2.2: In any RG-algebra $(X;*,0)$ (Omar, 2014), the following hold: For all $x, y, z \in X$,

(i) $0 * (y * x) = x * y$,

(ii) $0 * (0 * x) = x$,

(iii) $x * (x * y) = y$,

(iv) $x * y = (z * y) * (z * x)$,

(v) $x * y = 0$ if and only if $y * x = 0$,

(vi) $((x * y) * (x * z)) * (z * y) = 0$,

(vii) $x * x = 0$,

(viii) $x * 0 = 0$ implies $x = 0$.

Proposition 2.3: In any RG-algebra $(X;*,0)$ (Omar, 2014), the following hold: For all $x, y, z \in X$,

(i) $(x * y) * (0 * y) = (x * (0 * y)) * y = x$,

(ii) $x * (x * (x * y)) = x * y$,

(iii) $(x * y) * z = (x * y) * ((z * y) * (0 * y))$
$= ((x * z) * z) * (y * z)$
$= ((x * y) * y) * (z * y)$
$= (x * z) * y$.

Definition 2.4: Let $(X;*,0)$ be an RG-algebra. A nonempty subset A of X is called an *RG-sub algebra* of X if $(A;*,0)$ is itself an RG-algebra.

Definition 2.5: Let $(X;*,0)$ be an RG-algebra (Omar, 2014). A nonempty subset A of X is called an *ideal* or *RG-ideal* of X if:

(i) $0 \in A$ and

(ii) $x * y \in A$ and $0 * x \in A$ imply $0 * y \in A$ for all $x, y \in X$.

Remark: If $(X;*,0)$ is an RG-algebra, then $\{0\}$ and X are RG-ideals of X.

Definition 2.6: Let $(X;*,0)$ be an RG-algebra and A be an RG-ideal of X (Omar, 2014). The relation θ on X defined by $x\theta y$ if and only if $x * y \in A$ and $y * x \in A$ for all $x, y \in X$ is called *the relation defined by the ideal* A.

Remark: It is clear that θ is an equivalence relation on X.

Proposition 2.7: Let $(X;*,0)$ be an RG-algebra. If A is an RG-ideal of X (Omar, 2014), then A is an RG-sub algebra of X.

Theorem 2.8: Let A be an RG-ideal of an RG-algebra $(X;*,0)$ (Omar, 2014). If $x\theta y \in A$ and $x \in A$, then $y \in A$ for all $x, y \in X$, where θ is the relation defined by the ideal A.

Recall that if θ is the relation on the non empty-set X, then θ is called a *congruence* on X if and only if (i) θ is an equivalence relation on X and (ii) $x\theta y$ and $u\theta v$ imply $(xu)\theta(yv)$ for all $x, y, u, v \in X$.

Theorem 2.9: Let $(X;*,0)$ be an RG-algebra and A be an RG-ideal of X (Omar, 2014). If θ is the relation defined by the ideal A, then θ is a congruence on X.

Since θ is an equivalence relation on $(X;*,0)$, for all $x \in X$, the equivalence class of x is $C_x = \{y \in X | x\theta y\}$ and the family $\{C_x | x \in X\}$ form a partition of X which is denoted by $X | \theta$. We define the operation $\bullet$ on $X|\theta$ by $C_x \bullet C_y = C_{x*y}$ for all $x, y \in X$. It is easy to verify that $\bullet$ is well–defined on $X|\theta$ and $(X|\theta;\bullet,C_0)$ satisfies axioms (i) and (ii) but not (iii) of Definition 2.1 of RG-algebra. If the axiom holds for all the classes $C_x \in X\mid\theta,$ that is if the system $(X|\theta;\bullet,C_0)$ is an RG-algebra, then the congruence θ is called *regular*.

Theorem 2.10: Let $(X;*,0)$ be an RG-algebra and θ be a congruence on X (Omar, 2014). Then, $C_0=\{x\in X \mid 0\theta x\}$ is an RG-ideal of X.

Corollary 2.11: Let $(X;*,0)$ be an RG-algebra (Omar, 2014). Then, any RG-ideal in X can be determined by some congruence.

Theorem 2.12: A congruence on an RG-algebra X is regular if and only if it is defined by some RG-ideal (Omar, 2014).

Corollary 2.13: All congruences of a finite RG-algebra are regular and the theory of universal algebra yields (Omar, 2014).

Definition 2.14: Let $(X;*,0)$ and $(Y;*',0')$ be two RG-algebras (Omar, 2014). A mapping $f:X\to Y$ is called an *RG-hommorphism* if $f(x*y)=f(x)*'f(y)$ for all $x,y\in X$ and is called an *RG-antihommorphism* if $f(x*y)=f(y)*'f(x)$ for all $x,y\in X$. If f is an RG-hommorphism or RG-antihommorphism, then $\ker f=\{x\in X \mid f(x)=0'\}$.

Example 2.15: Let $X=\{0,a,b,c\}$ and $*$ be a binary operation on X defined by

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then, $(X;*,0)$ is an RG-algebra. Let $f:(X;*,0)\to(X;*,0)$ be a mapping defined by $f(x)=x*x$ for all $x\in X$. Thus, f is an RG-hommorphism and $\ker f=\{x\in X \mid f(x)=0\}$ $=\{x\in X \mid x*x=0\}=\{0,a,b,c\}=X$.

Definition 2.16: Let $(X;*,0)$ be an RG-algebra. A non empty-subset I of X is called a *closed set* of X if $a*b\in I$ for all $a,b\in I$.

Definition 2.17: Let $f:X\to Y$ be an RG-hommorphism, where $(X;*,0)$ and $(Y;*',0')$ are RG-algebras and let $\varnothing\neq I\subseteq X$ and $\varnothing\neq A\subseteq Y$. The *image of* I *in* X *under* f is $f(I)=\{f(x)\mid x\in I\}$ and the *inverse image of* A *in* Y is $f^{-1}(A)=\{x\in X \mid f(x)\in A\}$.

Theorem 2.18: Let $f:X\to Y$ be an RG-homomorphism, where $(X;*,0)$ and $(Y;*',0')$ are RG-algebras (Patthanangkoor, 2018). Then,

(i) $f(0)=0'$.

(ii) If $0*x=x$ for all $x\in X$, then $f(0)*'y=y$ for all $y\in f(X)$.

(iii) $\ker f$ is an RG-ideal of X.

(iv) $\ker f$ is an RG-sub algebra of X.

(v) $\ker f$ is a closed set of X.

(vi) $\ker f=\{0\}$ if and only if f is an injective.

(vii) If $x*y=0$, then $f(x)*'f(y)=0'$, where $x,y\in X$.

Theorem 2.19: Let $f:X\to Y$ be an RG-antihomomorphism, where $(X;*,0)$ and $(Y;*',0')$ are RG-algebras (Patthanangkoor, 2018). Then,

(i) $f(0)=0'$.

(ii) $f(0)*' y = y$ for all $y \in f(X)$.

(iii) $\ker f$ is an RG-ideal of X.

(iv) $\ker f$ is an RG-sub algebra of X.

(v) $\ker f$ is a closed set of X.

(vi) $\ker f = \{0\}$ if and only if f is an injective.

(vii) If $x * y = 0$, then $f(y)*' f(x) = 0'$, where $x, y \in X$.

Theorem 2.20: Let $f: X \to Y$ be an RG-homomorphism, where $(X;*,0)$ and $(Y;*',0')$ are RG-algebras (Patthanangkoor, 2018). Then,

(i) If I is a closed set of X, then $f(I)$ is a closed set of Y.

(ii) If I is an RG-ideal of X, then $f(I)$ is an RG-ideal of Y.

(iii) If f is an injective and I is an RG-sub algebra of X, then $f(I)$ is an RG-sub algebra of Y.

(iv) If A is a closed set of Y, then $f^{-1}(A)$ is a closed set of X.

(v) If A is an RG-ideal of Y, then $f^{-1}(A)$ is an RG-ideal of X.

(vi) $\operatorname{Im} f$ is a closed set of Y, where $\operatorname{Im} f = f^{-1}(X) = \{f(x) \mid x \in X\}$.

Recall that if $(X;*,0)$ is a finite RG-algebra and θ is a congruence on X, by Corollary 2.13, then θ is a regular and, using Theorem 2.12, θ is the relation defined by A, where A is an RG–ideal of X. It implies that $(X|\theta;\bullet,C_0)$ is an RG-algebra and $A = C_0$.

Definition 2.21: Let $(X;*,0)$ be a finite RG-algebra, θ be a congruence on X, and, for all $x \in X$, $C_x = \{y \in X \mid x\theta y\}$ be the equivalence class of x. Then, the family $X \mid \theta = \{C_x \mid x \in X\}$ form a partition of X and $(X|\theta;\bullet,C_0)$ is an RG-algebra, where the operation $\bullet$ on $X|\theta$ is defined as:

$C_x \bullet C_y = C_{x*y}$ for all $x, y \in X$. $(X|\theta;\bullet,C_0)$ is called the *quotient RG-algebra*.

Theorem 2.22: Let $(X;*,0)$ be a finite RG-algebra, A be a closed RG-ideal of X and θ be the relation defined by A (Patthanangkoor, 2018). If $f: X \to X \mid \theta$ is the map defined by $f(x) = C_x$ for all $x \in X$, then f is a surjective RG-homomorphism, we call f is the *natural RG-homomorphism*, and $\ker f = A$.

Theorem 2.23: (Patthanangkoor, 2018) Suppose that $(X;*,0)$ is a finite RG-algebra. Let $f: X \to Y$ be a surjective RG-homomorphism, A be an RG-ideal of X contained in $\ker f$ and θ be the relation defined by A. If g is the natural RG-homomorphism of X onto $X \mid \theta$, then there exists a unique RG-homomorphism h of $X \mid \theta$ onto Y such that $f = h \circ g$. Furthermore, h is an injective if and only if $A = \ker f$.

3. Main Results

We maintain the notation introduced in Section 2. Throughout, we let $(X;*,0)$ and $(Y;*',0')$ be two RG-algebras. The aim of this section is to describe the properties of RG-isomorphism.

Definition 3.1: An injective RG-homomorphism is called a *RG-monomorphism*. A surjective RG-homomorphism is called an *RG-epimorphism*. An *RG-isomorphism* is a bijective

RG-homomorphism. Two RG-algebras X and Y are *isomorphic* when there exists an RG-isomorphism from X onto Y, this relationship is denoted by $X \cong Y$.

Theorem 3.2: $f : X \to Y$ is an RG-isomorphism if and only if $f^{-1} : Y \to X$ is an RG-isomorphism.

Proof: Suppose that $f : X \to Y$ is an RG-isomorphism. Then, f is a bijection. Thus, $f^{-1} : Y \to X$ is also a bijection. Let $y_1, y_2 \in Y$. Then, $y_1 = f(x_1)$ and $y_2 = f(x_2)$ for some $x_1, x_2 \in X$. That is, $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$. Therefore,

$$\begin{aligned} f^{-1}(y_1 *' y_2) &= f^{-1}(f(x_1) *' f(x_2)) \\ &= f^{-1}(f(x_1 * x_2)) \\ &= (f \circ f^{-1})(x_1 * x_2) \\ &= x_1 * x_2 \\ &= f^{-1}(y_1) * f^{-1}(y_2), \end{aligned}$$

and then $f^{-1} : Y \to X$ is an RG-homomorphism. Thus, $f^{-1} : Y \to X$ is an RG-isomorphism.

Conversely, if $f^{-1} : Y \to X$ is an RG-isomorphism then f^{-1} is a bijection. Thus, $f : X \to Y$ is also a bijection. Let $x_1, x_2 \in X$. Then, $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$ for some $y_1, y_2 \in Y$. Hence, $f(x_1) = y_1$ and $f(x_2) = y_2$. It follows that

$$\begin{aligned} f(x_1 * x_2) &= f(f^{-1}(y_1) * f^{-1}(y_2)) \\ &= f(f^{-1}(y_1 *' y_2)) \\ &= (f^{-1} \circ f)(y_1 *' y_2) \\ &= y_1 *' y_2 \\ &= f(x_1) *' f(x_2). \end{aligned}$$

Therefore, $f : X \to Y$ is an RG-homomorphism and hence $f : X \to Y$ is an RG-isomorphism.

□

Remark: If $f : X \to Y$ is an RG-homomorphism, by Theorem 2.20, we have that $f(X)$ is an RG-ideal of Y. It follows by Proposition 2.7 that $f(X)$ is an RG-sub algebra of Y.

Theorem 3.2: Let $(X; *, 0)$ and $(Y; *', 0')$ be finite RG-algebras. If $f : X \to Y$ is an RG-homomorphism and θ is the relation defined by $\ker f$, then the quotient RG-algebra $X|\theta$ is isomorphic to $f(X)$.

Proof: Suppose that $f : X \to Y$ is an RG-homomorphism and θ is the relation defined by $\ker f$. By Theorem 2.18 (3), $\ker f$ is an RG-ideal of X. By Theorem 2.9, θ is a congruence on X. Using Corollary 2.13, θ is a regular. It implies that $(X|\theta; \bullet, C_0)$ is an RG-algebra. Let $K = \ker f$.

Assume that $\phi : X|\theta \to f(X)$ defined by $\phi(C_x) = f(x)$ for all $C_x \in X|\theta$. Let $C_x, C_y \in X|\theta$ be such that $C_x = C_y$. Then, $x\theta y$. It follows that $x * y \in K$ and $y * x \in K$. Thus, $f(x) *' f(y) = f(y) *' f(x) = 0'$. It implies that $f(x) = f(y)$, that is, $\phi(C_x) = \phi(C_y)$. Hence, ϕ is well-defined. Let $C_x, C_y \in X|\theta$. Therefore, we get $\phi(C_x \bullet C_y) = \phi(C_{x*y}) = f(x * y) = f(x) *' f(y) = \phi(C_x) *' \phi(C_y)$. This show that ϕ is an RG-homomorphism.

Let $C_x, C_y \in X|\theta$ be such that $\phi(C_x) = \phi(C_y)$. Then, $f(x) = f(y)$ and hence, $f(x * y) = f(x) *' f(y) = 0'$ and $f(y * x) = f(y) *' f(x) = 0'$. Thus, $x * y \in \ker f = K$ and $y * x \in \ker f = K$.

We have that $x\theta y$ and thus, $C_x = C_y$. Hence, ϕ is an injection. Let $y \in f(X)$. Then, there exists $x \in X$ such that $y = f(x)$ and $C_x \in X|\theta$. Therefore, $\phi(C_x) = f(x) = y$ and ϕ is a surjection. Hence, ϕ is an RG-isomorphism and $X|\theta \cong f(X)$. □

Lemma 3.3: Let A and B be RG-ideals of an RG-algebra X. Then,

(i) $A \cap B$ is an RG-ideal of X.

(ii) If $A \cup B$ is an RG-algebra, then A is an RG-ideal of $A \cup B$.

Proof: Suppose that A and B are RG-ideals of an RG-algebra X.

(i) Since $0 \in A$ and $0 \in B$, $0 \in A \cap B$ and then, $A \cap B \neq \varnothing$. Let $x, y \in X$ be such that $x * y \in A \cap B$ and $0 * x \in A \cap B$. Thus, $x * y, 0 * x \in A$ and $x * y, 0 * x \in B$. It follows that $0 * y \in A$ and $0 * y \in B$. Thus, $0 * y \in A \cap B$ and hence, $A \cap B$ is an RG-ideal of X.

(ii) Suppose that $A \cup B$ is an RG-algebra. Let $x, y \in A \cup B$ be such that $x * y \in A$ and $0 * x \in A$. Since $A \cup B \subseteq X$, $x, y \in X$. Therefore, $0 * y \in A$ and we get that A is an RG-ideal of $A \cup B$. □

Remark: Suppose that A and B are finite RG-ideals of an RG-algebra $(X; *, 0)$ and $A \cup B$ is an RG-algebra. Let θ be the relation defined by $A \cap B$ and μ be the relation defined by B. Therefore, $(A|\theta; \bullet, \overline{C}_0)$ is the quotient RG-algebra and $A \cap B = \overline{C}_0 = \{a \in A \mid a\theta 0\} = \{a \in A \mid a * 0, 0 * a \in A \cap B\}$. Similarly, since μ be the relation defined by B, $((A \cup B)|\mu; \bullet, C_0)$ is the quotient RG-algebra and $B = C_0 = \{x \in A \cup B \mid 0\mu x\}$.

Theorem 3.4: Let A and B be RG-ideals of a finite RG-algebra X. If $A \cup B$ is an RG-algebra, then the quotient RG-algebras $A|\theta$ and $(A \cup B)|\mu$ are isomorphic, where θ is the relation defined by $A \cap B$ and μ is the relation defined by B.

Proof: Suppose that A and B are RG-ideals of a finite RG-algebra X. and $A \cup B$ is an RG-algebra. By Lemma 3.3, B is an RG-ideal of $A \cup B$ Let $f : A \to (A \cup B)|\mu$, where μ is the relation defined by B, be a map defined by $f(a) = C_a$ for all $a \in A$, where $C_a = \{y \in A \cup B \mid a\mu y\}$. Let $a, b \in A$ be such that $a = b$. Then, $C_a = C_b$, it follows that $f(a) = f(b)$. Thus, f is well-defined. Let $C_x \in (A \cup B)|\mu$. Then, $x \in A \cup B$. If $x \in A$, then $C_x = f(x)$. If $x \in B$, then $x \in C_0$ and hence, $C_x = C_0 = f(0)$. Therefore, f is a surjection, that is, $f(A) = (A \cup B)|\mu$.

Let $a, b \in A$. Since $f(a * b) = C_{a*b} = C_a \bullet C_b = f(a) \bullet f(b)$, we then have f is an RG-homomorphism. Now, let $x \in \ker f$. Then, $f(x) = C_0$ and thus, $C_x = C_0$. It implies that $x \in C_0 = B$ and then, $\ker f \subseteq B$. Since $\ker f \subseteq A$, $\ker f \subseteq A \cap B$. On the other hand, let $x \in A \cap B$. Then, $x \in B = C_0$. It follows that $f(x) = C_x = C_0$, and we get that $x \in \ker f$. Therefore, $\ker f = A \cap B$. By Theorem 2.18, $A \cap B$ is an RG-ideal of A. Theorem 3.2 immediately gives us that $A|\theta$ is isomorphic to $f(A) = (A \cup B)|\mu$, where θ is the relation defined by $\ker f = A \cap B$. □

Theorem 3.5: Suppose that A and B are RG-ideals of a finite RG-algebra $(X; *, 0)$

with $A \subseteq B \subseteq X$. Let θ be the relation defined by A. Then,

(1) the quotient RG-algebra $B|\theta$ is an RG-ideal of the quotient RG-algebra $X|\theta$, and

(2) the quotient RG-algebra $(X|\theta)|\beta$ is isomorphic to $X|\mu$, where β is the relation defined by $B|\theta$ and μ is the relation defined by B.

Proof: Suppose that A and B are RG-ideals of a finite RG-algebra X such that $A \subseteq B \subseteq X$. Let θ be the relation defined by A

(i) It is clear that $B|\theta = \{C_b | b \in B\} \subseteq \{C_b | b \in X\} = X|\theta$ and $C_0 \in B|\theta$. Let $C_x, C_y \in X|\theta$ be such that $C_x \bullet C_y \in B|\theta$ and $C_0 \bullet C_x \in B|\theta$. Then, $C_{x*y} \in B|\theta$ and $C_{0*x} \in B|\theta$. It follows that $x*y \in B$ and $0*x \in B$. Since B is an RG-ideals of X, $0*y \in B$. That is, $C_{0*y} \in B|\theta$ or $C_0 \bullet C_y \in B|\theta$. Therefore, $B|\theta$ is an RG-ideal of $X|\theta$.

(ii) Suppose that μ is the relation defined by B. Let $f : X|\theta \to X|\mu$ be a map defined by $f(C_x) = C'_x$ for all $x \in X$, where $C'_x = \{y \in X | x\mu y\}$ is the equivalence class of $x \in X$. Let $C_x, C_y \in X|\theta$ be such that $C_x = C_y$. Then, $x\theta y$, that is $x*y \in A$ and $y*x \in A$. Since $A \subseteq B$, $x*y \in B$ and $y*x \in B$ Thus, $x\mu y$ and so $C'_x = C'_y$. It implies that $f(C_x) = f(C_y)$ and hence, f is well-defined. Next, let $C'_x \in X|\mu$. We see that $x \in X$. If $x \in B$, then $x \in C'_0$ and $C'_x = C'_0 = f(C_0)$. If $x \notin B$, then there exists $C_x \in X|\theta$ such that $f(C_x) = C'_x$. Therefore, f is onto. Since, for all $C_x, C_y \in X|\theta$, $f(C_x \bullet C_y) = f(C_{x*y}) = C'_{x*y} = C'_x \bullet C'_y = f(C_x) \bullet f(C_y)$. Hence, f is a RG-homomorphism.

Finally, we show that $\ker f = B|\theta$. Let $C_x \in \ker f$. Then, $f(C_x) = C'_0$, that is $C'_x = C'_0 = B$. Thus, $x \in B$ and we have $C_x \in B|\theta$. Therefore, $\ker f \subseteq B|\theta$. On the other hand, we let $C_x \in B|\theta$. Then, $x \in B = C'_0$ and we get $f(C_x) = C'_x = C'_0$. Thus, $C_x \in \ker f$ and hence, $B|\theta \subseteq \ker f$. Consequently, $\ker f = B|\theta$. By Theorem 3.2, $(X|\theta)|\beta$ is isomorphic to $f(X|\theta) = X|\mu$, where β is the relation defined by $\ker f = B|\theta$. □

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